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$1/f^{\alpha}$ noise in reaction times: A proposed model based on Piéron's law and information processing

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Piéron's law relates human reaction times to the intensity of a sensory stimulus by a power function. The neural processes responsible for this nonlinear behavior are not understood. A simple neural model based on the Brownian motion of spikes and information theory is presented. The model shows that Piéron's law is a transformation function in time. The shape of Piéron's law is invariant and scales into the intensity-response function of single neurons in a fractal-like process. The model also shows that Piéron's law gives rise to $1/f^{\alpha}$ noise together with a high-frequency thermal noise limit. It is proposed that the biophysical origin of reaction time variability is related to a form of noise-induced synchronization in weakly coupled neurons. The implications in visual-motor transduction are discussed.

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I. INTRODUCTION

Reaction time (RT) can be defined as the time elapsed from the onset of a test stimulus until a response occurs [1]. Applications are found in a wide variety of everyday tasks such as performance in sports, and air and land navigation. In the laboratory, RT has provided valuable information on the neural conduction-time dynamics over more than a century [1–3]. Two main aspects have been studied extensively. First, the precise duration of a response is not constant. Spectral methods have shown that the existence of variability from trial to trial causes flicker noise or $1/f^{\alpha}$ Fourier spectra ($\alpha \sim 1$), in most elementary cognitive processes [4–12]. This type of fluctuation is an example of long-range dependence in behavior. Current theories indicate that $1/f^{\alpha}$-type noise in RT exhibit fractal scaling from intrinsic neural activity [4,6,7,9–12]. Second, the expected mean RT value (t_{RT}) decreases as the intensity of the physical signal increases (e.g., luminance, sound pressure, odorant concentration, etc.) [1,2,13–18]. This is related to Piéron's law [19] and includes the sum of two separate processes as follows:

$$t_{\mathrm{RT}} = t_{\mathrm{RT}_0} + \mu I^{-p}, \tag{1}$$

where t_{RT_0}, μ, and p are constants greater than zero, the former representing the asymptotic plateau reached at higher intensities [1,19]. Piéron's law is an empirical psychophysical law. It has been verified in a wide range of experimental conditions and multiple observers. The importance of Piéron's law lies in that it represents a general form of sensory-motor transduction by a power law. Examples can be found in each modality (visual, auditory, olfactory, tactile, and gustatory), and fractional values of p are often reported [1,13,14,16,18–20]. Although some authors have examined the relation between RT and stimulus intensity [1,17,21–25], it is usually without consideration of flicker noise. Experimental power-law distributions have a close connection to a form of symmetry found in nature called scale invariance. Scale invariance maintains identical copies of the observed quantities, i.e., they look similar over different scales [21,26,27], and is related to anisotropic or self-affine transformations [26,27]. Power laws also occur in scale-free processes, where no scale is typical [28,29], and self-organized critical phenomena among other generating mechanisms [27,29,30]. These scaling properties can appear together with fractal processes and $1/f^{\alpha}$-type noises. The theory of noise in excitable systems explains, in part, the complexity of the brain activity at both threshold [31] and suprathreshold levels [32]. Noise affects the temporal dynamics and induces pattern formation and synchronization in coupled oscillators [31,32]. Neural synchronization is considered the fundamental coordination function at the large scale of integration [33,34]. The study of Piéron's law and reaction time variability is therefore important to elucidate the macroscopic consequences of the noisy neural activity, to better understand sensory communications and to produce better human-machine interfaces.

The aim of the present work is to examine the neural basis of Piéron's law by statistical physics. An analytical RT model is presented. The RT model exemplifies the theme of linking physiology and psychophysics, an issue that has not been solved yet. It is proposed as a Brownian transport of neural responses or spikes, showing that low-frequency flicker noise contributes to Piéron's law. In comparison with previous numerical simulations using neural networks in psychophysics [35,36], the present approach uses the principles of information theory as a general form of neural organization and communication [20,37,38]. The RT model is included in a theoretical framework that has been validated previously with reference to psychophysical data on sensory adaptation and threshold estimation and explains most of the empirical laws of sensation and perception including Fechner's, Weber's, and Stevens' laws [20,39,40]. The outline is organized as follows. In the first section, I show an alternative derivation of Piéron's law as a form of regularity in the spike trains related to multiplicative neural noise (i.e., signal-dependent intensity). In the second section, I argue that Piéron's law can be considered as an emergent (fractal) function in those noisy neural networks associated with sensory coding and motor processing and defines a scaling relationship. I show that both Piéron's law and the intensity-response function in

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single neurons, which is well fitted by Naka-Rushton or Hill-like equations, are the same. That is, both arise from the combination of flicker noise plus a white thermal noise limit, with the reciprocal of the exponent p in Eq. (1) as the corresponding slope. Finally, I discuss the theory in relation to the human visual-motor system.

