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Contribution of Reynolds stress distribution to the skin friction in compressible turbulent channel flows

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An exact relationship for the local skin friction is derived for the compressible turbulent wall-bounded flow (channel, pipe, flat plate). This expression is an extension of the compressible case of that derived by Fukagata *et al.* [Phys. Fluids **14**, L73 (2002)] in the case of incompressible wall-bounded flows. This decomposition shows that the skin friction can be interpreted as the contribution of four physical processes, i.e., laminar, turbulent, compressible, and a fourth coming from the interaction between turbulence and compressibility. Compressible numerical simulations show that, even at Mach number $M=2$, the main contribution comes from the turbulence, i.e., the Reynolds stress term.

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I. INTRODUCTION

The Reynolds number effects on the mean and statistical turbulence quantities have to be investigated to clarify the mechanism of wall-bounded compressible turbulent flows for engineering and industrial applications. For example, in the channel flows, it is well established that an increase of the turbulence intensity leads to an increase of the drag friction. Indeed, the frictional drag of a turbulent flow is usually much higher than that of a laminar one, leading to an increasing interest in near-wall turbulence control. This observation is explained by the dynamics of the near-wall vortical structures and the associated autonomous cycle [1]. Hence, many attempts have been made to reduce skin friction drag by controlling turbulence in wall-bounded flows. Most of these works have focused on suppressing or counteracting the near-wall coherent structures which are mainly responsible for the turbulence production. Numerous strategies have been adopted for incompressible flows, such as, for example, magnetic fluxes [2], active wall motions [3], transverse traveling waves [4], and polymer additives [5].

From a statistical point of view and in the incompressible case, Fukagata *et al.* [6] have derived a relation between the skin friction coefficient and the Reynolds stress tensor. This nonlocal splitting of the skin friction provides a deep insight into the physical mechanisms responsible for drag production in turbulent flows, and therefore makes it possible to optimize near-wall dynamics manipulation strategies.

The quantitative effect of compressibility on skin friction production remains to be analyzed in the same way. Indeed, we can find an abundant literature on compressible channel flows, e.g., Refs. [7–9]. But, most of these papers deal with budget equations coming from momentum and energy equations, and evaluate all the contributive terms. Nevertheless, a relationship between the skin friction and the statistical properties of the turbulence has not yet been written in the compressible case.

In the present paper, a direct relation between the Reynolds stress tensor and the skin friction coefficient for compressible channel flow is derived. Then, all the contributive terms to the drag are evaluated, for the compressible turbulent channel flows, at Mach number up to $M=2$.

II. EXACT RELATIONSHIP FOR SKIN FRICTION

First, we derive several dimensionless quantities and averaging operators. The drag coefficient C_f is the ratio between the boundary mean shear stress $\langle \tau_w \rangle$ and the mean kinetic energy per unit volume,

$$C_f = \frac{\langle \tau_w \rangle}{1/2 \rho_0 U_0^2}. \tag{1}$$

With nondimensionalized quantities, this can be written as

$$C_f = \left. \frac{2}{\mathrm{Re}} \frac{\partial \langle u \rangle}{\partial z} \right|_{z=-1}, \tag{2}$$

where $\mathrm{Re}=\frac{\rho_0 U_0 2\delta}{\mu(T_0)}$ is the bulk Reynolds number, δ is the channel half-width, ρ_0 is the initial uniform density, U_0 is the channel flow bulk velocity, and $\mu(T_0)$ is the molecular viscosity at the wall temperature T_0.

For a compressible turbulent flow, we need to define two averaging operators, namely the Favre $\{\cdot\}$ and the Reynolds $\langle \cdot \rangle$, defined as $\{f\}=\langle \rho f\rangle / \langle \rho \rangle$. The Favre average permits us to transform the mean of a product into the product of a mean as noted by Huang *et al.* [7,8]. In the case of the channel flow, described in Fig. 1, the Reynolds operator consists in averaging on x, y space variables and on t time variable. The single prime and the double prime denote the turbulent fluctuations with respect to Reynolds and Favre averages, respectively. The difference between the Reynolds- and the Favre-averaged quantities can be written as

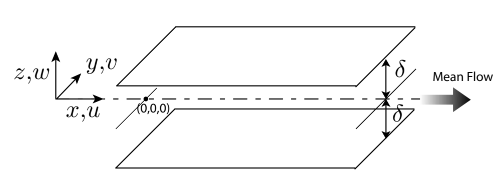

FIG. 1. Flow geometry.

$$\langle f\rangle - \{f\} = \langle f''\rangle = -\frac{\langle \rho' f'\rangle}{\langle \rho\rangle} = -\frac{\langle \rho' f''\rangle}{\langle \rho\rangle}.$$

The difference between Reynolds- and Favre-averaged quantities is mainly relevant in the near-wall region as shown by Huang *et al.* [8] by using direct numerical simulation (DNS).

