Electromagnetic boundary and its realization with anisotropic metamaterial

I. V. Lindell* and A. H. Sihvola†
Electromagnetics Group, Department of Radio Science and Engineering, Helsinki University of Technology, Box 3000, Espoo 02015TKK, Finland
(Received 17 October 2008; published 9 February 2009)

A set of boundary conditions requiring vanishing of the normal components of the **D** and **B** vectors at the boundary surface is introduced and labeled as that of DB boundary. Basic properties of the DB boundary are studied in this paper. Reflection of an arbitrary plane wave, incident with a complex propagation vector, is analyzed for the planar DB boundary. It is shown that waves polarized transverse electric (TE) and transverse magnetic (TM) with respect to the normal of the boundary are reflected as from respective perfect electric conductor and perfect magnetic conductor planes. The basic problem of current source above the planar DB boundary is solved by applying TE and TM decomposition for the source. Realization of the DB boundary in terms of an interface of uniaxially anisotropic metamaterial half-space with zero axial medium parameters is considered. It is also shown that such a medium with small axial parameters acts as a spatial filter for waves incident at the interface which could be used for narrowing the beam of a directive antenna. Application of DB boundary as an isotropic soft surface with low interaction between antenna apertures also appears possible.

DOI: 10.1103/PhysRevE.79.026604 PACS number(s): 03.50.De, 41.20.Jb, 42.25.Gy

I. INTRODUCTION

Electromagnetic problems defined by differential equations require boundary conditions for the fields to have unique solutions. Boundary conditions are expressed in the form of restricting conditions at the surface bounding the region of interest. For uniqueness they must have a proper form, being too loose makes the solution nonunique, being too tight makes the solution nonexistent. For time-harmonic electromagnetic field problems, proper boundary conditions appear to involve two scalar conditions at the boundary surface.

Boundary conditions[1] should not be confused with interface conditions, valid at interfaces of two media. The region behind the boundary does not affect the problem at all while the region behind the interface belongs to the region of interest and must be taken into account. However, since any boundary is a mathematical idealization, it must also have a relation to some material interfaces. This relation can be seen from two directions which can be called analytical and synthetical views.

The analytical view starts from the interface problem and calls for a boundary condition to approximate the interface in order to simplify the mathematical problem. In this case the boundary condition is an approximation. The synthetical problem is the opposite. It starts from the boundary condition, which is considered exact and calls for its realization in terms of an interface of some possible materials. Knowing the properties of certain boundary conditions on electromagnetic fields one has the possibility of synthesizing useful structures defined by boundary surfaces. To realize such a structure one must approximate the boundary in terms of an interface of suitable material which need not be unique. For example, surface-wave antennas have been first designed in terms of surfaces with reactive impedance boundary conditions after which the realization of such surfaces in terms of dielectric slabs above a metallic surface have been suggested [3].

In this paper we take the synthetical view by starting from a set of certain boundary conditions with interesting properties and considering its realization. Let us, however, first start by some well-known examples of boundary conditions. The perfect electric conductor (PEC) boundary condition for the electric field

$$\mathbf{n} \times \mathbf{E} = \mathbf{0} \tag{1}$$

is defined on a surface with unit normal vector **n**. The PEC boundary can be realized by a medium with limiting values of permittivity $\epsilon \to \infty$ and permeability $\mu \to 0$ [4]. The field inside the medium depends on the value of the limiting $\mu\epsilon$ while the field outside the medium is unique. Similarly, the perfect magnetic conductor (PMC) boundary with the condition

$$\mathbf{n} \times \mathbf{H} = \mathbf{0} \tag{2}$$

can be realized by an interface of a medium with $\mu \to \infty$ and $\epsilon \to 0$. Such a boundary is also known by the name "high-impedance electromagnetic surface" and structures for its realization have been constructed [5]. The perfect electromagnetic conductor (PEMC) boundary, introduced in [6] by

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = \mathbf{0}, \tag{3}$$

where M denotes the (pseudo) scalar admittance parameter, is a generalization of PMC ($M \to 0$) and PEC ($M \to \infty$) boundaries. Realization of the planar PEMC boundary $z=0$ can be achieved in terms of the interface of a certain limiting isotropic Tellegen medium as described in [6] or of an aniso-

*ismo.lindell@tkk.fi
†ari.sihvola@tkk.fi

[1]To emphasize the difference between boundary and its realization by a medium interface, a distinction between the concepts of boundary condition and interface condition is made following [1], pp. 71,74, in spite of common usage in the literature making no such distinction (see, e.g., [2]).

tropic medium with dyadic permittivity and permeability

$$\bar{\bar{\epsilon}} = \epsilon_z \mathbf{u}_z \mathbf{u}_z + \bar{\bar{\epsilon}}_t, \quad \bar{\bar{\mu}} = \mu_z \mathbf{u}_z \mathbf{u}_z + \bar{\bar{\mu}}_t, \tag{4}$$

$$\mathbf{u}_z \cdot \bar{\bar{\epsilon}}_t = \bar{\bar{\epsilon}}_t \cdot \mathbf{u}_z = 0, \quad \mathbf{u}_z \cdot \bar{\bar{\mu}}_t = \bar{\bar{\mu}}_t \cdot \mathbf{u}_z = \mathbf{0}, \tag{5}$$

as suggested in [7]. To yield (3), the axial parameters of (4) are required to satisfy $\epsilon_z \to \infty$ and $\mu_z \to \infty$ and the transverse medium dyadics $\bar{\bar{\mu}}_t, \bar{\bar{\epsilon}}_t$ are restricted by certain gyrotropic properties.