II. RESULTS

A. Definition of the entropy *H* function

In the present model, information is related to the reduction of entropy or uncertainty. This entropy should not be confused with the entropy of thermodynamics. Hence, the information-theory entropy or H function is calculated from the probability distribution function of the possible outcomes in the sensory system. It is assumed that the sensory system is represented by a neural network model, with peripheral sensory receptors (e.g., photoreceptors in vision, inner-hair cells in hearing, etc.), connected to the cortex in a noisy communication system or channel. Physical stimulation induces uncertainty in both sensory receptors and human perception. At both levels of processing, uncertainty is derived from fluctuations in stimulus and is represented simultaneously by the same entropy equation [20]. In the simplest case, the quantity of information transmitted per sensory stimulus depends on the intensity I of the external physical signal as well as the time t, i.e., $H=H(I,t)$ [20]. This steady signal is treated as a nonperiodic perturbation to the sensory neural network during a finite period of time (e.g., a brief sound, light flash, etc.). The entropy equation links both neural and perceptual levels though an optimization procedure. That is, the definition of RT is connected to the sensory neural network containing a source encoder or decoder. This is equivalent to asserting the existence of an informational threshold so that RT can be defined as the time needed to gather ΔH bits of information [20].

$$\Delta H = H(I,t_0) - H(I,t_{\mathrm{RT}}) \geqslant 0. \tag{2}$$

In contrast with previous models of RT [1,41], initially ($t < t_0$) the information does not accrue but the statistical structure of the external signals is assumed redundant [37,38]. In chronological order, t_0 indicates the first stage and represents the encoding time where H reaches a maximum, i.e., the state of maximum uncertainty or equivalently, maximum loss of information and minimum redundancy. t_{RT} is the required time to react (or RT), representing the final state of uncertainty that produces a gain of information of $\Delta H \geqslant 0$. Hence, the existence of a threshold device is linked with a transfer of information but only after the signal is decoded [20]. The transfer of information defined in Eq. (2) is evaluated during the temporal dynamics of the sensory neural network. Although this noisy communication channel contains nonlinear effects, they are considered relatively small and do not dominate over the time elapsed by the impulses or spikes through the nerve fibers (equivalently, short propagation distances). This affords the opportunity to treat the sensory network on the basis of the standard linear channel model defined by Shannon consisting of additive Gaussian noise. However, as shown later, the noise spectrum is not completely flat. It contains low-frequency colored (flicker) noise with white thermal noise as the asymptotic high-frequency limit. The existence of colored noise is explained by small deviations from the linear regime and incorporates the presence of nonlinear propagation effects weakly coupled (i.e., a quasilinear communication system). From information theory, the entropy H function of a Gaussian signal embedded in the presence of Gaussian noise as follows:

$$H = \frac{1}{2}\ln\left(1 + \frac{S}{N}\right) \quad \text{(natural units)}, \tag{3}$$

where (S/N) is the signal-to-noise variance ratio [20]. To evaluate this ratio, it is assumed that timing between spikes encodes the sensory information. That is, neurons are sensitive to the time of arrival between successive impulses and will respond as spike timing units detecting in which time interval one or the combined action of multiple spike trains is received [33]. Changes in the firing rates are considered noise and the reciprocal, the variability of the interspike time interval, the received signal. To describe noise fluctuations, each neuron in the sensory network is treated as a one-dimensional oscillator under external noisy stimulation characterized by its firing rate $m(t)$ (spikes per second). In a simple view, the instantaneous change in $m(t)$ can be described by a Langevin-type equation as follows:

$$\frac{dm(t)}{dt} = -am(t) + F(t). \tag{4}$$

The first term on the right-hand side in Eq. (4) indicates the restoring process. The coefficient a has dimensions of frequency (Hz) and governs the time scale of the firing rate. The second term $F(t)$ represents the stochastic perturbation due to the spontaneous activity of other neurons in the network. These noisy impulses have short memory. They are excitatory or inhibitory and are assumed zero-mean Gaussian delta correlated: $\langle F(t)F(t')\rangle = G\delta(t-t')$, $\langle F(t)\rangle = 0$. $\langle \ldots \rangle$ represents the average over the ensemble of impulses in the network and G is the coupling coefficient that determines the intensity of the impulses in $F(t)$ [42]. The noise variance or fluctuations in the average spike output σ^2 depends on $m(t)$ and gives the basis to evaluate the entropy H function: $\sigma^2 = \langle m^2\rangle - \langle m\rangle^2$. To solve for $m(t)$, it is proposed that a is a slow, frequency-varying function. That is, the coefficient a is, on average, approximately constant and equal to the mean value for each neuron in the network during the time needed to react [20,39]. Therefore, $m(t)$ reduces to the following equation:

