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Synchronization of modulated traveling baroclinic waves in a periodically forced, rotating fluid annulus

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Frequency entrainment and nonlinear synchronization are commonly observed between simple oscillatory systems, but their occurrence and behavior in continuum fluid systems are much less well understood. Motivated by possible applications to geophysical fluid systems, such as in atmospheric circulation and climate dynamics, we have carried out an experimental study of the interaction of fully developed baroclinic instability in a differentially heated, rotating fluid annulus with an externally imposed periodic modulation of the thermal boundary conditions. In quasiperiodic and chaotic amplitude-modulated traveling wave regimes, the results demonstrate a strong interaction between the natural periodic modulation of the wave amplitude and the externally imposed forcing. This leads to partial or complete phase synchronization. Synchronization effects were observed even with very weak amplitudes of forcing, and were found with both 1:1 and 1:2 frequency ratios between forcing and natural oscillations.

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INTRODUCTION

The addition of a periodic forcing to a system, which undergoes natural oscillations is a topic that has attracted interest in a wide variety of fields. It leads naturally to the concept of synchronization, which can be understood as an adjustment of rhythms of oscillating objects due to their weak interaction. The first scientist to document a synchronization phenomenon was probably Huygens in the 17th century [1], who discovered that a pair of pendulum clocks hanging from a common support moved consistently in antiphase with each other. If the motion was disturbed, the coupled motion reestablished itself after a short period of time—the clocks were synchronized. Recently, the ideas of synchronization have been applied to a wide variety of fields including physiology [2], neurology [3,4], zoology [5,6], chemistry [7], engineering, and physics [8–11].

The possible occurrence of regular fluctuations in the weather has fascinated weather watchers for a long time, motivating a sustained effort to demonstrate cyclic behavior in the climate system over a vast range of time scales from days to $O(10^5)$ years (see, e.g., Burroughs [12] for a review). Such cycles include stratospheric and tropospheric quasibiennial oscillations, the El Niño Southern Oscillation, the intraseasonal zonal index cycles, the North Atlantic Oscillation–Arctic Oscillation [13,14], and oceanic tropical instability waves [15]. On much longer time scales, examinations of ice and deep ocean cores reveal possible oscillations of the Earth's climate with periods from 10^3 to 10^5 years. It is widely thought that at least some aspects of these cycles are due to interactions between the internal variability of the climate and an imposed periodic forcing, such as the annual cycle or longer period variations in the Earth's orbit and rotation, leading, e.g., to the occurrence of certain flow changes occurring at a particular time of year. The mechanisms causing certain spatial patterns in a continuous flow with many potential degrees of freedom (DOFs) to synchronize spontaneously with external stimuli or other spatial modes within a complex flow are not well understood, however, in contrast to much simpler, discrete systems with small numbers of DOFs.

Studies of these and similar climate phenomena have typically been carried out using either direct observations of the climate system or various kinds of numerical models. However, an alternative approach to the study of such circulation systems is via well-controlled laboratory experiments on simple experimental analogs, such as the differentially heated, rotating annulus. In this system, a fluid is contained within the annular channel between two rotating, upright, coaxial cylinders and subject to *horizontal* differential heating between the inner and outer boundaries, thereby closely capturing some of the main physical attributes of potential energy-releasing instabilities in the mid-latitude atmospheric circulation of the Earth and other planets. This system has been studied extensively in this context for more than 50 years [16], and is well known to exhibit a rich variety of flow regimes and associated bifurcations, including steady, periodic, chaotic, and turbulent flows, depending upon the imposed parameters (principally the thermal contrast and background rotation speed).

Although most previous experimental studies using this system have been carried out using steady boundary conditions, a few have used cyclic forcing, either by varying the lateral boundary conditions of a differentially heated annulus or by varying the lid rotation of a two-layer, mechanically driven version of the experiment. However, these experiments [17,18] were largely concerned with wave number transitions, hysteresis effects, and the stability of particular wave modes. The periods of the forcing tended to be long and on the order of the (very slow) pattern drift period, rather than the much shorter periods associated with nonlinear amplitude modulations.