All of the previous boundary conditions are special cases of impedance conditions on a surface with unit normal vector $\mathbf{n}$ in the form [8,9,1]

$$[\mathbf{E} - \bar{\bar{Z}}_s \cdot (\mathbf{n} \times \mathbf{H})]_t = 0, \quad \mathbf{n} \cdot \bar{\bar{Z}}_s = \bar{\bar{Z}}_s \cdot \mathbf{n} = \mathbf{0}, \tag{6}$$

for some surface impedance dyadic $\bar{\bar{Z}}_s$. Realization of the general planar impedance boundary in terms of the uniaxial medium (4) with infinite axial parameters was suggested in [10].

Let us now introduce another set of boundary conditions in terms of the vectors $\mathbf{D}$ and $\mathbf{B}$ satisfying the following two scalar equations:

$$\mathbf{n} \cdot \mathbf{D} = 0, \quad \mathbf{n} \cdot \mathbf{B} = 0. \tag{7}$$

Boundaries with conditions (7) are labeled DB boundaries for brevity in this paper. The corresponding boundary conditions for the $\mathbf{E}$ and $\mathbf{H}$ vectors depend on the medium in front of the boundary. Assuming a simple isotropic medium with permittivity ϵ and permeability μ, (7) is equivalent to the conditions

$$\mathbf{n} \cdot \mathbf{E} = 0, \quad \mathbf{n} \cdot \mathbf{H} = 0. \tag{8}$$

The same conditions (8) are also valid for bi-isotropic media satisfying the medium equations

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \epsilon & \xi \\ \zeta & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \tag{9}$$

provided the medium matrix has an inverse which corresponds to the condition $\mu\epsilon - \xi\zeta \neq 0$. For bianisotropic media with dyadic medium parameters the DB-boundary condition for the $\mathbf{E}$ and $\mathbf{H}$ fields is not so simple.

Boundary conditions of the form (7) or (8) for the planar boundary $z=0$ were briefly introduced in [11] as following from the interface conditions for a half-space of certain exotic material labeled as uniaxial skewon-axion medium or IB medium [12,13]. Thus, conversely, the medium interface serves as a possible realization of the DB-boundary conditions. However, another, much simpler, realization for the DB boundary can be obtained in terms of the interface of the uniaxial anisotropic medium (4) or its axially symmetric special case

$$\bar{\bar{\epsilon}} = \epsilon_z \mathbf{u}_z \mathbf{u}_z + \epsilon_t \bar{\bar{\mathsf{I}}}_t, \quad \bar{\bar{\mu}} = \mu_z \mathbf{u}_z \mathbf{u}_z + \mu_t \bar{\bar{\mathsf{I}}}_t, \tag{10}$$

where the transverse unit dyadic is defined by

$$\bar{\bar{\mathsf{I}}}_t = \mathbf{u}_x \mathbf{u}_x + \mathbf{u}_y \mathbf{u}_y. \tag{11}$$

Because of the continuity of the normal components of the $\mathbf{D}$ and $\mathbf{B}$ vectors across the interface, the conditions (7) are obtained for vanishing axial parameters, $\epsilon_z \to 0$, $\mu_z \to 0$ while the values of the transverse parameters do not affect the fields outside the medium when the limits are attained. It is interesting to note that the uniaxial medium (4) can be applied for the realization of boundary conditions for either tangential (6) or normal (7) field components. In the former case the axial medium parameters must become infinite and in the latter case zero.

Because the conditions (7) have such a simple form and since they have not been treated in the literature known to these authors, they deserve to be studied in their own right, which serves as the basic motivation of this paper. Realization of the boundary in terms of an interface of the uniaxial medium (10) creates the problem how to realize the medium. The importance of this problem will grow if and when engineering applications for such a boundary will be found. Media with small permittivity and/or permeability have been of interest in the recent surge of metamaterial research because of their promising applications [14]. Since the work on metamaterials has produced structures with effectively negative values of permittivity and permeability, zero parameter values have become possible by combining regions of positive and negative parameter values [15–17]. For example, in the case of uniaxial anisotropic medium half-space the regions could be thin alternating layers of opposite-valued permittivity and permeability, all parallel to the plane $z=0$.

The first objective of the present paper is to study the effect of the ideal DB boundary conditions (7) on the electromagnetic field created by infinite sources (plane wave) and localized sources (field from a dipole). The second objective is to study the approximate realization of the DB boundary in terms of the uniaxial medium defined by (10) with small but finite axial parameter ϵ_z, μ_z values. In the analysis we assume that the medium above the DB boundary is isotropic with the parameters ϵ, μ, whence the ideal DB-boundary conditions can be taken in the form (8).

II. REFLECTION FROM DB BOUNDARY

The basic problem associated with the planar DB boundary $z=0$ is to find the reflection of a plane wave from the boundary. Let us assume incident and reflected fields of the form

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}^i e^{-j\mathbf{k}^i \cdot \mathbf{r}}, \quad \mathbf{H}^i(\mathbf{r}) = \mathbf{H}^i e^{-j\mathbf{k}^i \cdot \mathbf{r}}, \tag{12}$$

$$\mathbf{E}^r(\mathbf{r}) = \mathbf{E}^r e^{-j\mathbf{k}^r \cdot \mathbf{r}}, \quad \mathbf{H}^r(\mathbf{r}) = \mathbf{H}^r e^{-j\mathbf{k}^r \cdot \mathbf{r}}, \tag{13}$$

propagating in the isotropic half-space $z>0$ with the wave vectors

$$\mathbf{k}^i = -k_z \mathbf{u}_z + \mathbf{K}, \quad \mathbf{k}^r = k_z \mathbf{u}_z + \mathbf{K}, \tag{14}$$

satisfying the conditions

$$\mathbf{k}^i \cdot \mathbf{k}^i = \mathbf{k}^r \cdot \mathbf{k}^r = k_z^2 + \mathbf{K} \cdot \mathbf{K} = k^2, \quad k = \omega\sqrt{\mu\epsilon}. \tag{15}$$