$$m(t) = m(0)e^{-at} + e^{-at}\int_0^t e^{a\tau}F(\tau)d\tau. \tag{5}$$

After a long period of time, the initial responses in sensory receptors cells are progressively forgotten and the sensory neural network is completely adapted to the steady state. Hence, the first term on the right-hand side in Eq. (5) is dropped and $m(t)$ is mainly determined by postreceptoral mechanisms and the correlation between their spikes in $F(t)$ [42]. The time course of fluctuations σ^2 is determined by an ordinary Brownian process to the asymptotic value $G/2a$ as follows:

$$\sigma^2 = \frac{G}{2a}(1 - e^{-2at}). \tag{6}$$

The dissipation-fluctuation theorem [42] establishes

$$G = 2am_{\mathrm{eq}}, \tag{7}$$

where m_{eq} is a parameter to be evaluated at the steady state. Therefore, the reciprocal of σ^2 in Eq. (6) or fluctuations in the interspike time rate are associated with the signal-to-noise variance ratio in the H function

$$\frac{S}{N} = \frac{1}{\sigma^2}. \tag{8}$$

Although the power or energy spectrum of a classical Brownian motion is usually calculated by its autocorrelation function, here it is explicitly assumed that m_{eq} is not a constant value and a form of noise-coded signal emerges. Figure 1(a) shows the evolution of σ^2 in time for different values of a (m_{eq} fixed to unity).

Fluctuations in the firing rates accumulate over time describing an S-shaped curve (on a semilogarithmic scale), and eventually saturates at high processing times. Low a values (<0.5) move the S-shaped curve to higher processing times and vice versa. Because t is always higher than the encoding time t_0, the accumulation of noise in time is equivalent to increasing the frequency bandwidth of the process, defined proportional to the difference between the maximum and minimum frequencies available ($f_{\min}=1/t$)$<$($f_{\max}=1/t_0$). As a consequence, the variability of the interspike time interval decreases exemplifying the usual trade-off between the signal and the bandwidth in analog communication systems [43]. Figure 1(b) shows this relation plotting the signal-to-noise ratio [Eq. (8)] against the frequency bandwidth for different values of a ($t_0=30$ ms, $m_{\mathrm{eq}}=1$).

To overcome high noise fluctuations maintaining system performance, neurons must cooperate regenerating and amplifying the signal. A common procedure in band-limited communication systems consists of reducing the noise energy [43]. It is proposed that neurons promote synchronization preserving their relative phases and reaching the steady stage by phase locking [34,44]. This is a nonlinear process and indicates the presence of power-law scaling [36,44]. That is, m_{eq} impairs the growing of noise fluctuations in time modulating the ratio of the noise energy spectrum $E(f)$ to thermal noise by a power-law distribution as follows:

$$m_{\mathrm{eq}} \propto \left[\frac{E(f)}{k_{\mathrm{B}}T_{\mathrm{b}}}\right]^{-p} \leqslant 1. \tag{9}$$

p is a parameter greater than zero and sets the strength of the coupling: p values equal or higher than unity imply small coupling and vice versa [44]. $k_{\mathrm{B}}, T_{\mathrm{b}}$ are the Boltzmanns constant and the body temperature, respectively. The introduction of the quantum of thermal energy in Eq. (9) is justified by the fact that the stimulus intensity excites the local sensory network out of thermodynamic equilibrium. However, the network is still contained within a thermal bath generated by the whole brain. The thermal criterion establishes the baseline where nonlinear propagation starts and ensures stability. That is, fluctuations in the firing rate could easily dissipate in time to almost zero if the energy generated in the transduction process is far from $k_{\mathrm{B}}T_{\mathrm{b}}$, as in the absorption of

