Deformations of the Tracy-Widom distribution

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In random matrix theory, the Tracy-Widom (TW) distribution describes the behavior of the largest eigenvalue. We consider here two models in which TW undergoes transformations. In the first one disorder is introduced in the Gaussian ensembles by superimposing an external source of randomness. A competition between TW and a normal (Gaussian) distribution results, depending on the spreading of the disorder. The second model consists of removing at random a fraction of (correlated) eigenvalues of a random matrix. The usual formalism of Fredholm determinants extends naturally. A continuous transition from TW to the Weilbull distribution, characteristic of extreme values of an uncorrelated sequence, is obtained.

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I. INTRODUCTION

In the beginning of the 1990s, Tracy and Widom (TW) [1] derived the probability distribution of the largest eigenvalue of random matrices belonging to the three Gaussian ensembles: the Gaussian orthogonal ensemble (GOE), the Gaussian unitary ensemble (GUE), and the Gaussian symplectic ensemble (GSE). Few years later, Baik *et al.* [2] proved that the longest increasing subsequence of a random permutation fluctuates as the largest GUE eigenvalue. By doing this, they set a connection between combinatorics and random matrices and triggered TW applications in other areas such as growth processes in which the height can be identified as a longest increasing path [3–6] (see [7] for a review). The main ingredient in their derivation was the discovery that the random matrix formalism based on Fredholm determinants and Painlevé equations [8] which at the bulk of the spectrum is associated to integral equations with a sine kernel and at the border of the spectrum it is associated to integral equations with an Airy kernel. It is by now accepted that these distributions belong to a universal class of extreme values of correlated sequences. Deviations from the TW have been observed and studied. For instance, in a couple of papers Gravner *et al.* [9] using a simple model investigated a $1+1$ growth process in which at each step the probability of a random move is not fixed but sorted out an independent distribution. An external source of randomness is therefore superimposed to the process, a typical situation of disordered systems and spin glasses. They found that depending on the value of $\alpha = x/(t-x)$, the asymptotic behavior of the height function $h_t(x)$ when x and t go to infinity exhibits different regimes: a nonfluctuating deterministic one for large values of α; a competition between TW (quenched fluctuations) and Gaussian (pure fluctuations) at intermediate values of α; and, finally, for small values of α, there is a composite regime in which, depending on the scaling, the fluctuations are normal or exponential.

In this general context it is also important to establish links between TW and known universal distributions of extreme values of uncorrelated sequences. The physical motivation in making this connection is the interest in the transition random matrix theory (RMT)-Poisson statistics of quantum systems. It is well established that eigenvalues of physical systems whose classical analog is chaotic have the same statistical properties of the RMT correlated eigenvalues [10]. On the other hand, eigenvalues of classically regular systems fluctuate like uncorrelated variables of a Poissonian statistics [11]. Although universality is not expected in this chaos-order transition, there is some evidence of a generic statistics common to rather different quantum systems such as the pseudointegrable billiards [12] and the Anderson model in three dimensions [13]. Matrix models to describe this intermediate statistics have been proposed [14].

In the Poisson case, namely, for a sequence of N independent and identically distributed random variables x_i with $i = 1,2,\ldots,N$, the probability that the largest variable is less than a given value t is [15]

$$P(x_{\max} < t) = \prod_{i=1}^{N} P(x_i < t) = \left[1 - \frac{\int_t^\infty \rho(x)dx}{N}\right]^N = \exp\left[-\int_t^T \rho(x)dx\right], \tag{1}$$

where $\rho(x)$ is their density. Depending on the asymptotic behavior of the function $\rho(x)$, this probability distribution takes three forms. If it has an exponential or it is faster than exponential decay, in the scaled variable $y = \rho(\bar{t})(t-\bar{t})$ it becomes the Gumbel distribution [16] $\exp[-\exp(-y)]$, where $\bar{t}$ is the position at which the density distribution of the extreme peaks. If ρ decays with a power law such that $\rho \sim C/x^{\mu+1}$, it is the Fréchet distribution [17] $\exp(-1/y^{\mu})$ in the variable $y = (\mu/C)^{1/\mu}x$. Finally, if ρ has a bounded support such that near the extremum $x = L$, $\rho = (L-x)^{\nu}$, the probability of the extreme obeys the Weibull distribution [18] $\exp[-(L-x)^{\nu+1}/\nu]$.

