Dilemma game structure observed in traffic flow at a 2-to-1 lane junction

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(Received 24 October 2008; published 13 March 2009)

Using a cellular automaton traffic model based on the stochastic optimal velocity model with appropriate assumptions for both incoming and outgoing vehicle boundaries, the so-called bottleneck issue on a lane-closing section was investigated in terms of game theory. In the system, two classified driver agents coexist: C agents (cooperative strategy) always driving in the first lane and D agents (defective strategy) trying to drive in a lower-density lane whether the first or the second lane. In high-density flow, D agents' interruption into the first lane from the second just before the lane-closing section creates a heavier traffic jam, which reduces social efficiency. This particular event can be described with a prisoner's dilemma game structure.

DOI: 10.1103/PhysRevE.79.036104 PACS number(s): 89.40.−a

I. INTRODUCTION

Recently, traffic flow models have been studied increasingly. In order to understand significant traffic flow phenomena, traffic models such as the kinetic gas theory [1], fluid-dynamical model [2,3], car-following model [4,5], and cellular automaton (CA) model have been developed. In particular, since 1992, when the first CA traffic model, the Nagel-Schreckenberg model [6], was proposed, CA model research has made extensive progress because of the model's simplicity and features that can reproduce real traffic flows well. For example, the free-flow and jam phases can be approximately reproduced with the most basic CA model, called the asymmetric simple exclusion process (ASEP) [7]. However, ASEP cannot reproduce an unstable flow phase, called the metastable phase, which is commonly observed at the critical density when real traffic flow switches from the free-flow phase to congested flow. However, some advanced CA models, e.g., the slow-start model [8] or stochastic optimal velocity (SOV) model [9], can reproduce this phenomenon.

Although numerous previous models have analyzed simple systems without bottlenecks except at boundaries, study on bottlenecks in real traffic has attracted many researchers. For example, Li *et al.* [10] and Gao *et al.* [11] introduced a model considering the effect of on-ramps. In particular, the model introduced by Gao *et al.* [11] can reproduce complex traffic flow states discussed by Kerner [12].

Moreover, a series of studies revealed that Burgers' equation, which governs one-dimensional (1D) shock wave propagation, is exactly equivalent to the asymptotic behavior of elementary CA rule 184 [13] when Cole-Hopf transformation is applied to the ultradiscrete diffusion equation [14]. This theoretical consistency is one reason why CA traffic models are regarded as a powerful tool for analyzing traffic flow kinetics.

However, none of these previous studies provided a comprehensive understanding on real traffic flow phenomena because these models did not include the decision-making process of the drivers. In other words, most of the studies focus only on the kinetics of a self-driven many-particle system and ignore the effect of drivers' decisions on the entire system. Beyond those backgrounds, the objectives of this paper are to add a game theory framework as a rational decision process to the traffic model, construct a new CA model, and demonstrate that a bottleneck at a 2-to-1 lane junction has a dilemma game structure.

This paper is organized as follows: in Sec. II, the traffic model we use is explained along with basic simulation results of the SOV model with the open boundary condition. In Sec. III, the 2-lane model that we assume is presented. In Sec. IV, the results of the numerical experiment are shown and discussed. Finally, a brief conclusion is provided in Sec. V.

II. SOV MODEL

In this paper, we adopted the SOV model [9]. Considering the work by Gao *et al.* [11], SOV might be less plausible when dealing with bottleneck effects. However, here we focus on whether the dilemma game structure exists in traffic flow, not on the detailed physics of the traffic flow itself. On the other hand, we should not adopt a simpler model which cannot reproduce the metastable phase since bottleneck jams relate to vehicles' slow-start effect that is also bringing to reproduce the metastable phase. This compels us to adopt a relatively simple but acceptably accurate traffic model in this study. Hence, we choose the SOV model because it is simple to define a lane change by a driver using the model.

