

# Evoked magnetic fields of magnetoencephalography and their statistical property

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In an evoked magnetic field of magnetoencephalography a wave form is calculated by averaging. We propose that the wave form is deterministic in the case of 5 Hz periodical stimuli. We have found with statistical accuracy that the wave form of a somatosensory evoked magnetic field is deterministic in 5 Hz periodical median nerve stimuli, since any stationary process is decomposed into a deterministic part and a nondeterministic part from the Wold decomposition theorem. For the decorrelation method of blind source separation we have obtained several components which have nonzero wave forms. Via the selected components time series data of a somatosensory evoked magnetic field generated from somatosensory cortexes have been separated from background brain noise by using a  $T/k$  (fractional) type decorrelation method.

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## I. INTRODUCTION

There are two types of neuromagnetic fields in brain activities. One type is the spontaneous magnetic field and the other is the evoked magnetic field. The evoked magnetic field is classified by stimuli. There are mainly somatosensory, auditory, and visual stimuli for evoked magnetic fields. A sensory stimulus initially activates a small portion of the cortex. However, magnetic fields are generated by currents of various activities in a brain. Magnetic fields are detected by superconducting quantum interference devices (SQUIDs). Observed SQUIDs data are magnetoencephalography (MEG).

Evoked magnetic fields are often used to examine brain activities [1]. Amplitudes of evoked magnetic fields are smaller than those of spontaneous magnetic fields. Usually we use a wave form of MEG to examine the evoked magnetic field. The wave form is calculated from the average triggered by stimuli as will be seen in Eq. (1). Though we can obtain dynamical information from wave forms, there is still remaining dynamical information in MEG data. Therefore we should find a possibility to obtain dynamical information from MEG data of non-wave form. Along this direction we study statistical properties of MEG data in the present paper.

Let the number of SQUIDs be  $q$ , and that of active portions in a brain is  $r$ . When we want to examine brain activities, we must solve the inverse problem of  $r$  from  $q$ . Since there are a lot of background activities in the brain, usually  $q < r$ , that is, to examine brain activities is the underdetermined problem. In general it is difficult to examine brain activities from SQUID time series data inversely. In the case of an evoked magnetic field we can analyze a wave form obtained by averaging instead SQUID time series data. Since the wave form relates to a few activities in the brain, usually  $r < q$ , that is, activities of cortexes evoked by stimuli are studied by solving the overdetermined problem.

Evoked magnetic fields are responses for repeated stimuli, though the interstimulus interval is usually random to re-

move habituations. However, we can take advantage of the periodical property in mathematics, if stimuli are periodical. In 5 Hz periodical median nerve stimulus somatosensory activity is observed as a dipole pattern at the primary somatosensory cortex in the contralateral hemisphere [2], though somatosensory activities are known as contralateral primary somatosensory cortex, bilateral secondary somatosensory cortexes, and posterior parietal cortexes in random stimuli with interstimulus intervals more than 1 s [3].

In the present paper 5 Hz periodical median nerve stimuli for the somatosensory evoked field (SEF) were used, and statistical properties of SEF MEG data will be reported in Sec. III. From statistical properties some results will be found in a decorrelation method of blind source separation (BSS) in Sec. IV.

## II. SOMATOSENSORY EVOKED FIELD MAGNETOENCEPHALOGRAPHY

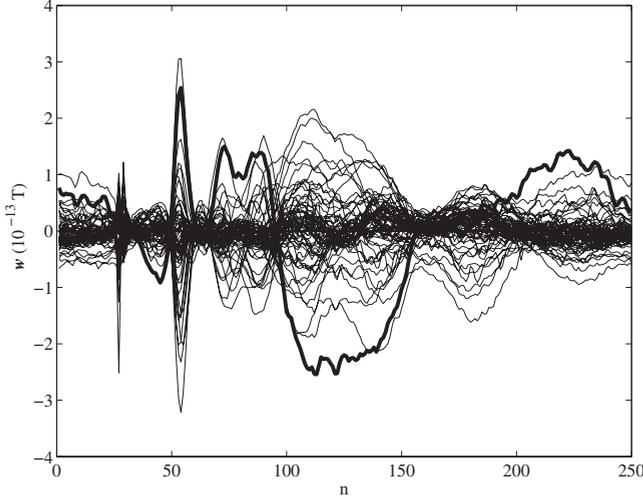
For seven healthy subjects the median nerve was stimulated electrically with a constant voltage, square-wave pulse of 0.2 ms duration delivered at the right wrist. Stimulus frequency was periodical 5 Hz ( $f_p=5$ ;  $T=200$  ms) and stimulus intensity was adjusted to the lowest level that would produce a twitch of the thumb. MEG data were recorded with a 64-channel whole-head MEG system (NeuroSQUID Model 100; CTF Systems Inc.). SQUID was the axial gradiometer type. MEG data were digitized with 1250 Hz sampling frequency ( $f_s=1250$ ). MEG data ( $N=125\,000$ ) during 100 s were recorded as a single sweep.