Largest eigenvalues of ensembles with independent but non-Gaussian matrix elements (Wigner matrices) have been the subject of recent investigations. It has been found that if the matrix elements are taken from a distribution with finite moments then the TW holds [19]. Considering instead the case in which the distribution of the matrix elements has long tails, it has been proven that when the second moment diverges, the largest eigenvalue and the largest matrix element follow a Fréchet distribution [17] with the same power $\mu \leq 2$ [20]. This result has more recently been extended until $\mu=4$ [21].

Models to describe deviations from TW have been discussed by Johansson [22]. In one model, he studied the behavior of the largest eigenvalue of a matrix model proposed by Moshe *et al.* [14] and found that it is described by a kernel that goes from a Poisson kernel [see Eq. (32) below] with an exponential density to the RMT Airy kernel. Accordingly, the distribution of the largest eigenvalue goes from Gumbel [16] to TW. In another model, a deformed GUE is considered in which the eigenvalue density fluctuates in such a way that the largest eigenvalue distribution goes from TW to Gaussian.

Following similar lines to those in [22] the purpose of this note is to investigate other models that describe deformations of the TW. The first model results from superimposing to the Gaussian fluctuations an external source of randomness [23]. This causes the eigenvalue semicircle density to fluctuate and results in features common to growth processes in random media. The second model is based on the recent recognition that the mathematical structure of RMT also describes the statistical properties of the eigenvalues of spectra when a fraction of eigenvalues is randomly removed [24]. As this operation reduces correlations, it describes intermediate statistics between RMT and Poisson statistics. Focusing on eigenvalues at the edge of the spectrum we show here that this leads to a transition from TW to a Weibull distribution [18].

Consider the Gaussian random matrix ensembles defined by a density distribution,

$$P_G(H;\alpha) = \left(\frac{\alpha\beta}{\pi}\right)^{f/2} \exp(-\alpha\beta \operatorname{tr} H^2), \tag{2}$$

where $f=N+\beta N(N-1)/2$ is the number of independent matrix elements and β is the Dyson index that takes the values 1, 2, and 4 for the orthogonal (GOE), the unitary (GUE), and the symplectic (GSE) ensembles, respectively. In Eq. (2), the normalization constant is calculated with respect to the measure $dH=\Pi_1^N dH_{ii}\Pi_{j>i}\Pi_{k=1}^{\beta}\sqrt{2}dH_{ij}^k$.

Let us start by recalling known facts about the eigenvalues of these Gaussian ensembles including some recent results regarding the behavior of their largest values. It is well known, for instance, that, to leading order, their eigenvalue density is given by Wigner's semicircle law,

$$\rho(\lambda) = \begin{cases} \dfrac{1}{2\pi\sigma^2}\sqrt{4N\sigma^2-\lambda^2}, & |\lambda| < 2\sigma\sqrt{N} \\ 0, & |\lambda| > 2\sigma\sqrt{N}, \end{cases} \tag{3}$$

where $\sigma=1/\sqrt{4\alpha}$ is the variance of the off-diagonal matrix elements. To study the behavior of the largest eigenvalues in the limit of large matrix size N, one introduces the scaling

$$\lambda = \left(2\sqrt{N} + \frac{s}{N^{1/6}}\right)\sigma, \tag{4}$$

which substituted in Eq. (3) leads to the N-independent density,

$$\rho(s) = \begin{cases} \dfrac{1}{\pi}\sqrt{-s}, & s \leq 0 \\ 0, & s > 0, \end{cases} \tag{5}$$

at the edge of the spectrum. In the scaled variable s, the probabilities $E(k,s)$ with $k=0,1,2,\ldots$ that the infinite interval (s,∞) has k eigenvalues are obtained from the generating function $G(s,z)$ through the relation [1,25]

$$G(s,z) = \sum_{n=0}^{\infty} (-1)^n (z-1)^n E(n,s) \tag{6}$$

such that

$$E(n,t) = \frac{(-1)^n}{n!}\left[\frac{\partial G(s,z)}{\partial z^n}\right]_{z=1}. \tag{7}$$