Here $\mathbf{K}$ is a vector transverse to $\mathbf{u}_z$ and it may have complex components. Let us, however, assume that $K^2 = \mathbf{K} \cdot \mathbf{K} \neq 0$, in which case any vector $\mathbf{a}$ can be expanded as

$$\mathbf{a} = \mathbf{u}_z(\mathbf{u}_z \cdot \mathbf{a}) + \frac{1}{K^2}\mathbf{K}(\mathbf{K}\cdot\mathbf{a}) + \frac{1}{K^2}\mathbf{u}_z \times \mathbf{K}(\mathbf{u}_z \times \mathbf{K}\cdot\mathbf{a}). \tag{16}$$

Now it is well known that any plane wave can be uniquely decomposed in two components, transverse electric (TE) and transverse magnetic (TM), polarized with respect to a direction in space not coinciding with that of the $\mathbf{k}$ vector. Assuming that the special direction is parallel to $\mathbf{u}_z$ we can express

$$\mathbf{E}^{i,r} = \mathbf{E}^{i,r}_{\mathrm{TE}} + \mathbf{E}^{i,r}_{\mathrm{TM}}, \quad \mathbf{H}^{i,r} = \mathbf{H}^{i,r}_{\mathrm{TE}} + \mathbf{H}^{i,r}_{\mathrm{TM}} \tag{17}$$

with

$$\mathbf{u}_z \cdot \mathbf{E}^{i,r}_{\mathrm{TE}} = 0, \quad \mathbf{u}_z \cdot \mathbf{H}^{i,r}_{\mathrm{TM}} = 0. \tag{18}$$

From the Maxwell equations the field amplitudes satisfy

$$\mathbf{k}^{i,r} \cdot \mathbf{E}^{i,r} = 0, \quad \mathbf{k}^{i,r} \cdot \mathbf{H}^{i,r} = 0. \tag{19}$$

From these and (18) we conclude that the field components can be expressed as

$$\mathbf{E}^{i,r}_{\mathrm{TE}} = E^{i,r}_{\mathrm{TE}}\mathbf{u}_z \times \mathbf{K}, \quad \mathbf{H}^{i,e}_{\mathrm{TM}} = H^{i,r}_{\mathrm{TM}}\mathbf{u}_z \times \mathbf{K}, \tag{20}$$

and the other components are obtained as

$$\mathbf{H}^{i,r}_{\mathrm{TE}} = \frac{E^{i,r}_{\mathrm{TE}}}{k\eta}[\mathbf{u}_z K^2 - (\mathbf{u}_z \cdot \mathbf{k}^{i,r})\mathbf{K}], \tag{21}$$

$$\mathbf{E}^{i,r}_{\mathrm{TM}} = -\frac{\eta H^{i,r}_{\mathrm{TM}}}{k}[\mathbf{u}_z K^2 - (\mathbf{u}_z \cdot \mathbf{k}^{i,r})\mathbf{K}], \tag{22}$$

with $\eta = \sqrt{\mu/\epsilon}$. One can easily check that the plane-wave equations are satisfied for TE and TM components separately.

Considering the total fields $\mathbf{E}^i_{\mathrm{TE}} + \mathbf{E}^r_{\mathrm{TE}}$ and $\mathbf{H}^i_{\mathrm{TM}} + \mathbf{H}^r_{\mathrm{TM}}$ one can notice that the DB-boundary conditions at $z=0$ in the form (8) are satisfied automatically. On the other hand, the remaining field components are subject to the conditions

$$\mathbf{u}_z \cdot (\mathbf{E}^i_{\mathrm{TM}} + \mathbf{E}^r_{\mathrm{TM}}) = -\frac{\eta}{k}(H^i_{\mathrm{TM}} + H^r_{\mathrm{TM}})K^2 = 0, \tag{23}$$

$$\mathbf{u}_z \cdot (\mathbf{H}^i_{\mathrm{TE}} + \mathbf{H}^r_{\mathrm{TE}}) = \frac{1}{k\eta}(E^i_{\mathrm{TE}} + E^r_{\mathrm{TE}})K^2 = 0, \tag{24}$$

whence the fields satisfy

$$\mathbf{u}_z \times (\mathbf{E}^i_{\mathrm{TE}} + \mathbf{E}^r_{\mathrm{TE}}) = -\mathbf{K}(E^i_{\mathrm{TE}} + E^r_{\mathrm{TE}}) = \mathbf{0}, \tag{25}$$

$$\mathbf{u}_z \times (\mathbf{H}^i_{\mathrm{TM}} + \mathbf{H}^r_{\mathrm{TM}}) = -\mathbf{K}(H^i_{\mathrm{TM}} + H^r_{\mathrm{TM}}) = \mathbf{0}. \tag{26}$$

Since the total TE field satisfies the PEC conditions and the total TM field satisfies the PMC conditions, one can say that the DB boundary appears as the PEC boundary for the TE component and the PMC boundary for the TM component of the plane wave. Since this is a linear relation valid for any plane wave, it is valid for any linear combination of plane waves and, eventually, for any field outside the sources because it can be expressed as a Fourier integral of plane waves. A proof without resorting to plane waves is suggested in the Appendix.