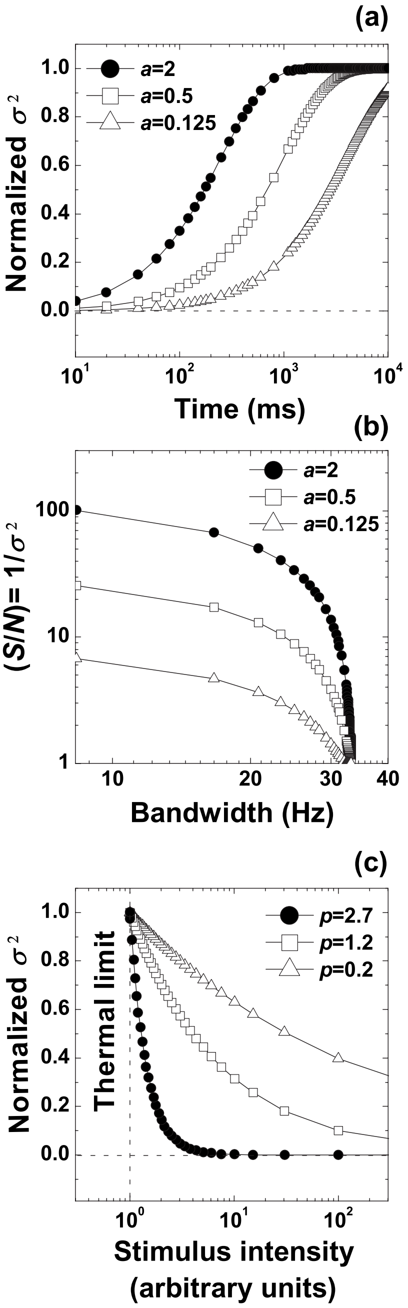

FIG. 1. (a) Semilogarithmic plot of the simulation of fluctuations in firing rates σ^2 (in normalized units) as a function of time for different values of a (m_{eq} fixed to unity). (b) Full logarithmic plot of the trade-off between the signal-to-noise ratio and the bandwidth (Hz) for different values of a (m_{eq} fixed to unity). (c) Semilogarithmic plot of the simulation of fluctuations in firing rates σ^2 (in normalized units) as a function of stimulus intensity for different values of p (very low, $p=2.7$; low, $p=1.2$; and moderate, $p=0.2$ neural synchronization, respectively). The temporal dependence was fixed to unity.

a photon or an odorant molecule [45]. The next important consideration is that the internal noise spectrum has a different dependence on changes in the stimulus strength. It contains multiplicative signal-dependent noise [31]. m_{eq} decreases as the stimulus intensity increases and noise remains constrained in the sensory network $m_{\mathrm{eq}} \propto [E(f)/k_{\mathrm{B}}T_{\mathrm{b}}]^{-p} = (\chi I/k_{\mathrm{B}}T_{\mathrm{b}})^{-p}$. χ is a constant. This form of power-law scaling resembles the reciprocal of Stevens' law in psychophysics [8,17,20,35,36,46]. From Eq. (6), the normalized σ^2 defines the functional steady state of the network as follows:

$$\sigma^2 = \left(\frac{k_{\mathrm{B}}T_{\mathrm{b}}}{\chi I}\right)^p (1 - e^{-2at}). \tag{10}$$

Figure 1(c) shows the evolution of σ^2 as a function of stimulus intensity for different values of p (very low, low, and moderate coupling at $p=2.7$, $p=1.2$, and $p=0.2$, respectively, $2at$ fixed). For suprathreshold stimulus intensities and low oscillatory synchronization ($p>1$), the evolution of fluctuations is bounded. Introducing the reciprocal of Eq. (10) into Eq. (8) it is now possible to evaluate the entropic H function [Eq. (3)] as follows:

$$H(I,t) = \frac{1}{2}\ln\left(1 + \frac{\gamma I^{\mathrm{p}}}{1 - e^{-2at}}\right), \tag{11}$$

where $\gamma=(\chi/k_{\mathrm{B}}T_{\mathrm{b}})^p$. For all modalities of sensation, Eq. (11) represents the Boltzmann-Shannon entropy function. The entropy H function describes the average flow of information transmitted by the sensory cells in the brain through a quasilinear channel with short-range perturbations [20,39]. Figures 2(a) and 2(b) show the evolution of $H(I,t)$ in the time and stimulus intensity for different values of a and p, respectively.

From Fig. 2(a), the entropy function decreases (a reduction of uncertainty) at higher processing times [20,39]. From Fig. 2(b), the entropy function increases (a gain of uncertainty), as a function of stimulus intensity [20,39]. The rise is attenuated changing the coupling coefficient from low ($p=1.2$) to moderate ($p=0.2$) neural synchronization. It is assumed that the H function is null at $t=0$. After reaching its maximum value at t_0, it decreases as the stimulus duration t increases ($t>t_0$), and therefore, there is a reduction of uncertainty and the sensory system gains information in accordance with Eq. (2). In a human reaction-time procedure, it is considered at intermediate times far from the saturation value (short-term memory process, $2at<1$), $1-e^{-2at}\cong 2at$ and for the entropy H function obtaining [20]

$$H(I,t) = \frac{1}{2}\ln\left(1 + \frac{\beta I^p}{t}\right), \tag{12}$$

where the parameter β includes $\beta=(1/2a)(\chi/k_{\mathrm{B}}T_{\mathrm{b}})^p$.