For the three symmetry classes, the generating functions $G_\beta(s,z)$ with $\beta=1$, 2, and 4 have been derived. Starting with the unitary case, $G_2(s,z)$ can be identified with the Fredholm determinant associated to the integral operator acting on the interval (s,∞) with kernel [26]

$$K(x,y) = \frac{\mathrm{Ai}(x)\mathrm{Ai}'(y) - \mathrm{Ai}(y)\mathrm{Ai}'(x)}{x-y}, \tag{8}$$

where $\mathrm{Ai}(s)$ is the Airy function. $G_2(s,z)$ is given by

$$G_2(s,z) = \exp\left[-\int_s^{\infty}(x-s)q^2(x,z)dx\right], \tag{9}$$

where $q(s,z)$ satisfies the Painlevé II equation

$$q'' = sq + 2q^3 \tag{10}$$

with boundary condition

$$q(s,z) \sim \sqrt{z}\mathrm{Ai}(s) \quad \text{when} \quad s\to\infty. \tag{11}$$

For GOE ($\beta=1$) and GSE ($\beta=4$) the generating functions are [27]

$$[G_1(s,z)]^2 = G_2(s,\bar{z})\frac{z-1-\cosh\mu(s,\bar{z})+\sqrt{\bar{z}}\sinh\mu(s,\bar{z})}{z-2} \tag{12}$$

and

$$[G_4(s,z)]^2 = G_2(s,z)\cosh^2\frac{\mu(s,z)}{2}, \tag{13}$$

where $\bar{z}=2z-z^2$ and

$$\mu(s,z) = \int_s^{\infty} q(x,z)dx. \tag{14}$$

We remark that the above expression for the GSE case was obtained in Refs. [26,27] using a scaling that assumes $N/2$ eigenvalues.

The above equations give a complete description of the fluctuations of the eigenvalues at the edge of the spectra of the Gaussian ensembles. In particular, for the largest eigenvalue, the TW distributions are expressed in terms of these generating functions as $E_{G,\beta}(E_{\max}<\lambda)=G_\beta(s,1)$.

We remark that the above set of equations also applies to largest eigenvalues at the edge of the Marchenko-Pastur density [28] of Laguerre ensembles [26]. Therefore the present analysis can easily be extended to the largest eigenvalue of Wishart matrices that has recently been investigated [29].

II. DISORDERED ENSEMBLES

To investigate the modifications these probabilities undergo when an external source of randomness is superimposed to the Gaussian fluctuations; we consider disordered ensembles whose matrices, $H(\xi,\alpha)$, are defined as [23]

$$H(\xi,\alpha)=\frac{H_G(\alpha)}{\sqrt{\xi/\bar{\xi}}}, \tag{15}$$

where H_G is a matrix of Eq. (2) and ξ is a positive random variable with distribution $w(\xi)$ with average $\bar{\xi}$ and variance σ_w. From Eqs. (2) and (15) it is deduced that the joint density distribution of the matrix elements is a superposition of the Gaussian ensemble distributions weighted with $w(\xi)$, namely,

$$P(H;\alpha)=\int d\xi w(\xi)P_G(H;\alpha\xi/\bar{\xi}). \tag{16}$$

Changing variables from matrix elements to eigenvalues and eigenvectors, it is also found, after integrating out the eigenvectors, that the joint probability distribution of the eigenvalues is obtained by averaging over the joint distribution of the Gaussian ensembles. As a consequence, measures of this average ensemble are averages over the Gaussian measures. The eigenvalue density, for instance, turns out to be an average over Wigner's semicircles with different radii, that is,

$$\rho(\lambda;\alpha)=\int d\xi w(\xi)\sqrt{4\sigma^2(\xi)N-\lambda^2}/[2\pi\sigma^2(\xi)], \tag{17}$$

where the ξ-dependent variance, $\sigma(\xi)$, is given by

$$\sigma(\xi)=\sigma\sqrt{\bar{\xi}/\xi}. \tag{18}$$

The probability that the largest eigenvalue $\lambda_{\max}$ is smaller than a given value t can be calculated by evaluating in the probability that the interval (t,∞) is empty. This is obtained by integrating the joint probability distribution of the eigenvalues in the interval $(-\infty,t)$ over all eigenvalues; we find

$$E_\beta(\lambda_{\max}<t)=\int d\xi w(\xi)E_{G,\beta}[S(\xi,t)] \tag{19}$$

with the argument of $S(\xi,t)$ obtained by plugging in Eq. (4) the above ξ variance [Eq. (18)], namely,

$$S(\xi,t)=N^{1/6}\left[\frac{t}{\sigma(\xi)}-2\sqrt{N}\right]. \tag{20}$$

Equations (19) and (20) give a complete analytical description of the behavior of the largest eigenvalue once the function $w(\xi)$ is chosen. But even without specifying $w(\xi)$, asymptotic results can be derived by comparing its localization, given by the ratio $\sigma_w/\bar{\xi}$, with that of $E_{G,\beta}$ considered as functions of the integrand variable ξ through Eq. (20). Since the widths of these last ones depend on the matrix size N, let us introduce a positive parameter z such that $\sigma_w/\bar{\xi}=N^{-z}$, which is kept fixed when the limit $N\to\infty$ is taken.