From (26) we obtain

$$\mathbf{u}_z \times (\mathbf{k}^i \times \mathbf{E}^i_{\mathrm{TM}} + \mathbf{k}^r \times \mathbf{E}^r_{\mathrm{TM}}) = k_z(\mathbf{E}^i_{\mathrm{TM}} - \mathbf{E}^r_{\mathrm{TM}})_t = \mathbf{0}, \tag{27}$$

whence combining with (25) a relation between the reflected and incident transverse field components can be expressed as

$$\mathbf{E}^r_t = \bar{\bar{\mathsf{R}}}_t \cdot \mathbf{E}^i_t, \tag{28}$$

with the reflection dyadic defined by

$$\bar{\bar{\mathsf{R}}}_t = \frac{1}{K^2}[\mathbf{K}\mathbf{K} - (\mathbf{u}_z \times \mathbf{K})(\mathbf{u}_z \times \mathbf{K})]. \tag{29}$$

As a check, the eigenvectors of the reflection dyadic ($\mathbf{K}$ and $\mathbf{u} \times \mathbf{K}$) are seen to correspond to the TM and TE waves with the respective eigenvalues $+1$ and -1 corresponding to reflections from respective PMC and PEC planes.

For any fields at the DB boundary we can write

$$\mathbf{n} \times (\mathbf{E} \times \mathbf{H}^*) = \mathbf{E}(\mathbf{n}\cdot\mathbf{H})^* - (\mathbf{n}\cdot\mathbf{E})\mathbf{H}^* = \mathbf{0}, \tag{30}$$

whence the complex Poynting vector is either normal to the boundary or zero. Conversely, if the complex Poynting vector is not zero and it satisfies (30) for any fields $\mathbf{E},\mathbf{H}$ at the boundary of an isotropic medium, the fields satisfy the DB conditions (8).

A boundary with no power propagation along the surface has been called an electromagnetic soft surface in the past [18]. Since this requires that the real part of the Poynting vector have no tangential component, a DB boundary is a soft surface but the converse is not necessarily true. Early realizations have required an anisotropic boundary surface representing the soft-surface property to waves propagating in a certain direction, for example, transverse to corrugations on a metal plane. Because the DB boundary has no component of the complex Poynting vector parallel to the surface, it can be called the isotropic soft surface. Isotropic soft surfaces have been realized in terms of band-gap structures [5,19]. There are engineering applications for the soft surface, like forming circularly symmetric radiation patterns for a horn antenna. An isotropic soft surface can be used to reduce coupling between antenna apertures in the surface so that they can be positioned closer to each other for a more compact array. The DB boundary serves as a simple model for the numerical implementation of problems involving certain soft surfaces regardless of their realization.

A parallel-plane waveguide with DB-boundary conditions on each plane supports modes consisting of plane waves reflecting from both planes. It is obvious that the modes can be split in two groups, those TE and those TM with respect to the normal of both planes. In this case, the TE modes are the same as those corresponding to two PEC planes while the TM modes correspond to two PMC planes.

In the previous study we have excluded the case $\mathbf{K}=\mathbf{0}$, i.e., the normally incident plane wave. This case appears somewhat strange, because the fields appear to satisfy identically the DB-boundary conditions (7). This means that an incident TEM wave does not see a planar DB boundary at all and, consequently, there is no reflection from it. For a source of limited extent, with radiation consisting of a continuous distribution of plane waves, this is not important, since the normally incident plane-wave component corresponds to a part of zero energy of the whole spectrum and can be easily neglected. A physical insight to this case can be obtained when considering a realization of the DB boundary in terms of the interface of anisotropic medium (10), as discussed in Sec. V

III. IMAGE THEORY

As a problem involving a localized source let us consider a current element above the planar DB boundary $z=0$. Decomposing the source in two parts radiating TE and TM fields in a homogeneous isotropic space splits the problem in two noninteracting parts because TE and TM fields do not couple at the DB boundary.

Because an axial (vertical) electric dipole source

$$\mathbf{J}=\mathbf{u}_z IL\delta(\mathbf{r}-\mathbf{u}_z h)=\mathbf{u}_z IL\delta(\boldsymbol{\rho})\delta(z-h), \quad \boldsymbol{\rho}=\mathbf{u}_x x+\mathbf{u}_y y, \tag{31}$$

radiates a TM field, the reflected field can be straightforwardly found by applying the image theory. In this case the image source yielding the reflected field equals that of the PMC plane,

$$\mathbf{J}^r(\mathbf{r})=-\mathbf{u}_z IL\delta(\mathbf{r}+\mathbf{u}_z h). \tag{32}$$

The problem is more complicated for the transverse (horizontal) electric dipole

$$\mathbf{J}=\mathbf{u}_x IL\delta(\mathbf{r}-\mathbf{u}_z h), \tag{33}$$

since its field contains both TE and TM components. However, we can now apply the theory described in [20] on how to decompose the source in two parts,

$$\mathbf{J}(\mathbf{r})=\mathbf{J}_{\mathrm{TE}}(\mathbf{r})+\mathbf{J}_{\mathrm{TM}}(\mathbf{r}), \tag{34}$$

so that $\mathbf{J}_{\mathrm{TE}}$ gives rise to a TE field and, $\mathbf{J}_{\mathrm{TM}}$, to a TM field. In this case the total reflection image can be obtained by combining the respective images in the PEC and PMC planes.