Therefore, the operator definitions, in the case of the turbulent channel presented in Fig. 1, gives the following relations:

$$\frac{\partial\langle\cdot\rangle}{\partial x} = \frac{\partial\langle\cdot\rangle}{\partial y} = \frac{\partial\langle\cdot\rangle}{\partial t} = 0 \text{ and } \frac{\partial\{\cdot\}}{\partial x} = \frac{\partial\{\cdot\}}{\partial y} = \frac{\partial\{\cdot\}}{\partial t} = 0. \quad (3)$$

Hereafter, we assume (i) constant flow rate, (ii) homogeneity in the streamwise (x) and the spanwise (y) directions, (iii) no-slip conditions at the wall surfaces $z=-1$ and $+1$, and (iv) symmetry with respect to the center plane. Then, the momentum equation averaged along homogeneous directions x,y and time t variables yields

$$\frac{\partial\langle\rho\rangle\{uw\}}{\partial z} = \frac{1}{\text{Re}}\frac{\partial\langle\tau_{xz}\rangle}{\partial z} - f_1, \quad (4)$$

where f_1 is the forcing gradient term, defined by the following relation:

$$f_1 = -\frac{1}{2}\left(\frac{2}{\text{Re}}\frac{\partial\langle u\rangle}{\partial z}\bigg|_{z=-1}\right) = -\frac{1}{2}C_f. \quad (5)$$

Moreover, as $\{w\}=0$ and $\{uw\}=\{u''w''\}$, we obtain

$$-\frac{C_f}{2} = \frac{\partial}{\partial z}\left[-\langle\rho\rangle\{u''w''\} + \frac{1}{\text{Re}}\langle\tau_{xz}\rangle\right]. \quad (6)$$

By taking $\tilde{u}=(1/2)\int_{-1}^{1}\langle u\rangle dz=\int_{-1}^{0}\langle u\rangle dz=1$, i.e., taking the bulk velocity $U_0=(1/2)\int_{-1}^{1}\langle u\rangle dz$ to nondimensionalize the velocities, and by integrating Eq. (6) between -1 and z, we obtain

$$-\frac{C_f}{2}z = -\langle\rho\rangle\{u''w''\} + \frac{1}{\text{Re}}\langle\tau_{xz}\rangle. \quad (7)$$

Equation (7) leads to

$$-\frac{C_f}{2}z = -\langle\rho\rangle\{u''w''\} + \frac{1}{\text{Re}}\left[(1+\langle\tilde{\mu}\rangle)\frac{\partial\langle u\rangle}{\partial z} + \left\langle \mu'\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right)\right\rangle\right], \quad (8)$$

with $\langle\tilde{\mu}\rangle=\langle\mu\rangle-1$. Then, by integrating twice Eq. (8), we obtain the following relationship:

$$C_f = \underbrace{\frac{6}{\text{Re}}}_{C_L} + \underbrace{6\int_{-1}^{0} z\langle\rho\rangle\{u''w''\}dz}_{C_T} + \underbrace{\frac{6}{\text{Re}}\int_{-1}^{0} -z\langle\tilde{\mu}\rangle\frac{\partial\langle u\rangle}{\partial z}dz}_{C_C} + \underbrace{\frac{6}{\text{Re}}\int_{-1}^{0} -z\left\langle\mu'\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right)\right\rangle dz}_{C_{CT}}. \quad (9)$$

TABLE I. Computation parameters.

Case	Re	Pr	M	γ	T_0	Re_τ
1	3121	0.72	0.4	1.2	293.5	197
2	3121	0.72	2	1.2	293.5	167

This relation shows that the skin friction coefficient can be split into four contributing terms: the laminar contribution C_L, the turbulent contribution C_T, the compressible contribution C_C, and the compressible-turbulent interaction term C_{CT}. The turbulent term C_T is proportional to the weighted average of the Reynolds stress, where the weight is linearly decreasing with the distance from the wall. As remarked by Fukagata *et al.*, this explains why the frictional drag observed in wall turbulence is mainly due to the turbulence wall structures which occur closer to the wall than the position of the maximum Reynolds stress. The compressible term C_C is proportional to the mean viscosity fluctuation due to thermal variations and to the mean wall normal velocity gradient. This term is obtained by weighted average with the linearly decreasing weight as for C_T. The last contributing term C_{CT} is proportional to the weighted average of the mean product between viscosity and velocity fluctuation gradients. The same relationships are given in Appendixes A and B for the turbulent pipe flow and the turbulent flat plane flow. However, in the following sections, the contributive terms C_L, C_T, C_{CT}, C_C are computed only for the turbulent channel flow case.