As $z>0$, when N increases, the $w(\xi)$ distribution becomes more and more localized and, if asymptotically, it can be approximated by a Gaussian by changing the integration variable to

$$\xi=\bar{\xi}-v\sigma_w. \tag{21}$$

Equation (19) can be written as

$$E_\beta(\lambda_{\max}<t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dv\,\exp\left(-\frac{v^2}{2}\right)E_{G,\beta}[S(v,t)], \tag{22}$$

where the argument $S(v,t)$, after neglecting higher order terms in $1/N$, takes the form

$$S(v,t)=N^{1/6}\left(\frac{t}{\sigma}-vN^{1/2-z}-2\sqrt{N}\right)=s-N^{2/3-z}v. \tag{23}$$

In the last step of Eq. (23), Eq. (4) was used and it was assumed that $z>2/3$. Taking now the limit $N\to\infty$, the vanishing of the second term in the right-hand side (rhs) of Eq. (23) makes $S(v,t)$ independent of v and the distributions $E_{G,\beta}(s)$, i.e., TW, are recovered. Values of the parameter z greater than 2/3 correspond to situations in which the distribution $w(\xi)$ collapses faster than $E_{G,\beta}$; for $z<2/3$, on the other hand, it is the opposite that happens. To be able to get N-independent results in this range of values of z, it is necessary to modify the scaling to

$$s=N^{z-1/2}\left(\frac{t}{\sigma}-2\sqrt{N}\right), \tag{24}$$

in which case the argument $S[v,t(s)]$ becomes

$$S[v,t(s)]=N^{2/3-z}(s-v). \tag{25}$$

Taking now the limit of $N\to\infty$, $E_{G,\beta}(S)$ becomes a step function centered at $v=s$ and the distribution goes to the normal distribution $N(0,1)$. Finally, at the critical value $z=2/3$, from both sides, Eq. (22) converges to the convolution of the normal distribution and TW. In summary we have the three regimes,

$$E_\beta(\lambda_{\max}<t)=\begin{cases} E_{G,\beta}(s), & z>2/3\\ \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty}dv\,\exp\left(-\frac{v^2}{2}\right)E_{G,\beta}(s-v), & z=2/3\\ \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{s}dv\,\exp\left(-\frac{v^2}{2}\right), & z<2/3.\end{cases} \tag{26}$$

Consider now the case in which the distribution $w(\xi)$ is independent of N. In this case, it is appropriate to make the parameter α equal to the matrix size N. With this scaling, the eigenvalues of the matrices of the average ensemble are located in the interval $(-1,1)$, and the ξ-dependent argument $S(\xi,t)$ [Eq. (20)] takes the simple form $S(\xi,t)=2N^{2/3}(t\sqrt{\xi/\bar{\xi}}-1)$. This expression makes evident that when the matrix size N increases, for the three invariant ensembles, the function $E_{G,\beta}$ becomes a step function centered at $\xi=\bar{\xi}/t^2$. Therefore, in this regime, the probability distribution for the largest eigenvalue converges to

$$E_\beta(\lambda_{\max} < t) = \int_{\bar{\xi}/t^2}^{\infty} d\xi w(\xi) \tag{27}$$

with density

$$\frac{dE_\beta(t)}{dt} = \frac{2\bar{\xi}w(\bar{\xi}/t^2)}{t^3}. \tag{28}$$

A special choice of $w(\xi)$ that has already appeared in previous studies of disordered ensembles [30,31] is that in which it is the one parameter family of gamma distributions,

$$w(\xi) = \exp(-\xi)\xi^{\bar{\xi}-1}/\Gamma(\bar{\xi}). \tag{29}$$

In this case, Eq. (28) becomes

$$\frac{dE_\beta(t)}{dt} = \frac{2\bar{\xi}^{\bar{\xi}}\exp(-\bar{\xi}/t^2)}{\Gamma(\bar{\xi})t^{2\bar{\xi}+1}}, \tag{30}$$

which defines a long-tailed distribution of extreme values of a correlated set of points (we remark that this distribution has recently been considered in random covariance matrices [32]).