In the SOV model, the velocity v_i^{t+1} of vehicle i at time t is defined by

$$v_i^{t+1} = (1-a)v_i^t + aV_i(\Delta x_i^t), \tag{1}$$

where a $(0 \le a \le 1)$ is a parameter, Δx is the headway, and function V_i is the optimal velocity function, which is defined as

$$V(\Delta x) = \frac{\tanh(\Delta x - c) + \tanh c}{1 + \tanh c}. \tag{2}$$

Here, c is a parameter. It is important to note that V_i is restricted to $[0,1]$ at any Δx and c; thus, v_i^t is also restricted to

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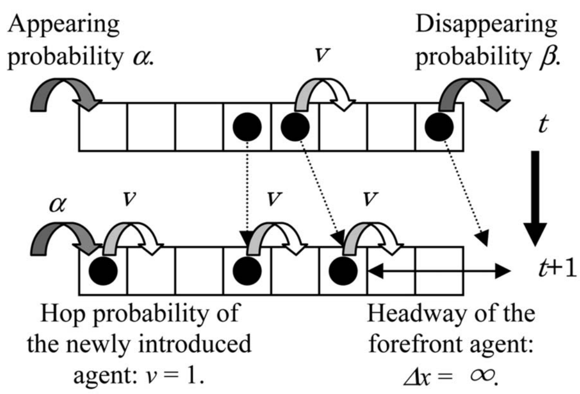

FIG. 1. Proposed SOV model considering both incoming and outgoing open boundary conditions.

$[0,1]$ if v_i^0 is less than 1. In other words, v_i^t expresses the hop probability (i.e., a normalized velocity).

The SOV model encompasses two fundamental stochastic submodels; i.e., when a in Eq. (1) is 0, this model is the same as ASEP [7], while in the case of $a=1$, it becomes the zero-range process [15]. In these two models, we can deduce the exact probability distribution for the configuration of vehicles in the stationary state by an analytical approach. Moreover, the SOV model has the same structure as the discrete OV model, which features coupled differential equations related to both inertia and the headway effect [16,17]. These two points highlighting the SOV's advantage in terms of theoretical robustness led us to adopt it.

When modeling, the way we set boundary conditions is important. In some previous models, a cyclical boundary condition was applied because of its simplicity and plausibility for tracing basic traffic flow features. However, in a road with a bottleneck, nonequilibrium flow can occur near the bottleneck. To apply the cyclical boundary condition leads to an idea that downstream of the nonequilibrium flow condition affects on the upstream flow of the bottleneck, which seems unrealistic. To avoid this, an open boundary condition is applied in this paper.

The scheme of the SOV model with the open boundary condition is shown in Fig. 1. When the cell in front of vehicle i is vacant, the vehicle can hop with probability v_i^{t+1}. A vehicle is created at the leftmost cell with probability α and deleted at the rightmost cell with probability β. The hop probability of the newly introduced vehicle is set as 1. Headway Δx of the furthest forward vehicle is set as ∞. The update rule is applied to parallel updates.

Figure 2 shows fundamental diagrams of real traffic data and our simulation result with $a=0.01$ in Eq. (1) and $c=3/2$ in Eq. (2). Note that the flux represented on the vertical axis is defined by the product of the velocity and density. Here, the system length is set as 200 cells. Figure 2(a) [18] indicates that four traffic flow phases can be observed in the real fundamental diagram. The most notable characteristic is that a metastable phase, in which the flow condition drops to that of the high-density phase even if a subtle disturbance is added, emerges between the free-flow and high-density phases. Some CA models cannot reproduce this feature, but