III. NUMERICAL SIMULATION

Direct numerical simulations of compressible channel flows are performed. A skew-symmetric formulation of the convective terms, as described by Blaisdell [10], is used to avoid aliasing errors without using filtering. In streamwise and spanwise directions, periodic conditions are applied. Therefore, a regular mesh is used along these two directions. First derivatives in space are discretized by using a sixth-order centered finite-difference method and second derivatives by using a second-order centered finite-difference method. In the wall-normal direction, the mesh is refined near the walls. The order of the centered scheme is decreased in the near-wall cells. A low storage compact third-order Runge-Kutta method is used for time stepping as in Lenormand *et al.* [11]. A fixed-point algorithm is used to force the pressure-driven flow. No-slip boundary conditions, constant temperature, and zero wall-normal gradient of velocity are imposed at the walls. The physical and numerical parameters of the simulations are given in Tables I and II, where $\text{Re}_\tau = \rho_0 U_\tau 2\delta/\mu(T_0)$ is the friction Reynolds number with $u_\tau = \sqrt{\tau_{\text{wall}}/\rho_0}$. The Prandtl number is Pr=0.7 and the specific-

TABLE II. Computational grid parameters.

L_x	L_y	L_z	N_x	N_y	N_z	z_0
3π	π	1	192	64	129	5×10^{-3}

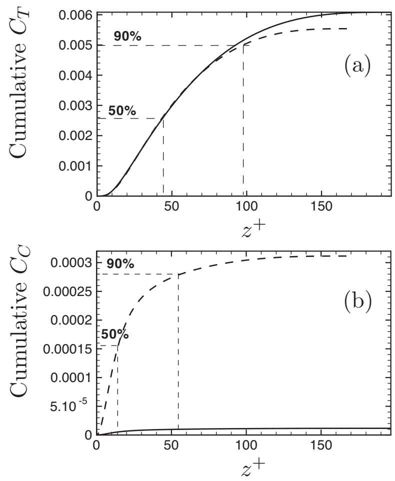

FIG. 2. Cumulative contribution of Reynolds stress (a) and of compressible terms (b) to skin friction for Mach number $M=0.4$ (—) and $M=2$ (- -).

heat ratio is $\gamma=1.2$. The dimensionless quantities L_x, L_y, and L_z define the computational box size, respectively, in the x, y, and z directions. N_x, N_y, and N_z are the grid point numbers in each direction and z_0 is the first grid size at the wall. The initial velocity field is given by superimposing three-dimensional random velocity fluctuations upon the laminar Poiseuille flow at uniform density. The thermodynamics variables are left unperturbed.

IV. RESULTS

Analysis of DNS results show that for fully developed compressible isothermal-wall channel flows, the main contribution to the skin friction is produced by turbulence, even at the supersonic regime for $M=2$. Indeed, the cumulative contribution C_T is 6×10^{-3}, whereas the corresponding compressible term C_C is 3×10^{-4}, as shown in Figs. 2(a) and 2(b). These figures show that 90% of the drag generated by compressibility effects C_C is located in the near-wall region $z^+<50$, i.e., essentially in the viscous sublayer of the turbulent boundary layer. On the other hand, the production area of drag by Reynolds stress is more extended. The cumulative C_T term reaches 90% at $z^+\sim100$, i.e., beyond the viscous sublayer. Moreover, we observe that the cumulative turbulent contribution is not quite modified by the Mach number variation, only 10%. At the same time, the cumulative compressible term varies from 1.169×10^{-5} to 3.15×10^{-4}, i.e., by a factor 27. Concerning the wall-normal distribution of the relevant terms, Figs. 3(a) and 3(b) confirm that the compressible contribution to the skin friction is concentrated in a region closer to the wall than that of the Reynolds stress contribution, which is more largely extended through the channel. This can be physically explained by the fact that the thermal boundary layer is thinner than the kinetic boundary layer. Note that C_{CT} is negligible compared to the other terms, as shown in Table III, and it fluctuates only in the near-wall region, as shown in Fig. 3(c).