From the three possible decompositions given in [20] we choose the one in terms of surface currents on the plane $z=h$,

$$\mathbf{J}_{\mathrm{TM}}(\boldsymbol{\rho},z)=\mathbf{u}_x IL\delta'(z-h)\frac{x}{2\pi\rho^2}, \tag{35}$$

$$\mathbf{J}_{\mathrm{TE}}(\boldsymbol{\rho},z)=\mathbf{J}(\mathbf{r})-\mathbf{J}_{\mathrm{TM}}(\boldsymbol{\rho},z), \tag{36}$$

in terms of which the reflection image can now be constructed as follows. The image of $\mathbf{J}_{\mathrm{TM}}$ equals that of the PMC plane,

$$\mathbf{J}^r_{\mathrm{TM}}(\mathbf{r})=\mathbf{u}_x IL\delta'(z+h)\frac{x}{2\pi\rho^2}. \tag{37}$$

The image of $\mathbf{J}_{\mathrm{TE}}$ equals that of the PEC plane,

$$\mathbf{J}^r_{\mathrm{TE}}(\mathbf{r})=-\mathbf{u}_x IL\delta(\mathbf{r}+\mathbf{u}_z h)+\mathbf{u}_x IL\delta'(z+h)\frac{x}{2\pi\rho^2}. \tag{38}$$

Thus, the total image has the form

$$\mathbf{J}^r(\mathbf{r})=-\mathbf{u}_x IL\delta(\mathbf{r}+\mathbf{u}_z h)+\mathbf{u}_x 2IL\delta'(z+h)\frac{x}{2\pi\rho^2}. \tag{39}$$

The derivative of the δ function indicates that the surface current is formed of two current layers flowing in opposite directions.

IV. PROPAGATION IN UNIAXIAL HALF-SPACE

Let us now consider the realization of the planar DB boundary in terms of the interface of a uniaxial anisotropic medium defined by permittivity and permeability dyadics (10). The ideal DB boundary is obtained in the limit $\epsilon_z\to 0$ and $\mu_z\to 0$. However, let us consider the approximate case with small values of these parameters.

Since the plane-wave fields in any medium satisfy the orthogonality conditions

$$\mathbf{E}\cdot\mathbf{B}=0, \quad \mathbf{H}\cdot\mathbf{D}=0, \tag{40}$$

for the anisotropic medium (10) we have

$$\mathbf{E}\cdot(A\bar{\bar{\mu}}+B\bar{\bar{\epsilon}}^T)\cdot\mathbf{H}=0 \tag{41}$$

for any coefficients A and B. Choosing $A=\epsilon_t$ and $B=-\mu_t$ this reduces to

$$(\epsilon_t\mu_z-\mu_t\epsilon_z)E_zH_z=0. \tag{42}$$

Assuming that the parenthetical term does not vanish (which corresponds to the special case of an affine-isotropic medium [1]), any plane wave must satisfy either $E_z=0$ or $H_z=0$, which means that the plane waves have either TE or TM polarization with respect to the axial direction. Thus, a TE wave incident at the interface of such a medium is reflected and transmitted as a TE wave and similar rule is valid for the TM wave.

Let us only consider the TE wave, the TM-wave case can be analyzed through similar steps. Assuming the wave vector of the form

$$\mathbf{k}=\mathbf{u}_x k_x+\mathbf{u}_z\beta, \tag{43}$$

the field vectors of the TE field can be expressed as

$$\mathbf{E}=\mathbf{u}_y E, \tag{44}$$

$$\mathbf{H}=\mathbf{u}_x H+\mathbf{u}_z H_z=H[\mathbf{u}_x-(k_x\mu_t/\beta\mu_z)\mathbf{u}_z]. \tag{45}$$

The last expression is due to the condition $\mathbf{k}\cdot\mathbf{B}=0$. The Maxwell equations now reduce to two scalar equations

$$\beta E=-\omega\mu_t H, \tag{46}$$

$$[\beta+(k_x^2\mu_t/\beta\mu_z)]H=-\omega\epsilon_t E, \tag{47}$$

whence the dispersion equation becomes

$$\beta^2+(\mu_t/\mu_z)k_x^2=\omega^2\mu_t\epsilon_t=k_t^2. \tag{48}$$

The k-vector diagram is a spheroid. For axial propagation $k_x=0$ (TEM wave) we have $\beta=k_t$ and for transverse propagation $\beta=0$, $k_x=k_t\sqrt{\mu_z/\mu_t}=\omega\sqrt{\mu_z\epsilon_t}$. In the limit $\mu_z/\mu_t\to 0$ the spheroid becomes a needle whence real β values are obtained only for small k_x values.

Solving the axial propagation factor β from (48) as

$$\beta = \pm\sqrt{k_t^2 - k_x^2\mu_t/\mu_z}, \tag{49}$$

real values for β are only obtained for real k_x values satisfying

$$k_x^2 < k_t^2\mu_z/\mu_t. \tag{50}$$

If μ_z/μ_t is small, real propagation is possible only in a narrow cone of k vectors around the z axis. For larger k_x values β becomes imaginary, and in the limit $\mu_z/\mu_t \to 0$ β obtains a large imaginary value

$$\beta \to jk_x\sqrt{\mu_t/\mu_z}. \tag{51}$$

The sign is chosen so that the field decays in the direction of negative z.

Denoting

$$\eta_t = \sqrt{\mu_t/\epsilon_t}, \tag{52}$$

the ratio of the transverse fields is

$$Z_{\mathrm{TE}} = E/H = -\eta_t k_t/\beta, \tag{53}$$

which becomes small and imaginary for small μ_z/μ_t,

$$Z_{\mathrm{TE}} \to j\omega\sqrt{\mu_t\mu_z}/k_x \to 0. \tag{54}$$

For the TM wave the results are dual to those of the TE wave [1]. The dispersion equation has the form

$$\beta^2 + (\epsilon_t/\epsilon_z)k_x^2 = k_t^2. \tag{55}$$

V. REFLECTION FROM THE INTERFACE

Let us now consider the plane wave (12) incident at the interface $z=0$ of the uniaxial medium. Because there is no coupling between the TE and TM waves at the interface, we can consider these components separately. Let us assume that the lateral component of the wave vector, $\mathbf{K}=\mathbf{u}_x k_x$ is real. Because of continuity of the transverse components of the total $\mathbf{E}$ and $\mathbf{H}$ fields across the interface, their ratio must equal the transverse wave impedance of the wave transmitted to the uniaxial medium. For the TE polarization we have

$$\frac{E_y^i + E_y^r}{H_x^i + H_x^r} = Z_{\mathrm{TE}} = -\eta_t k_t/\beta = -\frac{\eta_t k_t}{\sqrt{k_t^2 - k_x^2\mu_t/\mu_z}}. \tag{56}$$