Notice that in Eq. (28) the variable t is the eigenvalue itself without any edge scaling. This means that, in this case, fluctuations become of the order of the size of the average ensemble spectrum. Although Eq. (28) resembles a Fréchet distribution [17], the fact that the power of t in the exponent is fixed at the value of 2 makes it a different distribution. Nevertheless, for $\bar{\xi}=1$ it is indeed a Fréchet distribution [17]. We remark that $\bar{\xi}=1$ corresponds to the critical distribution of the family defined by Eq. (29) that separates the ones that converge from those that diverge at the origin. The asymptotical power-law decay of Eq. (30), similar to that of Fréchet distribution [17], suggests that in the asymptotic region the extreme value behaves independently of the other eigenvalues while, in the internal region, the presence of the others is felt.

III. FROM TRACY-WIDOM TO WEIBULL

Let us now turn to the case of a model to describe largest eigenvalues of spectra in the intermediate regime between RMT and Poisson. This model is brought on by the fact that the generating function [Eq. (6)] can be interpreted as a probability. Indeed, assuming with $0<z<1$ that the factor $(1-z)^n$ that multiplies $E(n,s)$ in Eq. (6) is the probability that the n eigenvalues in the interval (s,∞) have been removed, then summing all the terms gives the probability that there is no level in the interval.

A realization of this situation was considered in Ref. [33] in which the effect of removing at random a fraction $1-f$ of eigenvalues of RMT spectra was investigated. In this case, $1-f$ is the probability that a given eigenvalue has been dropped from the spectrum. Therefore with the identification of z with f the generating function [Eq. (6)] becomes the probability distribution of the largest eigenvalue for this kind of randomly incomplete spectra.

In Refs. [24,33], in studying the effect of incompleteness in the spectral statistics at the bulk, an interval of length s is increased by a factor of $1/f$ to compensate the reduction in the average number of levels inside it. Following the same idea at the edge, we want a scaling of the variable s such that the average number of eigenvalues in the interval (s,∞) remains the same when a fraction of levels is removed. This average is

$$\langle n\rangle = \frac{2}{3\pi}(-s)^{3/2}, \tag{31}$$

obtained by integrating Eq. (5) from s to ∞. Therefore, in order to keep it invariant when the density of eigenvalues is reduced by a factor of f, s has to be divided by $f^{2/3}$.

Using this scaling in the Airy kernel [Eq. (8)], we expect that when the limit $f\to 0$ is taken it converges to the Poisson kernel [3],

$$K_P(x,y) = \begin{cases} 0, & x \neq y \\ \rho(x), & x = y \end{cases} \tag{32}$$

with density given by Eq. (5). In fact, we have

$$\lim_{f\to 0}\frac{\mathrm{Ai}(x/f^{2/3})\mathrm{Ai}'(y/f^{2/3}) - \mathrm{Ai}(y/f^{2/3})\mathrm{Ai}'(x/f^{2/3})}{(x-y)/f^{2/3}} = K_P(x,y). \tag{33}$$

Denoting by $\hat{e}(s,f)$ the probability that the semi-infinite interval (s,∞) is empty, for the incomplete spectra it is given by

$$\hat{e}_\beta(s,f) = \sum_{k=0}^{\infty}(1-f)^k E_\beta(k,s/f^{2/3}) = G_\beta(s/f^{2/3},f), \tag{34}$$

which means that in an incomplete spectrum the largest eigenvalue can be anyone of the nth largest eigenvalues. The last equality in Eq. (34) follows from Eq. (6) and shows that the generating functions for the three symmetry classes contain a comprehensive description of the largest eigenvalues of complete and incomplete RMT spectra.