Let  $\mathbf{x}(n)$ ,  $n=1, \dots, N$  be sampled MEG data of  $q$  SQUIDs, i.e.,  $\mathbf{x} \in \mathbf{R}^{q \times N}$ . Here,  $q=64$ . Let an interval  $L:=Tf_s$ , and  $M:=N/L=500$  was a repetition number of 5 Hz periodical median nerve stimuli. Here, the interstimulus interval (ISI) was 200 ms, and  $L=250$ , since the sampling time  $\Delta t$  was 0.8 ms. For presignal processing of MEG data the mean value of each interval (200 ms) was set to be zero instead of a high pass filter: Since ISI was  $L$ ,  $\frac{1}{L} \sum_{m=1}^L \mathbf{x}(m)=0$  for each interval of MEG data. Then,  $\mathbf{x}(n)$  has zero mean.

### A. Wave form

After the presignal processing we can have a wave form defined by  $\mathbf{w} \in \mathbf{R}^{q \times L}$ ,

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FIG. 1. Wave form  $w(n)$  of SEF.

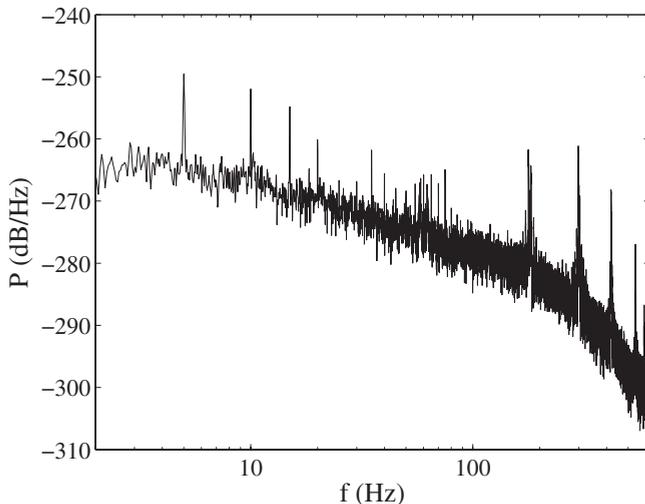
$$w(n) = \frac{1}{M} \sum_{m=1}^M x[n + (m-1)L] \quad \text{for } n = 1, \dots, L, \quad (1)$$

since ISI was fixed. The wave form of SEF is shown in Fig. 1 for one of seven subjects. Hereafter, we set  $\Delta t=1$  for the time representation of figures in the present paper. The electric stimulus time is at  $n=27$  (21.6 ms) in Fig. 1. The first peak of wave form at  $n=53$  is known as N20 [2,3] with latency 20.8 ms.

The bold line in Fig. 1 is a wave form at the 26th SQUID channel. The location of SQUID channels was reported in [4].

### B. Power spectral density

The power spectral density (PSD) of MEG at the 26th SQUID channel is calculated by the Welch method as  $P(f)$  and is illustrated in Fig. 2. PSD of MEG in 5 Hz periodical median nerve stimuli has a line spectrum at 5 Hz (fundamental) and its repeated higher harmonic frequencies. In PSD

FIG. 2. PSD  $P(f)$  of MEG at the 26th SQUID channel.

frequency modulation by the presignal processing is seen for 60 Hz power electrical noise. The effect of a 300 Hz low pass filter is also shown in PSD. If  $M$  times repeated wave form is defined by  $r_M w \in \mathbf{R}^{q \times N}$ , we have the periodicity of  $r_M w(n)$ ,

$$r_M w(n) = r_M w(n + mL), \quad m = 1, \dots, M-1. \quad (2)$$

Figure 2 suggests that  $r_M w(n)$  may be deterministic.

### III. WOLD DECOMPOSITION THEOREM AND SEF MEG

Let  $x(n)$  be a weakly stationary process. From the Wold decomposition theorem [5,6], a stationary process is uniquely decomposed into  $x(n)=y(n)+z(n)$ , where  $z(n)$  is nondeterministic and  $y(n)$  is deterministic. That is,  $y \perp z$ . The 5 Hz repetitive line spectra of Fig. 2 show us the effect of Eq. (2). This suggests that the wave form  $w(n)$  is deterministic, and lets us examine whether the repetitive wave form  $r_M w$  is deterministic or not.

The covariance between the 26th SQUID channel and any SQUID channel is defined by

$$V_{26*} = \frac{1}{N} \sum_{n=1}^N x_{26}(n)x_*(n) =: V(x_{26}, x_*), \quad (3)$$

under the ergodic assumption, where  $x_{26}$  means the 26th component of  $x$ . Here, the asterisk is a certain number of 1 ~ 64. If 500 times repeated wave form is defined by  $r_{500} w \in \mathbf{R}^{q \times N}$ , we have fluctuations,  $\delta x(n) := x(n) - r_{500} w(n)$ , by elimination of periodical functions. From the Wold decomposition theorem we have

$$\begin{aligned} V_{26*} &= V[(\delta x_{26} + r_{500} w_{26}), (\delta x_* + r_{500} w_*)] \\ &= V(\delta x_{26}, \delta x_*) + pV(r_{500} w_{26}, r_{500} w_*), \end{aligned} \quad (4)$$

where  $pV$  means the pseudocovariance for deterministic value. The direct sum of  $x(n)=y(n)+z(n)$  corresponds to Eq. (4). That is,  $V(\delta x, r_{500} w)=0$ .