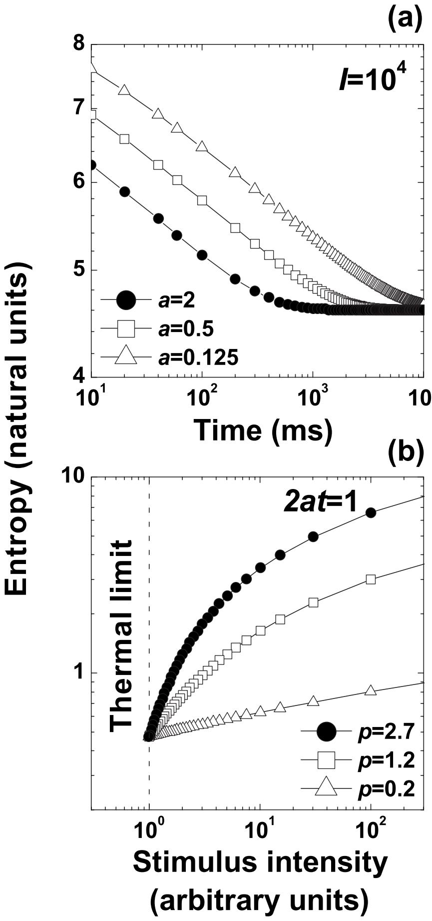

FIG. 2. Full logarithmic plot of the simulation of the entropy H function (in natural units) as a function of (a) time for different values of a. Stimulus intensity was fixed to 10 000 (arbitrary units) and (b) stimulus intensity for different values of p (very low, $p=2.7$; low, $p=1.2$; and moderate, $p=0.2$ neural synchronization, respectively). Time was fixed equal to the reciprocal of $2a$.

B. Modeling reaction times: Piéron's law

There is a correspondence between the transfer of information ΔH in Eq. (2) and the sensory or intensity threshold I_0 for which a response can start. Both are related in the limit when $t_{\mathrm{RT}}\to\infty$. The solution of Eq. (2) can also be rewritten using the asymptotic value t_{RT_0}, estimated in the limit when $I\to\infty$. From Eqs. (12) and (2), $t_{\mathrm{RT}}=t_{\mathrm{RT}_0}/[1-(I_0/I)^p]$ [20]. At suprathreshold intensities ($I_0 \ll I$), the denominator can be expanded in a binomial series and Piéron's law is derived if the constant μ in Eq. (1) is identified with $\mu=t_{\mathrm{RT}_0}I_0^p$ [20].

$$t_{\mathrm{RT}} = t_{\mathrm{RT}_0}\left[1 + \left(\frac{I_0}{I}\right)^p\right], \quad \forall\ I_0 \ll I. \tag{13}$$

Equation (13) was originally derived by Norwich using different arguments [20]. Indirectly, it has been partially verified in some studies in visual psychophysics, where the reciprocal of μ in Eq. (1) has been related with the human contrast sensitivity function and p fixed to unity [47–50]. Figure 3(a) represents a simulation of Piéron's law [Eq. (13)] for low and moderate neural synchronization ($p=1.2$ and $p=0.2$, respectively, solid symbols), over a range of intensities (e.g., in vision, luminance in cd/m^2), and their correction at near threshold conditions (open symbols) with $t_{\mathrm{RT}_0}=314$ ms and $I_0=15$.

The present RT model also describes the structure or how RT grows from Piéron's law. Figure 3(b) indicates a sche-

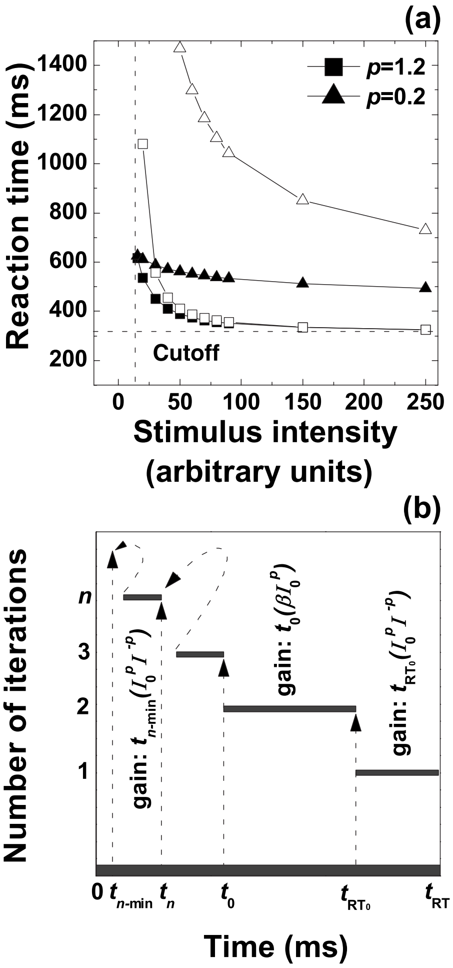

FIG. 3. (a) Reaction time as a function of the stimulus intensity (arbitrary units): simulation of Piéron's law (solid symbols) for different values of the exponent p (low, $p=1.2$ and moderate, $p=0.2$ neural synchronization, respectively, $t_{RT_0}=314$ ms, $I_0=15$). Open symbols represent the correction near the reference value I_0. (b) Schematic representation of a reaction time process in accordance with Piéron's law. For a fixed intensity, the mean RT or t_{RT} is represented as a horizontal segment in the time axis. The vertical axis indicates the cumulate number of different parts or segments that composes the mean RT. From right to left, t_{RT} is divided into smaller segments. Each time segment has a different size and grows from the previous one by a power function governed by the exponent p of Piéron's law (i.e., a gain factor). This has been represented by displacing each segment in the vertical axis as indicated by the arrows. At short processing times, the number of segments counted increases until the nth iteration but the power function remains.