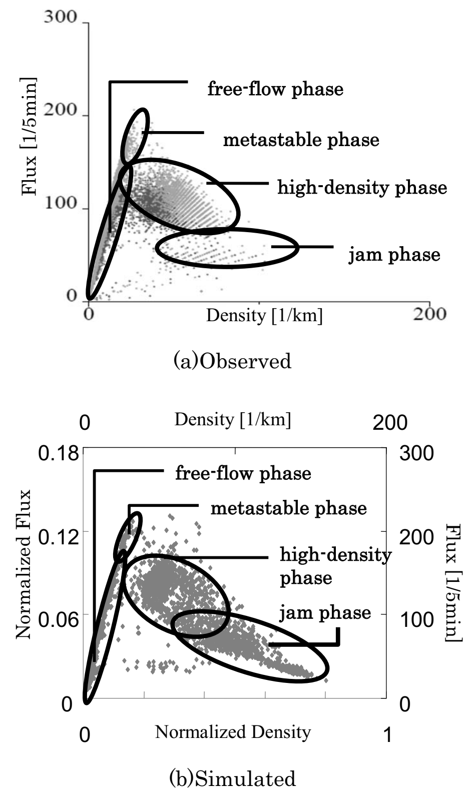

FIG. 2. Comparison of fundamental diagrams (a) field observed data to (b) simulated one by the proposed SOV model.

our SOV model with the open boundary condition can reproduce this particular characteristic, as shown in Fig. 2(b). Note that the normalized flux and density in Fig. 2(b) can be transformed to the real scale, as shown in the top and right axes, when we assume a cell size of 5 m and a maximum velocity of 100 km/h.

III. 2-LANE MODEL INCLUDING BOTTLENECK

In this paper, we analyze the bottleneck effect when closing lanes from double to single by means of the SOV model under the open boundary condition. The drivers' decision-making process is described by game theory; i.e., we assume that drivers have a strategy that is either cooperative or defective. Cooperative drivers (C agents) drive only in the first lane. The rules of defective drivers (D agents) are as follows: first, they are created in a lower-density lane; second, while in an overtaking area and if the second lane has lower density than the first, they will move to the second lane; and third, when they reach the interrupting area, they attempt to cut into the first lane. The rules of changing lanes are as follows: in the case of overtaking, drivers can change lanes only when other vehicles are not present in both the adjacent cell and one cell behind the adjacent cell; in the case of interrupting, even if another vehicle appears one cell behind the adjacent cell, they can change lanes with probability p. The rules are illustrated in Fig. 3. Of course, a D agent cannot be allowed to enter the first lane if the left adjacent cell is occupied by another agent.

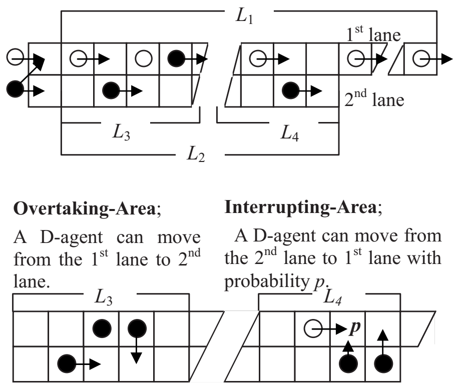

FIG. 3. Assumed model with both incoming and outgoing open boundary conditions. L_1, the length of the first lane; L_2, the length of the second lane; L_3, the length of the overtaking area; and L_4, the length of the interrupting area. An open circle indicates a C agent and a closed one is a D agent.

The updated rules of the system within a single time step are as follows:

(1) A vehicle is created with probability α. The strategy is determined to be C or D according to the cooperation fraction P_C, which is one of the simulation parameters.

(2) The velocity of all vehicles is calculated.

(3) All vehicles' moves are decided, i.e., hopping or not for C agent and hopping or not or changing lanes for D agent.

(4) Update all vehicles.

(5) If the newly created vehicle stays in the leftmost cell, it is deleted.

The following results are drawn from each 30-trial ensemble average observed from the 10 001th step to the 12 000th step. Lengths of $L_1=400$, $L_2=200$, $L_3=100$, and $L_4=20$ cells are assumed. The density and flux are measured for $L_2<x<L_1$ because the flow efficiency should be evaluated from amount the flux is in the downstream section of the bottleneck. It is confirmed that the effect of changing L_2, L_3, and L_4 is not very significant. We also confirm that there is a certain dependence of L_1-L_2 on the bottleneck effect, which is not sufficient to invalidate what we are trying to demonstrate below.