To understand that SEF MEG satisfy the decomposition of Wold theorem let us examine correlation functions of MEG in the following. The correlation functions of MEG between the 26th SQUID channel and any SQUID channel are defined by

$$C_{26*}(n) = \frac{1}{N} \sum_{m=1}^{N-n} x_{26}(m)x_*(m+n) =: C\{x_{26}(0), x_*(n)\}. \quad (5)$$

$C_{26*}(n)$  of the subject is shown in Fig. 3. The lag of correlation functions of Fig. 3 is from -150 (-120 ms) to 150 (120 ms).

The correlation functions  $D_{26*}$  defined by

$$D_{26*}(n) =: C\{\delta x_{26}(0), \delta x_*(n)\} \quad (6)$$

are also shown in Fig. 4.

We can define

$$R_{26*}(n) =: pC\{r_{500} w_{26}(0), r_{500} w_*(n)\}, \quad (7)$$

where  $pC$  means the pseudocorrelation function. From the periodical part of the somatosensory evoked magnetic field

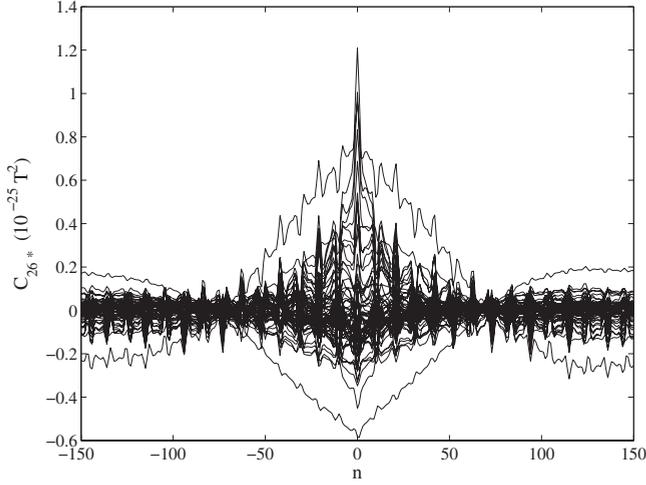


FIG. 3. Correlation functions  $C_{26*}(n)$  between the 26th and any SQUID channel.

$R_{26*}$  is calculated as in Fig. 5. From Figs. 4 and 5 correlations in fluctuations of MEG data are larger than those of repetitive wave forms. This means that amplitudes of background brain noises are more than ten times those of periodical evoked brain activities.

If we define

$$\Delta C_{26*}(n) := C_{26*}(n) - [D_{26*}(n) + R_{26*}(n)], \quad (8)$$

the Wold decomposition theorem means that  $\Delta C_{26*}(n) = 0$  theoretically.

$$\begin{aligned} \Delta C_{26*}(n) &= C\{\mathbf{x}_{26}(0), \mathbf{x}_*(n)\} - C\{\delta \mathbf{x}_{26}(0), \delta \mathbf{x}_*(n)\} \\ &\quad - pC\{r_{500} \mathbf{w}_{26}(0), r_{500} \mathbf{w}_*(n)\} \\ &= C\{\delta \mathbf{x}_{26}(0), r_{500} \mathbf{w}_*(n)\} + C\{r_{500} \mathbf{w}_{26}(0), \delta \mathbf{x}_*(n)\} = 0. \end{aligned}$$

Therefore  $\Delta C_{26*}(n) = 0$  means that  $\delta \mathbf{x}_{26}$  is perpendicular to  $r_{500} \mathbf{w}_*$ .

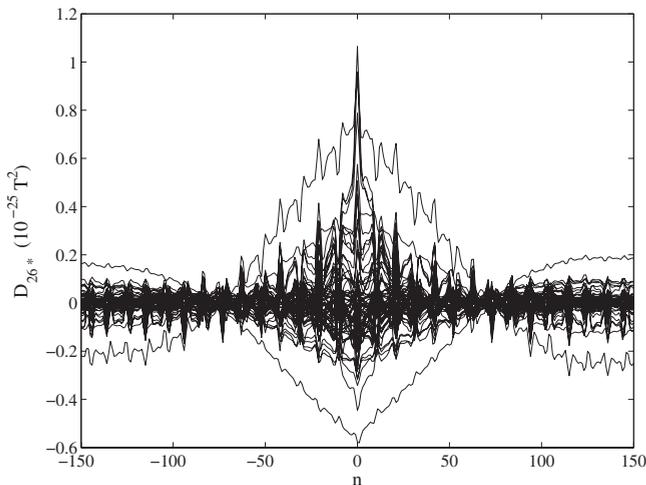


FIG. 4. Correlation functions  $D_{26*}(n)$  of fluctuations between the 26th and any SQUID channel.