matic representation. From a fixed intensity I, the mean RT is represented as a segment in the time axis and it is divided into smaller fragments. From Eq. (13), the mean RT grows from the asymptotic term t_{RT_0} with a gain factor that depends on the intensity ratio (I_0/I). That is, Eq. (13) can be rescaled to a proper power law as follows:

$$t_{RT} - t_{RT_0} = (t_{RT_0} I_0^p) I^{-p}. \tag{14}$$

Equation (14) establishes a scaling relationship in intensity. In a logarithmic plot, the length of the time segment $t_{RT}-t_{RT_0}$ decreases as a straight line as the intensity of the physical stimulus increases with slope $-p$. Shrinking iteratively in the time axis will move to shorter time segments but the power-law dependency prevails [21]. The next segment, the asymptotic term of Piéron's law t_{RT_0} is rescaled in relation to the encoding time t_0, and the time segment $t_{RT_0}-t_0$ depends on the following power law:

$$t_{RT_0} - t_0 = \beta I_0^p. \tag{15}$$

For a given pedestal or intensity background I_{bc}, I_0 is fixed but rises as I_{bc} rises. Over the middle range, both factors are related in accordance with Weber's law [17,20]. In a logarithmic plot, the term $t_{RT_0}-t_0$ decreases linearly as the reciprocal of I_0 increases with slope $-p$. In the asymptotic limit, it is possible to reach the response function of a single neuron. The Naka-Rushton or Hill-like functions give a good approximation of the firing state R (in spikes per second), and represent a form of gain control below the saturation value R_{max} [45,51–53]: $R=R_{max}/[1+(I_0/I)^q]$, where I_0 and q are the corresponding intensity threshold and the response exponent, respectively. Gain control mechanisms adjust the cell's responses to different input signals: low intensities are enhanced in relation to high intensity variations and neurons are sensitive to a wide range of intensity changes [52,53]. Taking the reciprocal of the neural response R as the interspike time interval in the nth iteration $t_n=R^{-1}$, an inverted Naka-Rushton equation follows Piéron's law if the minimum time is equal to the reciprocal of R_{max}, $t_{n-\min}=R_{max}^{-1}$, and q is equal to p as follows:

$$t_n - t_{n-\min} = (t_{n-\min} I_0^p) I^{-p}. \tag{16}$$

The reduced time fragments are not exact copies and differ from each other except for the form of the power functions [Eqs. (14)–(16)] are identical with exponent p. There is no single scale. Equations (14)–(16) are rescaled by different amounts and shifted in accordance with affine time segments. Therefore, Piéron's law involves self-affine scaling behavior, and the fractional values of the exponent p define the fractal dimension of the process.

C. Flicker noise at the threshold and suprathreshold levels

The intensity ratio in Eq. (13) defines a measure of the power spectrum in the noisy state m_{eq}. That is, $E(f) \propto I$ and the normalized energy spectrum $\tilde{P} \equiv (I/I_0) > 1$. As previously defined, the linear frequency is taken as the reciprocal of the time and can be expressed in normalized units: $(f_{\min}=1/t_{RT}) < (f=1/t_{RT_0})$, $\tilde{f}=(f/f_{\min}) \equiv (t_{RT}/t_{RT_0}) > 1$. From Eq. (13), in a logarithmic frequency scale Piéron's law acquires the form

$$\ln(\tilde{P}) = -(1/p)\ln(\tilde{f}-1), \quad \forall\, (I_0 \ll I). \tag{17}$$

Equation (17) indicates a general form of flicker noise in sensory systems with the reciprocal of the exponent p in Piéron's law as the corresponding slope. At high frequencies, the power spectrum in Eq. (17) should be corrected. In accordance with the exact solution derived from Eq. (12),