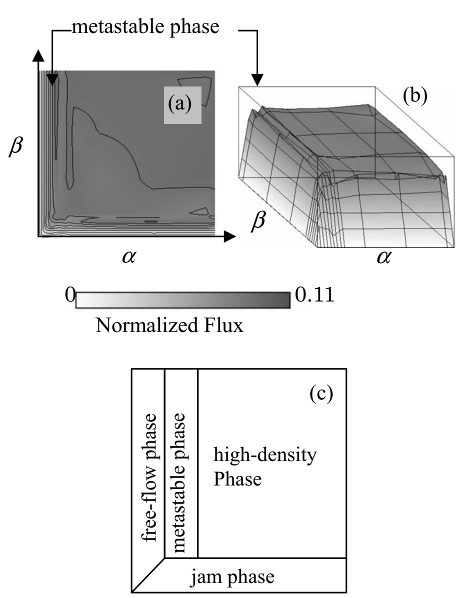

FIG. 4. (a) Two-dimensional (2D) and (b) 3D flow-α-β diagrams; gray-scale contour indicates normalized flux with an assumption of cooperation fraction $P_C=1$. (c) Indicates four representative traffic flow phases based on a sketch observation on (a).

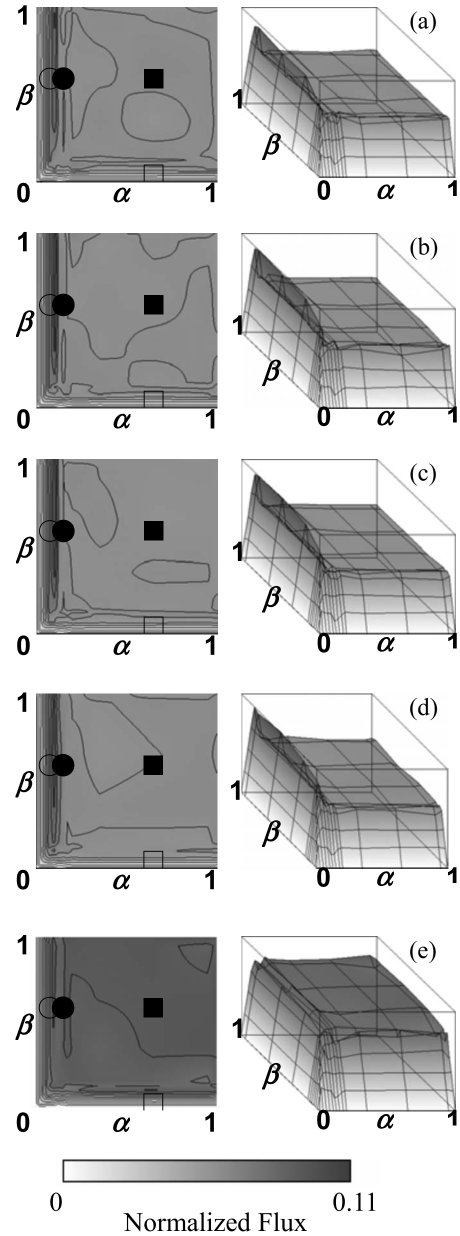

FIG. 5. Flow-α-β diagram; gray-scale contour indicates normalized flux. Parameters are (a) $P_c=0$, (b) $P_c=0.3$, (c) $P_c=0.5$, (d) $P_c=0.7$, and (e) $P_c=1$. Four symbols (open and closed circles and open and closed squares) are representative points of the free-flow ($\alpha=0.05$, $\beta=0.65$), jam ($\alpha=0.65$, $\beta=0.05$), metastable ($\alpha=0.10$, $\beta=0.65$), and high-density ($\alpha=0.65$, $\beta=0.65$) phases shown in Fig. 7.