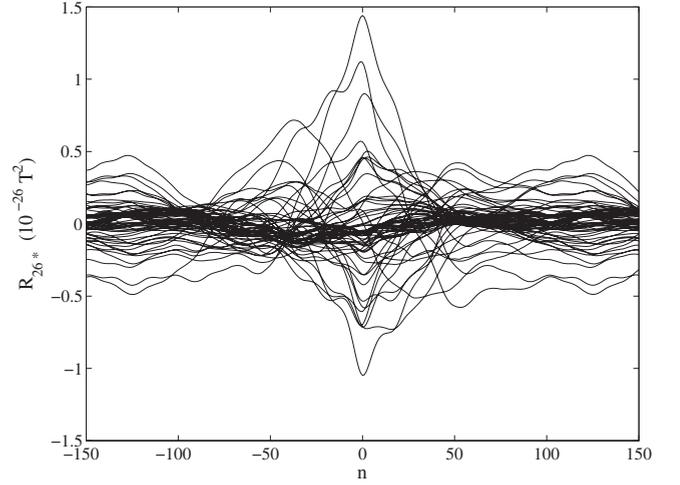


FIG. 5. Pseudocorrelation functions  $R_{26*}(n)$  of repeated wave forms between the 26th and any SQUID channel.

Let us examine  $\Delta C_{26*}(n)$  in SEF MEG. The absolute value of  $\Delta C_{26*}(n)$  is bounded by a small value as in Fig. 6, since the number of times of repeated stimuli was 500.

If  $M$  is smaller than 500, the estimation of wave form  $\mathbf{w}(n)$  becomes less poor than that of  $M=500$ . This means that there is a bias from the true value in the case of small  $M$ . Hence the small value is dependent on the number of  $M$ .

The Wold decomposition theorem teaches us that  $\Delta C_{26*}(n)$  of MEG data are considered to be a kind of error, since  $\Delta C_{26*}(n) = 0$  theoretically. Finally, let us examine statistical effects on  $\Delta C_{26*}(n)$  by changing  $M$ . If 64 standard deviations of  $\Delta C_{26*}(n)$  within  $-150 \leq n \leq 150$  are denoted by  $S_{26}$ , the  $M$  dependence of  $S_{26}$  is shown at  $M=30, 50, 100, 250,$  and  $500$  of Fig. 7.

These changes of  $\Delta C_{26*}(n)$  are due to the statistical convergence of the wave form. Inversely, Figs. 6 and 7 of  $\Delta C_{26*}(n)$  with  $M=500$  supports an assertion that the somatosensory evoked magnetic field of MEG corresponds to the

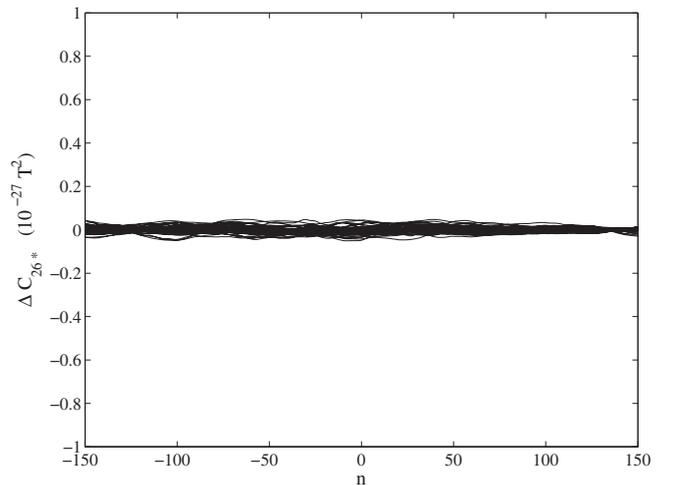
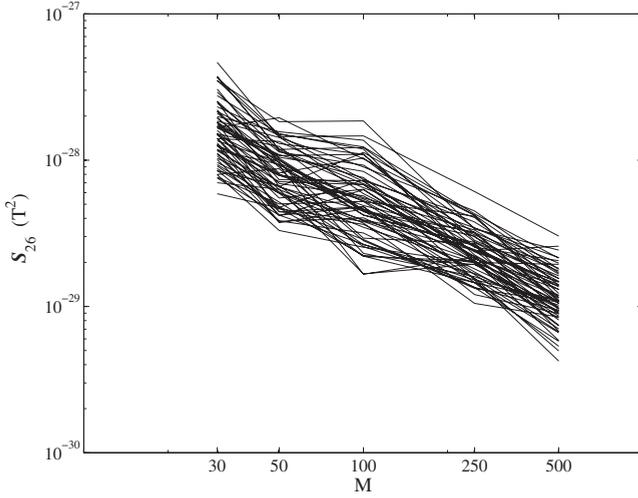


FIG. 6. Differences  $\Delta C_{26*}(n)$  of Eq. (8) in  $M=500$ .

FIG. 7.  $M$  dependence of standard deviations of  $\Delta C_{26*}$ .

deterministic part of the Wold decomposition theorem. Therefore it is concluded that the wave form  $\mathbf{w}(n)$  is deterministic with statistical precision of MEG data.