To validate these results, the drag friction coefficient C_f is estimated by two different methods. The first one consists in summing the contributions of each term appearing on the right-hand side (rhs) of the relationship (9). The second method uses the expression $C_f=2\rho(T_0)(\mathrm{Re}_\tau/\mathrm{Re})^2$ based on the definition of both the Reynolds numbers and on the choice of the characteristic quantities used to nondimensionalize the equations. Hence, the error on the C_f coefficient computed by these two methods is about 0.4% for the $M=0.4$ case and about 4% for the $M=2$ case, respectively. Then, the relationship (9) seems to be well founded and the estimation of each contributive term is quite good within a few percent maximal error at the $M=2$ regime.

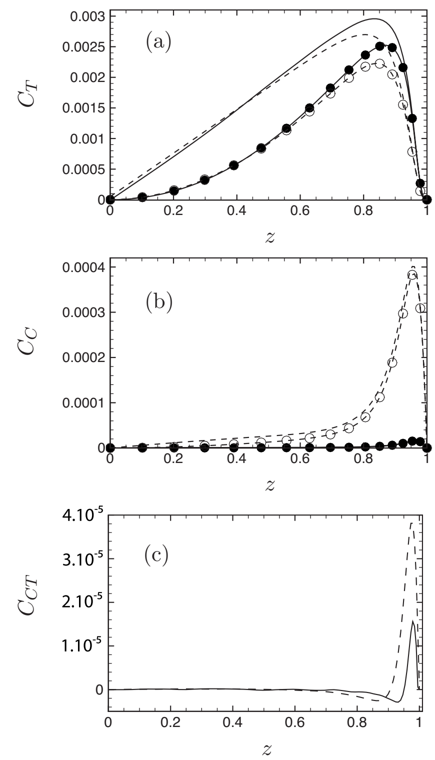

FIG. 3. Reynolds stress terms (a), compressible terms (b), and compressible/turbulent terms (c) contribution to skin friction for Mach number $M=0.4$ (—) and $M=2$ (- -), and its weighted contributions (with circle).

V. CONCLUSION

To conclude, we have obtained a relationship giving the contribution of different physical mechanisms to the skin friction. This derivation is straightforward, although the result is suggestive and useful for analyzing the effects of Reynolds stress on the frictional drag at different Mach numbers. Indeed, we have used compressible channel flow DNS to show that even at the $M=2$ regime, the compressible effects on the skin friction are quasinegligible compared to the turbulence action. This relationship enables us to physically analyze drag reduction devices in the compressible boundary layer case.

APPENDIX A: CYLINDRICAL PIPE FLOW

A similar relationship can be derived in the case of cylindrical pipe flow. By writing the longitudinal momentum

TABLE III. Contribution to the skin friction in the subsonic and supersonic cases.

Mach	C_T	C_C	C_{CT}	C_L	C_f
0.4	6.083×10^{-3}	1.169×10^{-5}	1.037×10^{-6}	1.922×10^{-3}	8.018×10^{-3}
	75.87%	1.4×10^{-1}%	1.19×10^{-2}%	23.97%	
2	5.538×10^{-3}	3.15×10^{-4}	8.653×10^{-6}	1.922×10^{-3}	7.78×10^{-3}
	71.18%	4.05%	1.11×10^{-1}%	24.70%	

equation in the cylindrical coordinates, averaged along homogeneous directions θ, z and time t variables yields

$$\frac{1}{r}\frac{\partial r\langle\rho\rangle\{u_r u_z\}}{\partial r} = \frac{1}{\mathrm{Re}}\frac{1}{r}\frac{\partial r\langle\tau_{rz}\rangle}{\partial r} - f_1, \tag{A1}$$

where the driving gradient term f_1 is given by

$$f_1 = \left(\frac{2}{\mathrm{Re}}\frac{\partial\langle u_r\rangle}{\partial r}\bigg|_{r=1}\right) = C_f$$

and with

$$\tau_{rz} = \mu\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right).$$

By a similar triple integration process, we obtain the following relationship:

$$C_f = \underbrace{\frac{16}{\mathrm{Re}}}_{C_L} + 8\int_0^1 r^2\langle\rho\rangle\{u_r'' u_z''\}dr + \frac{8}{\mathrm{Re}}\int_0^1 - r^2\langle\tilde{\mu}\rangle\frac{\partial\langle u_z\rangle}{\partial r}dr + \frac{8}{\mathrm{Re}}\int_0^1 - r^2\left\langle\mu'\left(\frac{\partial u_r'}{\partial z} + \frac{\partial u_z'}{\partial r}\right)\right\rangle dr, \tag{A2}$$

where the relationship $\int_0^1 r\langle u_z\rangle dr \equiv 1$, coming from the adimensionalization, is used. The definition of the dimensionless quantities is similar to that in the channel case, but δ is replaced by the pipe radius R and the Reynolds number is $\mathrm{Re}=\frac{\rho_0 U_0 2R}{\mu(T_0)}$. The weighting factors for the Reynolds stress and for the additional terms coming from the compressibility are proportional to the square of the radial distance from the pipe axis.

APPENDIX B: PLANE BOUNDARY LAYER

In the case of the plane boundary layer with zero mean streamwise pressure gradient, the physical quantities are nondimensionalized by the free stream velocity U_∞ and density ρ_∞, by the boundary layer thickness δ, and by the wall temperature T_0. The flat plane is located at $z=0$ with the same coordinate system as in the channel case shown in Fig. 1. The skin friction C_f is defined by $C_f=\frac{2}{\mathrm{Re}_\delta}\langle\tau_{xz}\rangle|_{z=0}$, with $\mathrm{Re}_\delta=\rho_\infty U_\infty\delta/\mu(T_0)$. In this case, $\langle\cdot\rangle$ denotes the averaging operator along the spanwise direction. We assume (i) a constant free stream velocity and (ii) $(\partial u/\partial x)\sim 0$ at $z=1$. The skin friction can then be written as

$$C_f(x,t) = \frac{4}{\mathrm{Re}_\delta}[1-\delta_d] + 4\int_0^1 (1-z)\langle\rho\rangle\{u''w''\}dz + \frac{4}{\mathrm{Re}_\delta}\int_0^1 (1-z)\langle\tilde{\mu}\rangle\frac{\partial\langle u\rangle}{\partial z}dz + \frac{4}{\mathrm{Re}_\delta}\int_0^1 (1-z) \times\left\langle\mu'\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right)\right\rangle dz - 2\int_0^1 (1-z)^2 \times\left(\langle I\rangle + \frac{\partial\langle\rho u\rangle}{\partial t}\right)dz, \tag{B1}$$

where $\delta_d=\int_0^1(1-\langle\rho u\rangle)dz$ is the displacement thickness normalized by δ, and $\langle I\rangle$ is the contributive term when the flow is inhomogeneous in the streamwise direction, i.e.,

$$\langle I\rangle = \frac{\partial\langle\rho u^2\rangle}{\partial x} + \frac{\partial\langle\rho\rangle\{u\}\{w\}}{\partial z} - \frac{1}{\mathrm{Re}_\delta}\frac{\partial\langle\tau_{xx}\rangle}{\partial x} - \frac{1}{\mathrm{Re}_\delta}\frac{\partial}{\partial z}\left(\langle\mu\rangle\frac{\partial\langle w\rangle}{\partial x}\right).$$

As noted by Fukagata *et al.* [6], the first term on the rhs does not correspond to the purely laminar contribution. Indeed, this quantity is a function of the mean turbulent velocity profile through the δ_d term. As in the channel case, the weighting factors for the Reynolds stress and for the additional terms coming from the compressibility are linear.

[1] J. Jiménez and A. Pinelli, J. Fluid Mech. **389**, 335 (1999).
[2] J. Lim *et al.*, Phys. Fluids **10**, 1997 (1998).
[3] S. Kang and H. Choi, Phys. Fluids **12**, 3301 (2000).
[4] Y. Du *et al.*, J. Fluid Mech. **457**, 1-34 (2002).
[5] S.-Q. Yang and G. Dou, Phys. Fluids **17**, 065104 (2005).
[6] K. Fukagata *et al.*, Phys. Fluids **14**, L73 (2002).
[7] G. N. Coleman *et al.*, J. Fluid Mech. **305**, 159 (1995).
[8] P. G. Huang *et al.*, J. Fluid Mech. **305**, 185 (1995).
[9] R. Lechner *et al.*, J. Turbul. **2**, 1 (2001).
[10] G. A. Blaisdell *et al.*, Appl. Numer. Math. **21**, 207 (1996).
[11] E. Lenormand *et al.*, AIAA J. **38**, 1340 (2000).