From the Maxwell equations the incident and reflected wave amplitudes satisfy

$$k_z^i E_y^i = -k\eta H_x^i, \quad k_z^i E_y^r = k\eta H_x^r. \tag{57}$$

Defining

$$k_z^i = k\cos\theta^i, \quad k_x = k\sin\theta^i, \tag{58}$$

where θ^i is the angle of incidence (see Fig. 1), the reflection coefficient

$$R_{\mathrm{TE}} = E_y^r/E_y^i = -H_x^r/H_x^i, \tag{59}$$

can be solved from (53) as

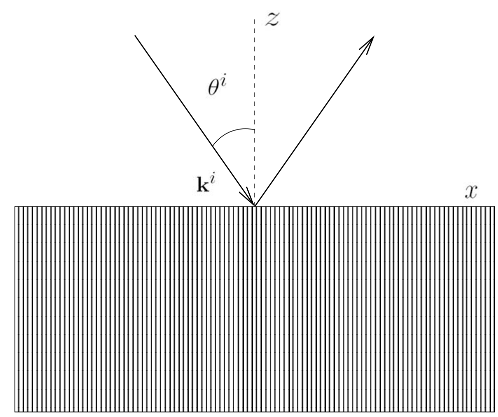

FIG. 1. Geometry of the reflection from a DB surface.

$$R_{\mathrm{TE}} = \frac{Z_{\mathrm{TE}}\cos\theta^i + \eta}{Z_{\mathrm{TE}}\cos\theta^i - \eta} = \frac{\eta_t\cos\theta^i - \eta\sqrt{1-\sin^2\theta^i\mu\epsilon/\mu_z\epsilon_t}}{\eta_t\cos\theta^i + \eta\sqrt{1-\sin^2\theta^i\mu\epsilon/\mu_z\epsilon_t}}. \tag{60}$$

It is of interest to consider the reflection coefficient for different angles of incidence. For grazing incidence, $\theta^i=\pi/2$, we obviously have $R_{\mathrm{TE}}=-1$ or the boundary acts as a PEC plane. For smaller angles R_{TE} is complex with $|R_{\mathrm{TE}}|=1$ until the square root in (60) becomes zero, which happens at the angle $\theta^i=\theta_{\mathrm{TE}}$ defined by

$$\sin^2\theta_{\mathrm{TE}} = \frac{\mu_z\epsilon_t}{\mu\epsilon}. \tag{61}$$

For this angle we have $\beta=0$ and $R_{\mathrm{TE}}=+1$, or the boundary acts as a PMC plane. For still smaller angles R_{TE} becomes real and smaller than +1 until for normal incidence $\theta^i=0$ we have

$$R_{\mathrm{TE}} = \frac{\eta_t - \eta}{\eta_t + \eta}. \tag{62}$$

If $\eta_t=\eta$, this limit is zero, whence there is no reflection for TE waves. Obviously, this limit is also valid for TM waves because the field is TEM for normal incidence. For small values of μ_z/μ_t the angle θ_{TE} is small as

$$\theta_{\mathrm{TE}} \approx \sqrt{\frac{\mu_z}{\mu_t}}\frac{k_t}{k}, \tag{63}$$

which means that the above-mentioned change in the reflection coefficient happens within a small cone of incidence angles. Figure 2 illustrates the absolute value and the phase angle of the reflection coefficient for the TE-polarized field for three values of the relative axial permeability μ_z/μ_t. It can be clearly seen that when μ_z/μ_t decreases, the angular filtering region around $\theta^i=0$ becomes narrower. In Fig. 3 the reflection coefficient is shown in the complex plane. Considering the impedance (53) of the TE wave which for oblique incidence can be approximated by (54) and comparing with (56), we can interpret the (almost) total reflection for large angles of incidence as being due to the impedance mismatch while the opposite is true for normal incidence when $\eta_t=\eta$.

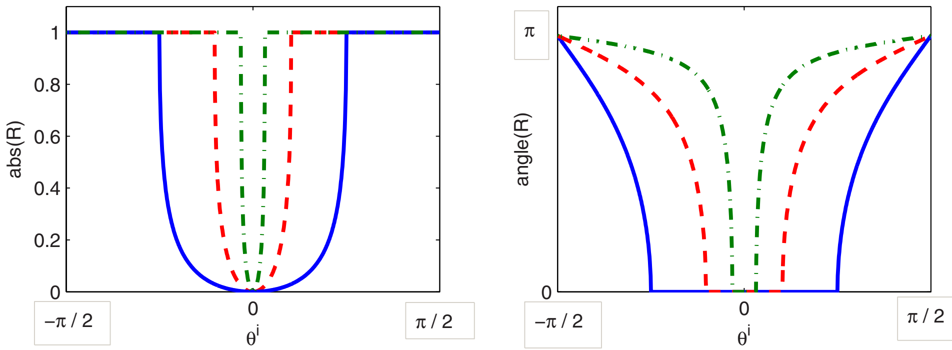

FIG. 2. (Color online) The TE reflection coefficient from a DB surface with parameters $\epsilon_t = \epsilon$, $\mu_t = \mu$, and varying axial values of the relative permeability $\mu_z/\mu_t = 0.5$ (solid line), 0.1 (dashed line), and 0.01 (dotted-dashed line).