To investigate the limit $f\to 0$ we remark that as the scaling factor $f^{2/3}$ appears in the denominator, for small f the function $q(x,z)$ can be replaced by its asymptotic form at $x\to\pm\infty$. For $x>0$, this is given by the exponential decay of the asymptotic behavior of the Airy function. So, at this positive side, $q(x,z)$ vanishes and $\hat{e}_\beta(s,f)=1$. For $s<0$, by the same argument, the integrals can be performed from s to zero with $q(x,z)$ replaced by its asymptotic expression for large negative values. For $0<z<1$, this expression has been worked out by Hastings and McLeod [34], who found that

$$q(x,z) \sim d(-x)^{-1/4} \sin[\tfrac{2}{3}(-x)^{3/2} - \tfrac{3}{4}d^2 \log(-x) - c], \tag{35}$$

where $d^2 = -\frac{1}{\pi}\log(1-z)$. Using this asymptotic expression we find that the Fredholm determinant [Eq. (9)] in this asymptotic regime can be written as

$$\log G_2 = -\frac{2d^2(-s)^{3/2}}{3\pi f} + \frac{d^2}{2f}\int_s^0 \frac{dx}{\sqrt{-x}}(x-s)\cos\left[\frac{4(-x)^{3/2}}{3f} - \frac{3d^2}{4}\log(-x/f^{2/3}) - 2c\right], \tag{36}$$

where the relation $2\sin^2 x = 1 - \cos 2x$ was used, the integral of the nonoscillating term was performed, and a substitution of variable removed the dependence on f in the limit of integration. On the other hand, the oscillations of the cosine in the second term in Eq. (36) increase as f decreases such that the integral averages out to zero and can be neglected. For the same reason, in the same limit, the function $\mu(s,f)$ [Eq. (14)] vanishes. Taking all this into account we find that in the limit $f \to 0$ the largest eigenvalue distributions, for the three symmetry classes, converge to the Weibull distribution [18],

$$\hat{e}_\beta(s) = \begin{cases} \exp\left[-\dfrac{2g_\beta}{3\pi}(-s)^{3/2}\right], & s \le 0 \\ 1, & s > 0, \end{cases} \tag{37}$$

where $g_\beta = 1$ for $\beta = 1,2$ and $g_\beta = 1/2$ for $\beta = 4$ (this parameter reflects the fact mentioned above that, in the GSE case, we are using a scaling with $N/2$). This Weibull distribution [18] describes the extreme value of a set of uncorrelated points with a semicircle density distribution. In Fig. 1, the transition from TW ($f=1$) to Weibull [18] ($f=0$) is illustrated for the GUE case ($\beta=2$). The oscillations at intermediate values are clearly seen.

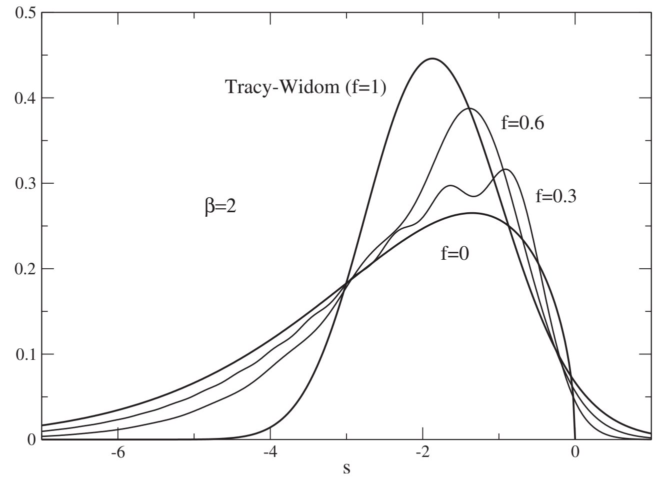

FIG. 1. Density distribution of largest eigenvalue for GUE ($\beta=2$): for complete sequence ($f=1$; Tracy-Widom), for uncorrelated sequence (limit $f \to 0$ from Eq. (37); Weibull [18]), and for partially incomplete sequences [$f=0.3, 0.6$ from Eq. (34)].

IV. CONCLUSION

In conclusion, we have investigated the behavior of the largest eigenvalue of two models that generalize the RMT Gaussian ensembles. In the first one, by superimposing an external source of randomness in the Gaussian ensemble, another ensemble is obtained that shows features typical of disordered systems and spin glasses. It is found, at the edge of the spectrum, asymptotic regimes similar to those of growth processes in a random media. We can expect that this ensemble can model extreme values of correlated random sequences submitted to an external source of fluctuation. In the second model, the correlations among the RMT eigenvalues are progressively reduced. It is then observed a continuous transition from the TW to the Weibull distribution [18], characteristic of uncorrelated variables. At the intermediate regime, the model has a distribution with oscillations, a prediction to be compared with other models of intermediate statistics, for instance, the semi-Poisson model [35].

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