The effect of more rapid modulations of the forcing in such systems has not so far been investigated in the laboratory, although some earlier studies have been carried out using simple, low-dimensional numerical models of nonlinear baroclinic waves [19,20]. Both Hart [19] and Eccles *et al.* [20] have studied the effects of modulating the zonally symmetric differential heating or vertical shear on such systems. These model studies revealed the possibility of a rich variety of synchronization phenomena when the period of forcing is comparable to that of the intrinsic nonlinear variability, including frequency entrainment of natural amplitude modulations, complete or partial phase synchronization (PS), and suppression or stimulation of chaotic behavior.

In the present work, laboratory experiments have been carried out by periodically varying the imposed thermal contrast in rotating annulus experiments on time scales that are directly comparable to natural oscillatory periods associated with internal nonlinear variability of baroclinic instabilities. We have also considered effects of much smaller amplitude perturbations than in previous work, in order to investigate possible synchronization phenomena by analogy with model studies [19,20]. In the following sections we briefly describe the experimental setup, and present results that demonstrate frequency entrainment and complete and partial PS that are directly analogous to these model studies.

THE ROTATING ANNULUS EXPERIMENT

The apparatus used is largely as described by Früh and Read [21] and comprises an aqueous fluid placed in a cylindrical annular channel that can be rotated about its vertical axis of symmetry. The vertical side walls, which can be heated and cooled, consist of two coaxial brass cylinders with boundaries at radii $a=25$ mm and $b=80$ mm and the horizontal lid and base (both in contact with the convecting fluid) are thermally insulating. The annulus is mounted on a turntable such that its vertical axis of symmetry coincides with the axis of rotation. The apparatus rotated anticlockwise with the outer wall usually warmer than the inner wall, at a rotation rate Ω between 0.5 and 4.0 rad s^{-1}. The annulus itself and some of the heating elements were placed in a temperature-controlled enclosure, to provide a thermal environment stable to about ± 0.5 K, and the laboratory was also temperature controlled to within about ± 1.5 K. The thermal boundary conditions (leading to thermal contrast ΔT) were under computer control via a commercial process controller (Eurotherm 2704), allowing independent, real-time control of the temperature of each boundary over a range of up to ± 5 K via in-line heaters and platinum resistance thermometer (PRT) temperature sensors in the coolant circuit. During the course of the experiment the boundary conditions of the annulus (inner and outer wall temperatures), rotation rate, coolant flow rates, and environment temperatures were continuously measured and recorded via a data logger in the rotating frame. The set point temperature for each boundary could be varied in real time on command from a PC, allowing sinusoidal variations in temperature with a prescribed amplitude and frequency of the form $\Delta T = \Delta T_0 + \epsilon \sin \gamma t$. In practice, periods as short as $\tau_\gamma = 2\pi/\gamma = 150$ s were attainable, although the amplitude ϵ of the actually imposed boundary variations was reduced significantly with periods much less than 200 s, owing to the thermal inertia and transit time of coolant through the apparatus.

Experiments were typically carried out by initial spin-up to a fixed point in parameter space, in order to establish a desired initial flow regime. The boundary temperatures were then periodically modulated with a given amplitude and frequency for several hours. Subsequent experiments were carried out, incrementally changing either the amplitude or frequency of forcing, while the temperature response of the fluid was measured using a symmetric array of 32 thin-wire thermocouple probes, equally spaced in azimuth and located at mid-height and mid-radius in the annular channel. Thus, the instantaneous amplitude and phase of each azimuthal wave number component could be determined at regular intervals (typically every 10–12 s) throughout each run.