#### IV. APPLICATION TO BLIND SOURCE SEPARATION

To retrieve SEF MEG, we have used the second-order correlation functions for periodical types of blind source separation (BSS), i.e., the decorrelation method of BSS. The decorrelation method was developed by Molgedey and Schuster [7], Ziehe *et al.* [8], Murata *et al.* [9], and briefly summarized in [4]. The decorrelation method of BSS has two procedures. One procedure is the sphering for orthogonalizing time series data. Since  $\mathbf{x}(n)$  has zero mean, let us define a covariance matrix

$$D = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}(n)\mathbf{x}(n)^T, \quad (9)$$

and MEG data are transformed into  $\mathbf{z}(n) := \sqrt{D^{-1}}\mathbf{x}(n)$ . The other procedure is the rotation for removing the off-diagonal elements of the correlation matrices at several time delays,

$$C_{zz}(\tau_m) = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{z}(n)\mathbf{z}(n + \tau_m)^T, \quad m = 1, \dots, k. \quad (10)$$

The main process of rotation is to determine a square matrix  $U$  in the problem of minimization of the cost function  $J$ ,

$$J(U) = \sum_{m=1}^k \sum_{i \neq j} | [UC_{zz}(\tau_m)U^T]_{ij} |^2, \quad (11)$$

where  $[UC_{zz}(\tau_m)U^T]_{ij}$  denotes the  $ij$  element of matrix  $UC_{zz}(\tau_m)U^T$ . The Jacobi-like algorithm proposed by Cardoso and Souloumiac [10] has been used to solve approximately the simultaneous diagonalization problem on  $k$  normalized correlation matrices.

However, BSS performance is strongly dependent on the choice of time delayed parameters. The temporal decorrelation method of BSS has an open problem in the choice of the time delayed parameters [8,11].

In SEF MEG data there are periodical components. Power electric noise is a typical periodical example. Especially, periodical stimuli induce a repetitive wave form of an evoked field. Taking advantage of a periodical evoked magnetic field, we have developed the temporal decorrelation method useful for the periodical case by minimizing the absolute sum of off-diagonal elements of correlation matrices at particular time delays. As to a period  $T = 1/f_p$ , the time delayed parameters can be defined by

$$kT \text{ type: } \tau_m = m \left\lfloor \frac{f_s}{f_p} \right\rfloor, \quad m = 1, 2, \dots, k. \quad (12)$$

Here  $\lfloor c \rfloor$  rounds the value to the nearest integer. This choice of time delayed parameters has been called the kT type of decorrelation method as in Kishida *et al.* [4,12]. Although we could extract periodical components related to contralateral primary somatosensory cortex by using the kT type of decorrelation method, we could not find any other of periodical brain activities. In 5 Hz periodical median nerve stimuli PSD of MEG has a line spectrum of 5 Hz and its repeated higher harmonic modes are illustrated in Fig. 2. Then, improvements with time delayed parameters defined by

$$T/k \text{ type: } \tau_m = \left\lfloor \frac{f_s}{f_p} \right\rfloor / m, \quad m = 1, 2, \dots, k, \quad (13)$$

should be made on the BSS as mentioned in Ref. [13]. In the BSS, the absolute sum of off-diagonal elements of normalized correlation matrices are minimized at times corresponding to 5 Hz and its higher harmonic frequencies. From the above two procedures we can define a matrix,  $A^{-1} := U\sqrt{D^{-1}}$  to determine the blind source separation with  $T/k$  type of time delays:

$$\mathbf{x}(n) = A\mathbf{s}(n). \quad (14)$$

where  $\mathbf{s}(n)$  is a vector of the normalized BSS components. Let us call the temporal BSS with Eq. (13) by the  $T/k$  type of decorrelation method.

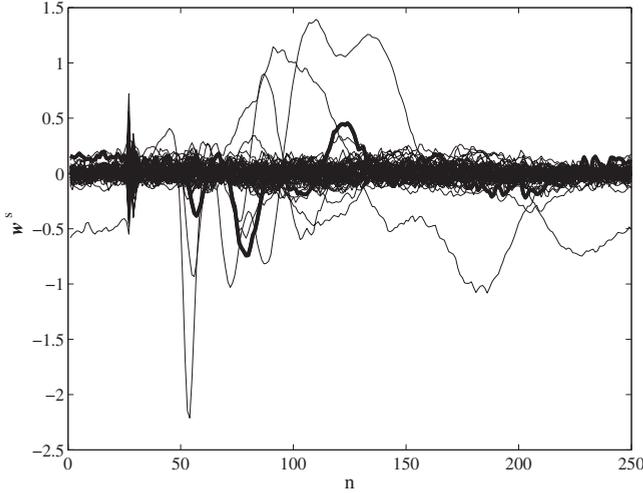
For MEG data we have used the  $T/k$  type of BSS method based on temporal structure to extract 5 Hz periodical SEF components. Here, we used parameters of the  $T/k$  type of BSS,  $f_s = 1250$ ,  $f_p = 5$ , and  $k = 8$ . The wave forms of BSS components are defined by

$$\mathbf{w}^s(n) := \frac{1}{M} \sum_{m=1}^M s[n + (m-1)L] \quad (15)$$

and shown in Fig. 8 for the subject. The superscript  $s$  denotes components of BSS.