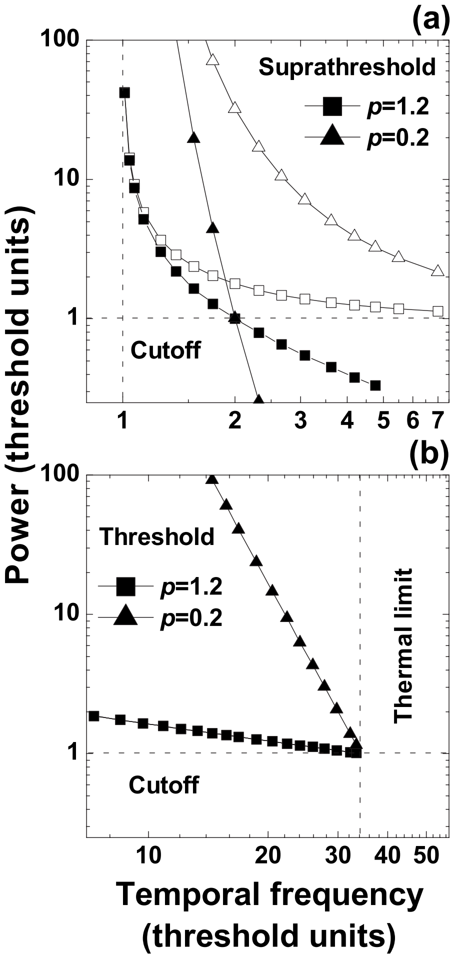

FIG. 4. Double logarithmic plot of the simulation of the normalized power spectra (in threshold units) derived from Piéron's law (solid symbols) for different values of the exponent p (low $p=1.2$ and moderate $p=0.2$ neural synchronization, respectively) at (a) suprathreshold conditions with $t_{\mathrm{RT}_0}=314$ ms (open symbols represent the correction at high frequencies) and (b) threshold conditions with $t_0=30$ ms and $a=0.5$.

$\ln(\tilde{P})=-(1/p)\ln[1\text{-}(1/\tilde{f})]$. Hence, $E(f)$ deviates from linearity and drops to the reference value defined by the threshold I_0. Figure 4(a) represents in a double logarithmic plot, a simulation of the normalized power spectrum at suprathreshold conditions [Eq. (17)] based on Piéron's law (solid symbols) and its correction at high frequencies (open symbols), with $t_{\mathrm{RT}_0}=314$ ms and for low and moderate neural synchronization ($p=1.2$ and $p=0.2$, respectively).

The functional dependence of the power spectrum at the threshold is derived from the asymptotic term in Eq. (15): $t_{\mathrm{RT}_0}=t_0+\beta I_0^p$. The normalized power is defined as the reciprocal of the ratio between I_0 to the intensity background I_{bc}, $\tilde{P}' \equiv (I_{\mathrm{bc}}/I_0) > 1$. The linear frequency is now normalized by the minimum frequency: $(f'_{\min}=1/t_{\mathrm{RT}_0}) < (f'=1/t_0)$, $\tilde{f}'=(f'/f'_{\min}) \equiv (t_{\mathrm{RT}_0}/t_0) > 1$. There is a maximum value defined by $\tilde{f}'_{\max}=2at_0$ and the energy spectrum at the threshold results as follows:

$$\ln(\tilde{P}') = -\frac{1}{p}\ln[\tilde{f}'_{\max}(\tilde{f}'-1)] + \ln\left(\frac{\chi I_{\mathrm{bc}}}{k_{\mathrm{B}}T_{\mathrm{b}}}\right). \qquad (18)$$

Equation (18) represents again a form of flicker noise. As the normalized frequency moves to the maximum value $(2at_0)$, the power spectrum flattens. Under the latter condition, the intensity background converges to I_0; i.e., the absolute sensory threshold. The power spectrum is modulated by the rightmost term in Eq. (18). The criterion of stability [Eq. (9)] prevents a power spectrum less than unity. Hence, fluctuations are made against the quantum level $k_{\mathrm{B}}T_{\mathrm{b}}$ and merge into thermal noise. Figure 4(b) indicates in a double logarithmic plot, a simulation of the normalized power spectrum at the threshold [Eq. (18)] for different exponent p values ($p=1.2$ and $p=0.2$ or low and moderate oscillatory coupling, respectively), with $t_0=30$ ms and $a=0.5$.

III. DISCUSSION

A. Fractal time processes in Piéron's driven neural networks

In the present study, Piéron's law has been reformulated using methods from statistical physics and information theory. The RT model uses the Langevin theory of Brownian motion to show that variability in the firing rate has a functional significance in sensory coding as noise. Noise fluctuations accumulate in time and approach a steady state in short-range interaction processes [Eqs. (4)–(10)]. It is proposed that spontaneous oscillatory synchronization reduces multiplicative noise. Hence, the steady state constrains the internal noise energy as a function of stimulus intensity [Eqs. (7) and (9)]. A plausible mechanism of response variability with stimulus intensity may be the membrane potential noise of neurons in the cortex [54–56]. The RT model defines Piéron's law as a transformation function in the time axis from the inverted intensity-response function of single neurons. Each time segment in Piéron's law represents a different stage and is normalized by different factors to look similar to the whole [Eqs. (14)–(16)]. There is no single scale and fractional values of the exponent p are often reported [1,13,16,18–20]. The present study shows that Piéron's law can be rewritten in the frequency domain [Eqs. (17) and (18)], and connects the gain of information [Eq. (2)] with low-frequency flicker noise, with the reciprocal of the exponent p as the slope of the power spectra (Fig. 4). Therefore, the RT model suggests the existence of a fractal (self-affine) process in time [26], a signature of complexity also reflected in heartbeat dynamics [57].