Similar analysis can be applied to the TM component with interchanged permittivity and permeability parameters. Taken together we can state that, for $\eta_t = \eta$ the medium acts as a spatial filter for incident plane waves. For small μ_z and ϵ_z the fields are completely reflected by the interface for angles of incidence greater than θ_{TE} and θ_{TM}. Within the narrow cones defined by these angles the field components are let through as multiplied by the corresponding transmission functions $T_{TE} = 1 - R_{TE}$ and $T_{TM} = 1 - R_{TM}$. A slab of this kind of medium may have application in narrowing radiation beams of directional antennas or reducing their sidelobes, with the expense of lowering the effectivity due to reflection of the radiated power outside the filtering angular cone. A similar filtering effect has been previously reported for isotropic epsilon-near-zero metamaterials in [15,16]. This effect is different from that due to the metamaterial layer described in [21] which is based on concentrated refraction in which the sidelobes are refracted instead of being filtered out. Similar effect in terms of a layer of magnetoelectric chiral medium was described in [22].

In the present analysis we have not taken into account the limitations of real media as being dispersive and lossy. However, for a narrow frequency band with relatively constant permittivity and permeability values the dispersion and loss effects may be neglected as the first approximation.

VI. CONCLUSION

Boundary conditions requiring vanishing of the **D** and **B** vector components normal to the boundary were studied in this paper. The conditions, labeled here as those of DB boundary, were briefly introduced in [11] as arising at an interface of an exotic bianisotropic medium. In the present paper realization of the same conditions is considered in terms of the interface of a simple uniaxial anisotropic medium in the case when the axial permittivity and permeability parameter values tend to zero. Fabrication of such media appears possible by alternating layers of metamaterials with positive and negative values of the permittivity and permeability parameters. The most interesting property of a planar DB boundary is that it appears as a perfect electric and magnetic conductor (PEC and PMC) boundary for fields with respective TE and TM polarizations. This allows one to solve the basic problem of a source above a DB plane through image theory by applying the known TE and TM decomposition theory for sources. Finally, wave reflection from an interface of the uniaxial anisotropic half-space with small axial components is analyzed. It is shown that, for this approximate realization of the DB boundary, there is a narrow cone around the normal incidence where the wave is transmitted through the interface. Since a layer of such a medium acts as a spatial filter it may have application for narrowing the beam or reducing the sidelobes of an antenna. Also, since the Poynting vector does not have a component along the DB boundary, interaction of aperture antennas on such a surface can be reduced.

APPENDIX: GENERAL FIELDS AT DB BOUNDARY

To study fields from general sources above a planar DB boundary, we apply the known property that any field in a homogeneous isotropic medium outside the sources can be expressed in terms of two scalar potential quantities $\psi(\mathbf{r})$, $\phi(\mathbf{r})$ as [4,23,24]

$$\mathbf{E}(\mathbf{r}) = \nabla \times (\mathbf{u}_z \psi) + \nabla \times [\nabla \times (\mathbf{u}_z \phi)], \qquad \text{(A1)}$$

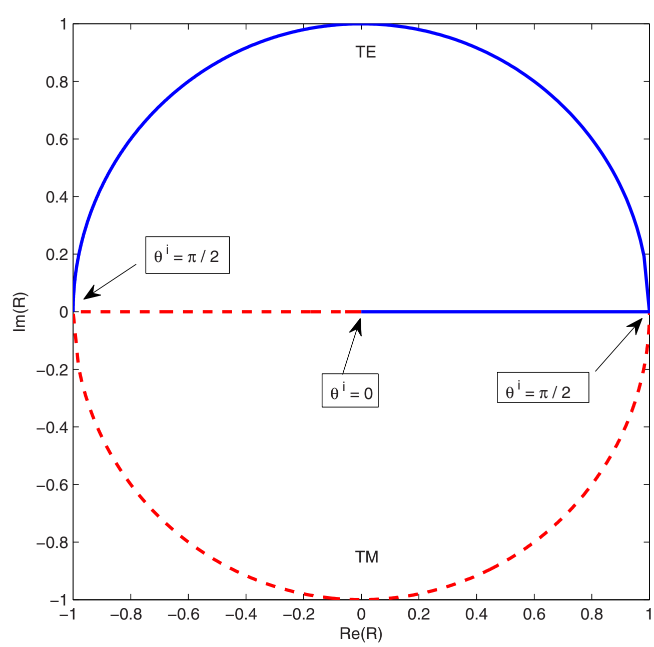

FIG. 3. (Color online) The TE reflection coefficient from a DB surface with parameters $\epsilon_t = \epsilon$, $\mu_t = \mu$ in the complex plane (solid line). Note that the TM reflection (dashed line) follows the same path multiplied by −1.

$$\mathbf{H}(\mathbf{r}) = \frac{j}{\omega\mu}\nabla \times [\nabla \times (\mathbf{u}_z\psi)] + j\omega\epsilon\nabla \times (\mathbf{u}_z\phi). \quad \text{(A2)}$$

To satisfy the Maxwell equations, the potential functions must satisfy outside the sources the Helmholtz equation

$$(\nabla^2 + k^2)\psi = 0, \quad (\nabla^2 + k^2)\phi = 0. \quad \text{(A3)}$$

Obviously, the ψ function defines the TE component, and the ϕ function the TM component, of the electromagnetic field,

$$\mathbf{E}_{\mathrm{TE}}(\mathbf{r}) = \nabla \times (\mathbf{u}_z\psi), \quad \text{(A4)}$$

$$\mathbf{E}_{\mathrm{TM}}(\mathbf{r}) = \nabla \times [\nabla \times (\mathbf{u}_z\phi)], \quad \text{(A5)}$$

$$\mathbf{H}_{\mathrm{TE}}(\mathbf{r}) = \frac{j}{\omega\mu}\nabla \times [\nabla \times (\mathbf{u}_z\psi)], \quad \text{(A6)}$$

$$\mathbf{H}_{\mathrm{TM}}(\mathbf{r}) = j\omega\epsilon\nabla \times (\mathbf{u}_z\phi). \quad \text{(A7)}$$