RESULTS

It is generally found that the strongest synchronization effects occur between two weakly coupled oscillators when their natural frequencies of oscillation are close to a 1:1 ratio. In the present case, therefore, periodic modulation of the boundary temperatures was investigated using a sinusoidal function with a period close to that of the natural period τ_v of amplitude modulation found in the *amplitude vacillation* (AV) regime of the rotating annulus. These AV regimes are believed to be primarily produced by nonlinear wave–mean flow interactions, perhaps modified by wave-wave interactions as suggested in previous experiments in the rotating annulus using high-Pr fluids [16,21,22]. However, the detailed mechanisms for these phenomena are still not fully understood in any system. A rough estimate of time scales can be obtained from the advection time scale for the Ekman thermal boundary layer circulation $\sim \mathrm{Ra}^{-1/2} L^2/\kappa$, where L is the apparatus dimension, κ is the thermal diffusivity, and Ra the Rayleigh number. Given typical values for this experiment ($\mathrm{Ra} \sim 10^7$, $L \sim 10$ cm, $\kappa = 10^{-7}$ m^2 s^{-1}), this suggests time scales of several tens of seconds, comparable to the observed vacillation time scales. Both forcing frequency and amplitude were varied to examine the range of these parameters over which complete or partial synchronization could be observed.

Figure 1 illustrates an example of an amplitude vacillation subject to periodic forcing of varying frequency γ at a fixed amplitude $\epsilon = 0.5$ K with $\Delta T_0 \approx 5.6$ K. At each time step of 6 s, an azimuthal wave number spectrum of temperature was obtained by discrete Fourier transform of the 32 near-simultaneous temperature measurements, from which a time series of the amplitude of each wave number could be obtained. Figure 1(a) shows the frequency spectrum obtained for the dominant wave number $m=3$ with steady boundary conditions ($\epsilon = 0$), in which the frequency of amplitude modulation is clearly seen around 5×10^{-3} Hz ($\tau_v \simeq 180$ s). A strong peak is also observed around 2.5×10^{-4} Hz, matching the drift frequency of the $m=3$ flow pattern around the apparatus. This is believed to be an artifact, and appears in the $m=3$ amplitude spectrum mainly because of small departures from azimuthal symmetry and uniform radial position in the thin-wire array of probes.

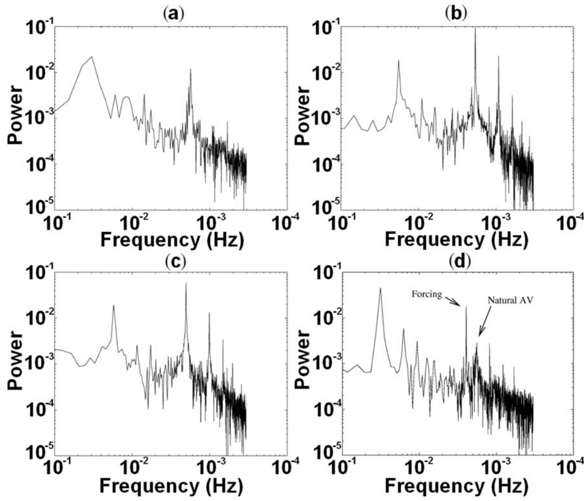

FIG. 1. Frequency spectra of variations of the temperature amplitude of $m=3$ in the rotating annulus within the 3AV regime at $\Delta T_0 \approx 5.6$ K, (a) without periodic forcing, and (b)–(d) with periodic forcing of amplitude $\epsilon=0.5$ K ($\sim 0.08\Delta T$) and period τ_γ (b) 180, (c) 200, and (d) 240 s. The total duration of the acquisition for each case, from where these spectra were calculated, was of 21 600 s.

Figures 1(b)–1(d) show the corresponding frequency spectra of the amplitude of $m=3$ but with periodic forcing of amplitude $\epsilon=0.5$ K and periods $\tau_\gamma=180$, 200, and 240 s. Separate peaks corresponding to both the forcing and natural AV modulation are clearly apparent in Fig. 1(d), together with various higher harmonics, suggesting that the two oscillations are uncoupled. In Figs. 1(b) and 1(c), however, they are seen to merge into a single, sharp peak of significantly enhanced amplitude (again with higher harmonics). The latter two cases would seem to suggest an active, resonant interaction between the forced and natural oscillations, possibly indicative of synchronization. The drift frequency peaks, too, are shifted to somewhat higher frequencies in the coupled cases of Figs. 1(b) and 1(c), indicating a complex dynamical response to the applied forcing.