The bold line in Fig. 8 is a wave form of the 23rd BSS component,  $s_{23}$ . PSD of the 23rd BSS is shown in Fig. 9. From Fig. 8 there are several BSS components which have wave forms of the evoked magnetic field, and the other BSS components have no wave forms of evoked magnetic field.

Hereafter, we focus cross-correlations of BSS components from the point of figure presentation, since  $\mathbf{s}(n)$  is normalized by  $\sqrt{D^{-1}}$ . Let us examine a cross-correlation function between the 23rd BSS and the other BSS defined by


 FIG. 8. Wave form  $w^s(n)$  of BSS components.

$$C_{23^\circ}^s(n) = \frac{1}{N} \sum_{m=1}^{N-n} s_{23}(m)s_\circ(m+n) := C\{s_{23}(0), s_\circ(n)\}. \quad (16)$$

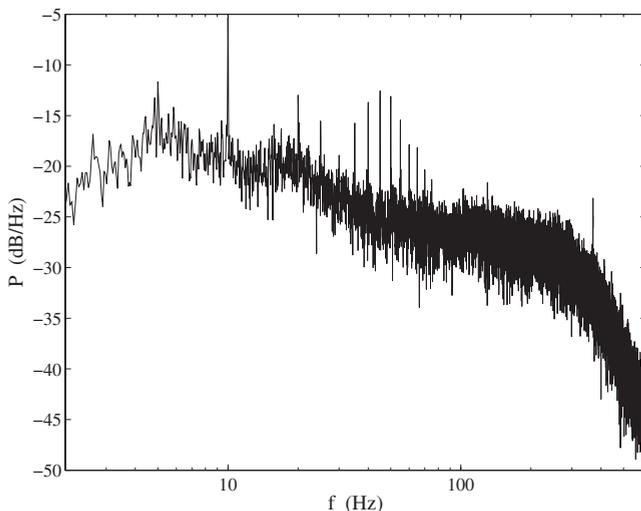
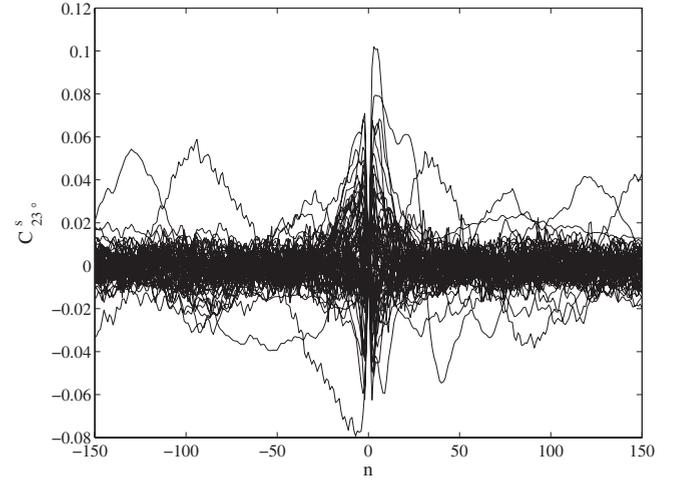
The 63 cross-correlation functions,  $C_{23^\circ}^s(n)$ , are shown in Fig. 10. Here the symbol  $^\circ$  is the other number except 23.

If  $M$  times repeated wave form of  $w^s(n)$  is defined by  $r_M w^s(n)$ , we have  $\delta s(n) := s(n) - r_M w^s(n)$ . The cross-correlation function  $D_{23^\circ}^s(n)$  defined by

$$D_{23^\circ}^s(n) := C\{\delta s_{23}(0), \delta s_\circ(n)\} \quad (17)$$

is also shown in Fig. 11.

This means that the decorrelation method is not the independent component analysis but the blind source separation, since there remain small correlations between BSS components. For a trivial example there are two BSS components necessary to express power electric noises, which have correlations and are not independent. That is, the temporal decorrelation method is not perfectly the independent component

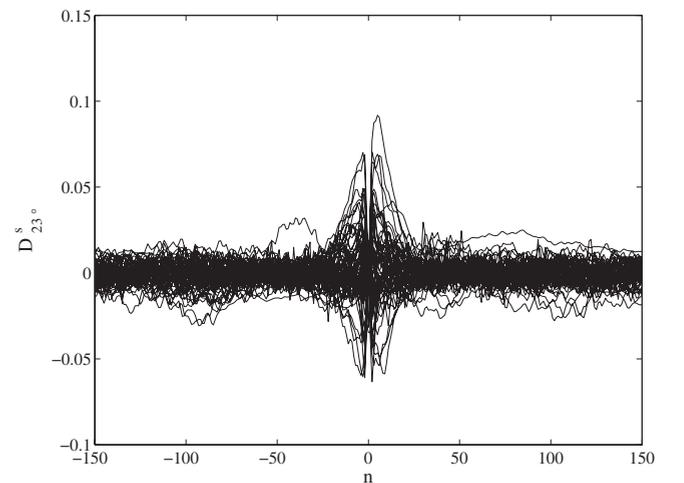

 FIG. 9. PSD  $P(f)$  of the 23rd BSS component.