Let us study the DB boundary conditions on the plane $z=0$ for the TE part. Since the electric field satisfies $\mathbf{u}_z \cdot \mathbf{E}_{\mathrm{TE}}=0$ everywhere, we must require that the condition

$$-j\omega\mathbf{u}_z \cdot \mathbf{B}_{\mathrm{TE}} = \mathbf{u}_z \cdot \nabla \times \mathbf{E}_{\mathrm{TE}} = \mathbf{u}_z \cdot \{\nabla \times [\nabla \times (\mathbf{u}_z\psi)]\} = \partial_z^2\psi - \nabla^2\psi = 0 \quad \text{(A8)}$$

be valid at the boundary $z=0$. Applying (A3) yields the condition

$$\nabla_t^2\psi(\mathbf{r}) = 0, \quad \mathbf{u}_z \cdot \mathbf{r} = 0. \quad \text{(A9)}$$

Since there are no sources at the boundary, the potential cannot have any singularities on the plane $z=0$. Also, assuming that the sources have finite extent close to the z axis, the field must reduce to zero for increasing distance $\sqrt{x^2+y^2}$. It is known that a function $f(x,y)$ satisfying the Laplace equation in a region on a plane has its maxima and minima at the boundary curve. Taking the boundary far from the axis shows us that the minima and maxima must tend to zero. This leaves us with the following solution at the boundary:

$$\psi(\mathbf{r}) = 0, \quad \Rightarrow \mathbf{u}_z \times \mathbf{E}_{\mathrm{TE}}(\mathbf{r}) = 0, \quad \mathbf{u}_z \cdot \mathbf{r} = 0. \quad \text{(A10)}$$

Thus, the TE field $\mathbf{E}_{\mathrm{TE}}(\mathbf{r})$, $\mathbf{H}_{\mathrm{TE}}(\mathbf{r})$ satisfies the PEC conditions at the boundary. The magnetic field at the boundary becomes

$$\mathbf{H}_{\mathrm{TE}}(\mathbf{r}) = \frac{j}{\omega\mu}\nabla \times [\nabla \times (\mathbf{u}_z\psi)] = \frac{j}{\omega\mu}\nabla_t\partial_z\psi, \quad \mathbf{u}_z \cdot \mathbf{r} = 0, \quad \text{(A11)}$$

which is tangential to the boundary and does not vanish in the general case.

After a similar analysis one can show that the TM field $\mathbf{E}_{\mathrm{TM}}(\mathbf{r})$, $\mathbf{H}_{\mathrm{TM}}(\mathbf{r})$ satisfies the PMC condition at the boundary $z=0$.

[1] I. V. Lindell, *Methods for Electromagnetic Field Analysis* (Oxford University Press, Oxford, 1992).
[2] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), p. 34.
[3] R. E. Collin and F. J. Zucker, *Antenna Theory, part 2* (McGraw-Hill, New York, 1969), Chap. 21.
[4] R. F. Harrington, *Time-Harmonic Electromagnetic Fields* (*McGraw-Hill*, New York, 1961), p. 34.
[5] D. Sievenpiper *et al.*, IEEE Trans. Microwave Theory Tech. **47**, 2059 (1999).
[6] I. V. Lindell and A. H. Sihvola, IEEE Trans. Antennas Propag. **53**, 3005 (2005).
[7] I. V. Lindell and A. H. Sihvola, IEEE Trans. Antennas Propag. **53**, 3012 (2005).
[8] D. J. Hoppe and Y. Rahmat-Samii, *Impedance Boundary Conditions in Electromagnetics* (Taylor and Francis, Washington, 1995).
[9] G. Pelosi and P. Y. Ufimtsev, IEEE Antennas Propag. Mag. **38**, 31 (1996).
[10] I. V. Lindell and A. H. Sihvola, IEEE Trans. Antennas Propag. **54**, 3669 (2006).
[11] I. V. Lindell and A. H. Sihvola, IEEE Trans. Antennas Propag. (to be published).
[12] F. W. Hehl and Yu. N. Obukhov, *Foundations of Classical Electrodynamics* (Birkhäuser, Boston, 2003).
[13] I. V. Lindell, Electromagn. Waves: Progress Electromagn. Res. **PIER 57**, 1 (2006).
[14] *Metamaterials: Physics and Engineering Explorations*, edited by N. Engheta and R. Ziolkowski (IEEE Press, Wiley, New York, 2006).
[15] M. Silveirinha and N. Engheta, Phys. Rev. B **75**, 075119 (2007).
[16] A. Alù, M. G. Silveirinha, A. Salandrino, and N. Engheta, Phys. Rev. B **75**, 155410 (2007).
[17] R. W. Ziolkowski, *Proceedings of Metamaterials 2008* (Pamplona, Italy, 2008).
[18] P. S. Kildal, Electron. Lett. **24**, 168 (1988).
[19] P. S. Kildal and A. Kishk, Appl. Comput. Electromagn. Soc. J. **18**, 32 (2003).
[20] I. V. Lindell, IEEE Trans. Antennas Propag. **36**, 1382 (1988).
[21] S. Enoch, G. Tayeb, P. Sabouroux, N. Guerin, and P. Vincent, Phys. Rev. Lett. **89**, 213902 (2002).
[22] C. W. Qiu, H. Y. Yao, L. W. Li, S. Zouhdi, and T. S. Yeo, Phys. Rev. B **75**, 155120 (2007).
[23] D. S. Jones, *The Theory of Electromagnetism* (Pergamon, Oxford, 1964), p. 19.
[24] I. V. Lindell and G. Dassios, J. Electromagn. Waves Appl. **15**, 1281 (2001).