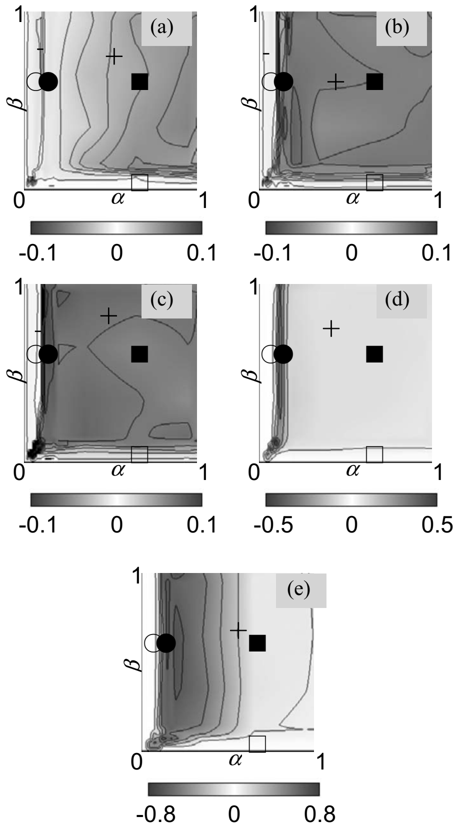

FIG. 6. Payoff difference-α-β diagram. The payoff indicates hop probability (normalized velocity) difference between D and C agents. Each subgraph from (a) to (e) shows the result for P_C =0.1–0.9. Four symbols (open and closed circles and open and closed squares) are representative points of the free-flow (α =0.05, β=0.65), metastable (α=0.10, β=0.65), jam (α =0.65, β=0.05), and high-density (α=0.65, β=0.65) phases shown in Fig. 7. The + and − signs indicate positive and negative areas, respectively.

IV. RESULTS AND DISCUSSIONS

In this section, we analyze the flow state with changing $\alpha-\beta$, β, and the fraction of cooperators P_C.

First, consider Fig. 4. Figures 4(a) and 4(b) show the α-β phase diagram in two and three dimensions with P_C=1, i.e., the system contains only C agents. Figure 4(c) is a sketch of the resulting classified flow phases. First, the area characterized by small α and insensitivity to β can be classified as the free-flow phase. Second, the area characterized by small β and insensitivity to α can be classified as the jam phase. Third, the area characterized by insensitivity to both α and β can be classified as the high-density phase. Finally, the area with the highest flux can be classified as the metastable phase.

Figure 5 shows the α-β phase diagram with a changing fraction of cooperators P_C, where we can observe normalized flux ahead of the bottleneck. Likewise, Fig. 6 shows the payoff difference between D agents and C agents in several cooperation fractions. In this diagram, if the payoff difference is positive, defecting is more rational than cooperating. Figure 7 shows payoff structure functions of four representative