Such a picture is confirmed in Fig. 2(i), which shows time sequences of the phase difference $\Delta\phi$ between the external forcing γt and that of the natural AV modulation. $\Delta\phi$ is almost constant in time in Figs. 2(a)–2(c), apart from some weak fluctuations on the time scale of individual vacillation periods, strongly indicating almost complete PS of the vacillation with the forcing, though the strength of the phase fluctuations increases slightly with detuning. In Fig. 2(d), however, with $\tau_\gamma=240$ s, $\Delta\phi$ appears to remain constant for short intervals but then decreases rapidly to another value in an irregular series of small steps. Such a behavior is typical of *partial* PS [11] when the oscillations are sufficiently detuned for synchronization to break down intermittently. This breakdown of synchronization with increasing detuning is also clearly apparent in histograms of $\Delta\phi$, as shown in Fig. 2(ii), for which the strongly phase-synchronized cases Figs. 2(a)–2(c) show sharply peaked histograms, gradually shifting in mean phase with detuning, in contrast to the much broader peak found with weakly synchronized case Fig. 2(d).

Based on the time variations of $\Delta\phi$, we can therefore classify cases (a)–(c) in Fig. 2 as complete PS, and case (d)

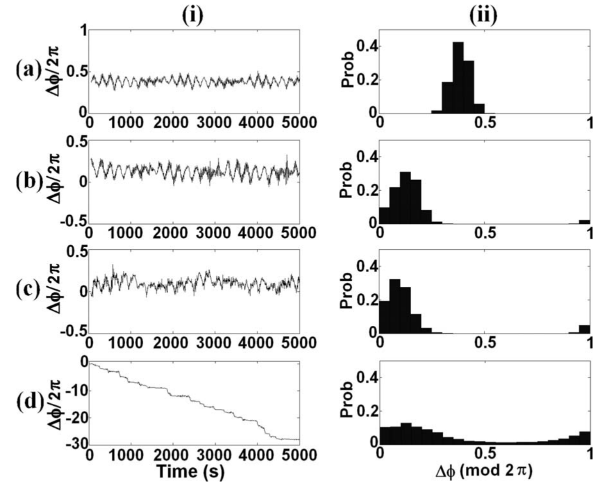

FIG. 2. (i) Time variation of the phase difference $\Delta\phi$ between the external forcing and the amplitude modulation of $m=3$, and (ii) histograms of $\Delta\phi$ (mod 2π), for periodic forcing with $\epsilon=0.5$ K and period τ_γ (a) 180, (b) 200, (c) 220, and (d) 240 s, showing the breakdown of complete to partial phase synchronization.

as partial PS. In Fig. 3 we show results of a set of experiments covering a range in both ϵ and τ_γ. These results cover cases for which $\tau_\gamma \geqslant \tau_v$ since, for practical reasons, shorter forcing periods were not possible to achieve with the present apparatus.

The results clearly show a broadening of the fully synchronized region with increasing forcing amplitude ϵ, much as expected for a classical Arnol'd tongue. The fully synchronized region is bounded by a narrow strip characterized by partial PS or phase slips, before giving way to a broad region without synchronization. Although this appears at first sight as a classical Arnol'd tongue, the full PS region does not appear to extend precisely to zero with decreasing ϵ,

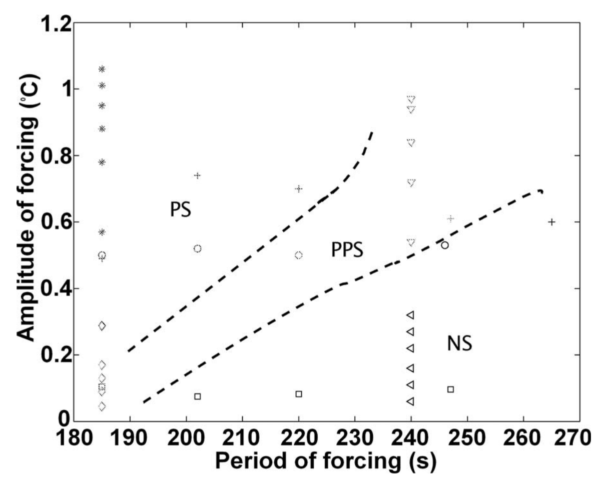

FIG. 3. Parametric variation of partial and complete phase synchronization between the periodic forcing and temperature amplitude of $m=3$ as a function of amplitude ϵ and frequency ω of the forcing, close to the 1:1 ratio between forcing and the natural vacillation frequencies. Dotted lines represent boundaries between different synchronized states: PS denotes full (phase) synchronization, PPS partial phase synchronization, and NS unsynchronized behavior. Different symbols denote different experimental runs.