 FIG. 10. Cross-correlation functions  $C_{23^\circ}^s(n)$  between the 23rd and the other BSS component.

analysis but the blind source separation, i.e., not diagonal but block-diagonal matrix type.

We can define

$$R_{23^\circ}^s(n) := pC\{r_M w_{23}^s(0), r_M w^s(n)\}. \quad (18)$$

Pseudocorrelation functions of repeated wave forms between 23 and the other BSS,  $R_{23^\circ}^s(n)$ , are shown in Fig. 12. It should be noted that Fig. 12 indicates the number of BSS components which have nonzero wave forms. This means that the SEF MEG is given by the several BSS components which have the nonzero wave forms in Fig. 12. By selecting the several BSS components and transforming them to SQUID data we can extract time series data which have repetitive wave forms and their fluctuations of the evoked magnetic field from MEG data. From Fig. 12 the number of BSS components with wave forms was eight in the subject under statistical accuracy of MEG data. That is, SEF MEG of the subject is expressed by a block of eight BSS components of 5, 15, 23, 26, 35, 39, 41, and 48. The number is equal to that


 FIG. 11. Cross-correlation functions  $D_{23^\circ}^s(n)$  of fluctuations between the 23rd and the other BSS component.

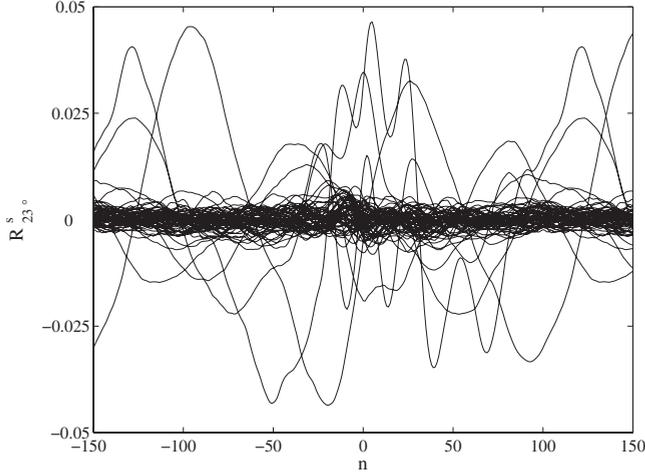


FIG. 12. Pseudocorrelation functions  $R_{23}^s(n)$  of repeated wave forms between the 23rd and the other BSS component.

with nonzero wave forms in Fig. 8. This property holds for all subjects, though the number is not fixed in subjects. This property will be useful in checking whether any BSS has a wave form or not. Hence it should be noted that SEF MEG is not diagonal but block-diagonal in BSS.

Let us define

$$\Delta C_{23}^s(n) := C_{23}^s(n) - [D_{23}^s(n) + R_{23}^s(n)] \quad (19)$$

to examine that SEF MEG satisfy the decomposition of Wold theorem in the decorrelation method. The absolute value of  $\Delta C_{23}^s(n)$  is bounded by a small value as in Fig. 13. Therefore BSS components of MEG satisfy the decomposition of Wold theorem with statistical precision.

It can be concluded from Fig. 13 that periodic data  $r_M \mathbf{w}(n)$  or  $r_M \mathbf{w}^s(n)$  are deterministic and that

$$C_{23}^s(n) = D_{23}^s(n) + R_{23}^s(n). \quad (20)$$

These properties will be useful for MEG analysis with BSS as in [12,14,15].

By selecting eight BSS components with wave forms in the subject time series SQUID data of the somatosensory

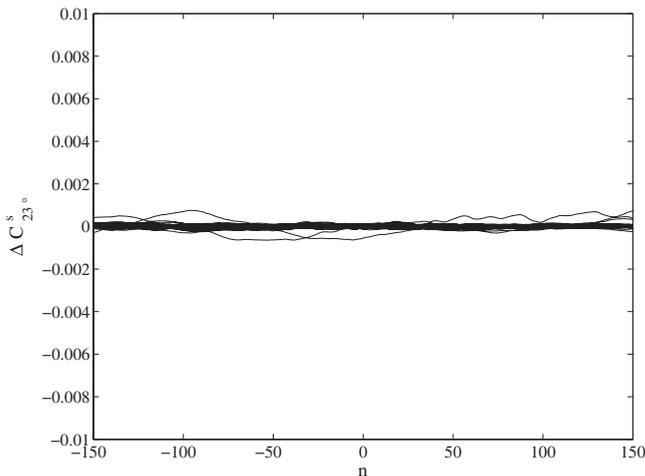


FIG. 13. Differences  $\Delta C_{23}^s(n)$  of Eq. (19).