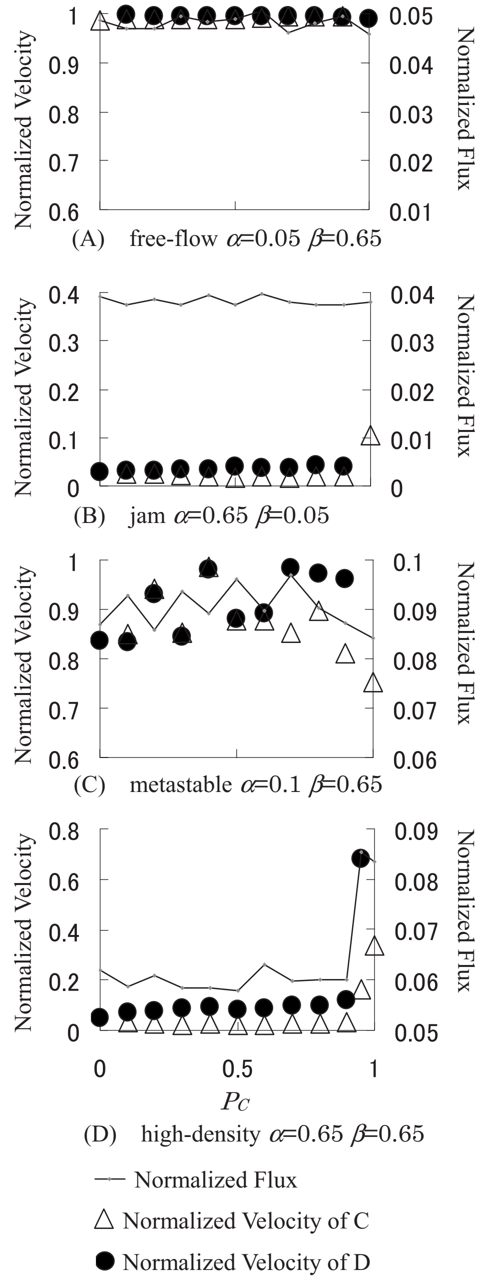

FIG. 7. Payoff structure functions of both C (triangles) and D (gray circles) agents with social average (line). The payoff implies averaged hop probability (normalized velocity) of agents. The social average indicates a normalized flux of the traffic. (a) Free-flow (α=0.05, β=0.65), (b) jam (α=0.65, β=0.05), (c) metastable (α =0.10, β=0.65), and (d) high-density (α=0.65, β=0.65) phases. Those four points are shown in both Figs. 5 and 6 by open and closed circles and open and closed squares, respectively.

points that indicate the free-flow (α=0.05, β=0.65), jam (α=0.65, β=0.05), metastable (α=0.1, β=0.65), and high-density (α=0.65, β=0.65) phases (these four points are also shown in both Figs. 5 and 6). The payoffs of both strategies are evaluated by hop probabilities measured within the L_2 area. Note that employing the higher payoff strategy is more rational than using the opposite strategy from the viewpoint of individual benefit. We evaluate the social profit by flux observed ahead of the bottleneck. We can observe both individual and social benefits simultaneously in Fig. 7, which indicates social dynamics. For example, if we have a situation where the D-agent payoff is always larger than the C-agent payoff, this inevitably leads to fewer C agents in the society and eventually no C agents (P_C=0). In addition, if the social maximum payoff is not consistent with this situation, game theory defines this as a social dilemma [19].

In the free-flow phase, there is no difference between C- and D-agents' payoffs, and social profit does not depend on the fraction of cooperation, as indicated in Fig. 7(a). Thus, this has a trivial game structure, where no social dilemma occurs [19].

Figure 7(b) indicates the jam phase. We may consider this as an n-player prisoner's dilemma (nPD) since the D strategy always dominates over the C strategy [19] because the D agents' profit is higher than that of C agents at all fractions of cooperation. However, this structure can never be nPD since the social profit is almost constant, regardless of the fraction of cooperation. This means that the jam phase has a trivial game structure, where the strategy dynamics do not affect the social efficiency.

Figure 7(c) shows the metastable phase. In $P_C > 0.6$, because the D agents' payoff is higher than that of C agents, there is an incentive for C agents to drive in the second lane (i.e., to convert to the D strategy). In $P_C < 0.6$, however, there is no velocity difference between strategies (i.e., no incentive for C agents to change their strategy). Therefore, the strategy dynamics based on each individual's payoff leads to a midway cooperation fraction around P_C=0.6. The society cannot avoid D agents' incursions, but the equilibrium favors coexisting C agents and D agents. One should note that this internal equilibrium fraction around P_C=0.6 is consistent with the social maximum payoff point. In this sense, this particular game class also does not contain any social dilemmas. This implies that in the metastable phase, driving in the second lane to a moderate extent can improve flow efficiency rather than all driving in only the first lane (P_C=1).

Finally, Fig. 7(d) shows the high-density phase. Here, the strategy payoff structure is "D dominant over C" dynamics, similar to the jam phase, but the social maximum point is at the fraction of cooperation P_C=0.95. This situation can be exactly the same as nPD, namely, the social maximum payoff appears around P_C=1. However, the society inherently has a robust incentive to increase D agents since D agents can obtain higher payoff than C agents at any cooperative fraction. Thus, D agents always increase, finally reaching absorbed equilibrium P_C=0, where no C agents can survive. This structure can be recognized by observing the high-density phase area in Figs. 5 and 6. Figure 6 explicitly shows that in the high-density phase, the D agents' payoff is always higher than that of C agents. Moreover, Fig. 5 explicitly shows that the social payoff at P_C=0 is the lowest among all P_C.