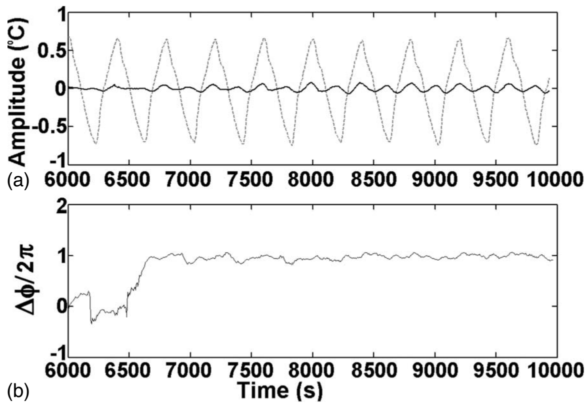

FIG. 4. (a) Extract of the temperature variation of the baroclinic wave (continuous line) and the imposed external forcing (broken line). (b) Time variation of the phase difference $\Delta\phi$ between the external forcing and the amplitude modulation of $m=3$, for periodic forcing with $\epsilon=0.6$ K and period $\tau_\gamma=400$ s. As before, $\tau_v \simeq 180$ s.

suggesting that internal noise and fluctuations in the experimental control may be sufficient to interrupt the full PS state when the forcing amplitude is very weak.

Evidence for similar synchronization effects was also found with much slower forcing frequencies, close to the case where $\tau_\gamma \simeq 2\tau_v$, in which the AV would synchronize with the *first superharmonic* of the forcing frequency. In this case, $\Delta\phi$ was redefined as $\Delta\phi = 2\phi_\gamma - \phi_v$, where $\phi_\gamma = \gamma t$ and ϕ_v is the phase of the AV amplitude modulation. Again, complete synchronization was found when τ_γ was sufficiently close to $2\tau_v$, manifest by the approximate stationarity of $\Delta\phi$. An example of this state is presented in Fig. 4.

DISCUSSION

In this study we have investigated the effects of imposing a small amplitude periodic modulation in the thermal boundary conditions of a differentially heated, rotating annulus experiment, operated in a regime that favors nonlinear, traveling baroclinic waves with a spontaneous periodic amplitude modulation. When the frequency of the imposed periodic modulation (or its harmonic) approaches the natural modulation frequency, entrainment, and synchronization are found to occur in a manner that is consistent with recent model studies [20] of periodically forced, baroclinic wave-zonal flow oscillators. Depending on the strength of the forcing and detuning, perturbations were found not only to the amplitude variations of the wave, influencing the extent to which it remains quasiperiodic or chaotic, but also its propagation speed. There was also some evidence for the largest Lyapunov exponent of the resulting flow, measured from temperature time series following [23] (see also [22]), to vary depending on the strength of the forcing, though results were found to be quite sensitive to experimental noise. This study clearly serves to confirm and generalize the applicability of simple synchronized oscillator models to systems of traveling baroclinic waves and, by implication, to other forms of nonlinear traveling wave system in continuous fluids and plasmas.

The potential implications for other disciplines are manifold, including areas as diverse as climate dynamics, meteorology and oceanography, aerodynamics and plasma physics. In the case of the Earth's climate system, evidence for "teleconnections" in the form of correlated variations between meteorological variables in widely separated geographical regions is well known, though it has typically been attributed in the past to propagating planetary waves. The process identified here would extend the range of possible feedbacks and spread of influence between disparate regions unconnected by coherent wave patterns, with important implications for climate models and predictability.

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