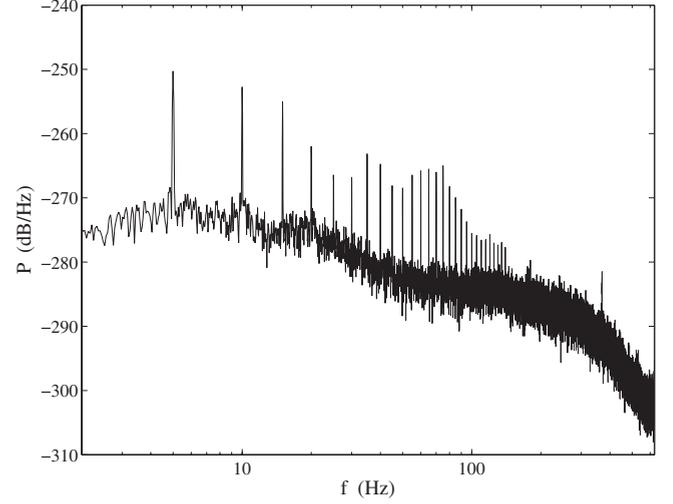


FIG. 14. PSD  $P(f)$  of SEF at the 26th SQUID channel.

evoked magnetic field generated from somatosensory cortexes,  $\mathbf{b}_e(n) := A \mathbf{s}_{\square}(n)$ , can be separated from background brain noise of MEG data, since SEF wave forms are generated from currents of cortexes related to somatosensory activities. Here the symbol  $\square$  indicates that eight BSS components of 5, 15, 23, 26, 35, 39, 41, and 48 with wave forms leave the original ones and that the other 56 BSS components without wave form are set as zero vectors. PSD of the reconstructed somatosensory evoked field  $\mathbf{b}_e(n)$  at the 26th SQUID channel is illustrated in Fig. 14 (cf. Fig. 2).

In this way, stationary time series SQUID data of fluctuations of SEF MEG as  $\delta \mathbf{b}_e(n) := A \delta \mathbf{s}_{\square}(n) = A[s_{\square}(n) - r_M \mathbf{w}_{\square}^s(n)]$  is obtained by elimination of the repeated SEF wave form. PSD of  $\delta \mathbf{b}_e(n)$  at the 26th SQUID channel is also illustrated in Fig. 15.

By comparison with Figs. 14 and 15 PSD over 150 Hz shows background noises of MEG, and dynamical information of fluctuations of stationary SEF is included in PSD under 150 Hz, though the effect of a 300 Hz low pass filter is also shown in PSD.

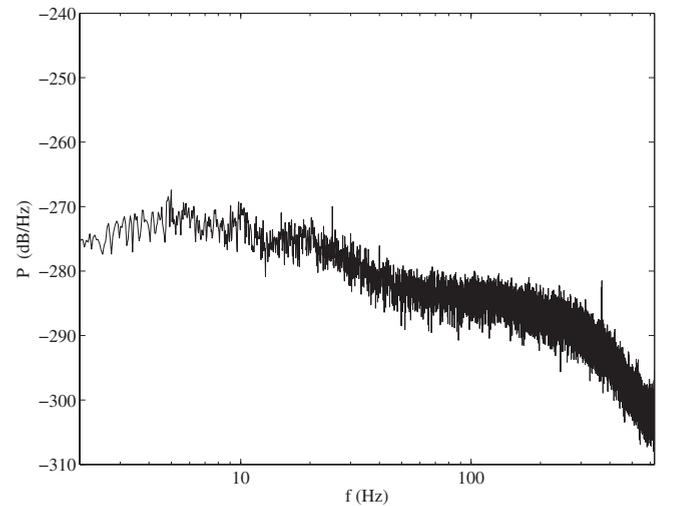


FIG. 15. PSD  $P(f)$  of fluctuations of SEF at the 26th SQUID channel.

Wave forms are usually used in MEG analysis. Though dynamical information is obtained from wave forms, there still remains dynamical information in MEG data. That is, SEF MEG shown by Fig. 14 have repetition of a SEF wave form and SEF fluctuations around it. It is a future problem to have dynamical information from SEF fluctuations of Fig. 15.

## V. CONCLUSION

When MEG data with evoked magnetic fields in 5 Hz median nerve stimuli satisfy the decomposition of Wold theorem, it can be concluded that wave forms of a somatosensory evoked magnetic field are deterministic with statistical precision of MEG data.

Results can be summarized for the  $T/k$  type of the decorrelation method.

(1) Stationary SEF MEG data are decomposed into the deterministic part and nondeterministic part.

(2) Time series data of a somatosensory evoked magnetic field generated from somatosensory cortexes can be separated from background brain noises of MEG data with statistical precision as in Fig. 15.

Though wave forms that have actives in the somatosensory evoked magnetic field are found usually from Eq. (1) or Eq. (15), there is a possibility to find a nonzero small wave form [13] that has actives in addition to the somatosensory evoked magnetic field by comparison with correlation functions of BSS components mentioned as in Sec. IV. Our approach to this direction will be useful for understanding dynamical brain functions.

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