In the high-density phase, traffic flow seems so fragile that exogenous impacts can easily lead to congestion. The best situation having high social efficiency from an egalitarian point of view is that all drivers stay in the first lane. The society, however, cannot avoid the entry of a defecting agent who is willing to drive in the second lane because he can enjoy a higher payoff than C agents by adopting the D strategy.

V. CONCLUSION

To clarify the social dilemma structure in traffic flow that is one of the pure physical processes, we built a CA model based on the SOV model with the open boundary condition and applied it to the bottleneck problem caused by reducing lanes from double to single. The established model contains a game theory framework to deal with drivers' decision-making processes.

We found that the four traffic flow phases have various game structures. Precisely speaking, the nPD game structure arises in high-density phase areas, which account for most ranges of car-creating probability α and car-deleting probability β. On the other hand, no dilemma exists at other flow phases. In the free-flow and jam phases, there is no incentive to drive in the overtaking lane. In situations of quite high flow, in the high-density phase for instance, selfish drivers changing lanes can obtain a higher payoff than altruistic drivers staying in the first lane, but they cause a remarkable decrease in social efficiency. In contrast to the high-density phase, in the metastable phase, the social efficiency is likely to increase when drivers use the overtaking lane.

What we present here implies that the social dilemma structure that *game theorists* use may underlie a traffic flow phenomenon that is believed to be a typical physics problem, which might be interesting.

In this paper, we analyzed the bottleneck effect at closing lanes from double to single. However, there might be lots of other application cases. Potential bottlenecks might be one of the examples. For example, a question if frequent lane changes in a 1D like homogenous road (without any other obvious bottlenecks such as lane-closing, uphill, or tunnel) may also bring another social dilemma might be interesting. We are presuming that "changing lanes" itself could call the dilemma in a traffic flow. To figure out this is our next challenge.

[1] M. J. Lighthill and G. B. Whitham, Proc. R. Soc. London, Ser. A **229**, 317 (1955).
[2] B. S. Kerner and P. Konhäuser, Phys. Rev. E **50**, 54 (1994).
[3] D. Helbing, Phys. Rev. E **51**, 3164 (1995).
[4] L. A. Pipes, J. Appl. Phys. **24**, 274 (1953).
[5] C. Gazis, R. Herman, and R. W. Rothery, Oper. Res. **9**, 545 (1961).
[6] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).
[7] N. Rajewsky, L. Santen, A. Schadschneider, and M. Schreckenberg, J. Stat. Phys. **92**, 151 (1998).

[8] M. Takayasu and H. Takayasu, Fractals **1**, 860 (1993).
[9] M. Kanai, K. Nishinari, and T. Tokihiro, Phys. Rev. E **72**, 035102(R) (2005).
[10] F. Li, Z. Y. Gao, and B. Jia, Physica A **385**, 333 (2007).
[11] K. Gao, R. Jiang, S. X. Hu, B. H. Wang, and Q. S. Wu, Phys. Rev. E **76**, 026105 (2007).
[12] B. S. Kerner, *The Physics of Traffic* (Springer, Berlin, 2004).
[13] S. Wolfram, *Theory and Applications of Cellular Automata* (World Scientific, Singapore, 1986).
[14] K. Nishinari and D. Takahashi, J. Phys. A **31**, 5439 (1998).
[15] F. Spitzer, Adv. Math. **5**, 246 (1970).
[16] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, Phys. Rev. E **51**, 1035 (1995).
[17] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, J. Phys. I **5**, 1389 (1995).
[18] Y. Sugiyama, Nagare **22**, 95 (2003) (in Japanese).
[19] J. Tanimoto and H. Satara, BioSystems **90**, 105 (2007).