The presence of small values of multiplicative flicker noise [Eq. (9)] validates the use of the underlying quasilinear communication system, with Piéron's law supporting moderate levels of neural synchronization $(p > 0.1)$ [1,16–18,20,48–50,52]. Multiplicative noise may induce phase transitions [31] and neural activity by power laws might be related with the presence of self-organized critical states [7,58], with the reciprocal of the exponent of Stevens's law as the critical exponent [36]. These self-organized states may also improve the internal coherence of spikes, where the variability of the interspike time rate will grow with stimulus intensity (equivalently, internal noise energy), at mediumhigh levels [Eqs. (8) and (10)] [31]. These issues merit further investigation.

B. Implications for visual-motor transduction

Unlike previous RT models [1,17,22–25,41], the present approach based on Piéron's law is a three-serial-stage sensory-motor model and resolves the separation of RT into the sensory latency and the motor component [1]. The entropy model guarantees that the time hierarchy in Piéron's law corresponds to a generic communication system (a source generator or encoder comprising a transmitter, a noisy transmission channel, and finally, a receiver). That is, the asymptotic limit t_{RT_0} does not contain the motor component but those latencies associated with the two earliest visual stages: the encoding time t_0 and the time associated with the formation of the threshold condition, $t_{\mathrm{RT}_0}=t_0+\beta I_0^p$. In chronological order, it is suggested that the asymptotic term $(t_{\mathrm{RT}_0}>t_0)$ can be identified as the visual latency and the motor processing time may be associated within the factor, $t_{\mathrm{RT}_0}(I_0/I)^p$ [Eq. (13)]. The analysis of flicker noise at the threshold [Eq. (18)] asserts that RT is not a scale-free process, and the existence of white noise at high frequencies could not be ascribed to the motor component [4,6–8,10] but to thermal fluctuations [Fig. 4(b)]. The thermal noise limit defined by the body temperature indicates the characteristic scale from nonlinear propagation starts [Eq. (9)]. It is interesting to note that for a fixed stimulus intensity I, the asymptotic term t_{RT_0} indicates that RT decreases as the body temperature increases as confirmed previously [59]. Therefore, the present RT model indicates that the motor component also follows $1/f^{\alpha}$-type noise [Eq. (17)]. This is in agreement with the existence of blood oxygenation level dependent (BOLD) fluctuations in the motor cortex that follows $1/f$ power spectrum [60].

In vision research, the experimental magnitude of t_{RT_0} (usually higher than 200 ms) [48–50], contrasts with the motor conduction time (less than 50 ms) [61]. The present RT-motor model [Eq. (13)] is also in agreement with an extreme but important condition: if a very intense signal is evoked without previous adaptation such as in night driving, glare effects become important. In this situation, the threshold to extract any relevant information from the environment (e.g., target identification, an intersection in a road, etc.) rises [50,62] and delays any voluntary motor movement [50].

It is interesting to note that an exponent p equal to unity involves $1/f$ noise [Eqs. (17) and (18)]. Light adapted conditions improve transient behavior and provide shorter and less variable latencies [15]. From psychophysics, p values close to unity have been reported (low synchronization) [48–50]. On the contrary, dark-adapted eyes involve longer and more variable latencies and p values lower than unity (moderate synchronization) [1,20]. Light adaptation may help optimal transfer of information in the visual cortex through $1/f$ noise [63], and may enhance the processing of natural scenes in relation to dark adaptation. This has an advantage in the coherence transport of spikes, reducing noise fluctuations to a low level while uncertainty does not rise too much [Figs. 1(c) and 2(b)].

IV. CONCLUDING REMARKS

Piéron's law defines a sensory-motor model based on flicker noise not restricted to the analysis of error sequences [4,6–10]. The RT model developed in this study presents, possibly for the first time, a plausible mechanism between flicker noise and the gain of information in human perception though oscillatory neural synchronization. The fact that the Naka-Rushton-type equation can be modeled as a form of low-frequency flicker noise should be incorporated into network models that analyze neural dynamics of the cortex [36,52,53,55]. Finally, it is possible to apply the RT model not only to vision but to other sensory modalities [13,14,16,18,20]. This suggests that Piéron's law represents a general form of flicker noise in sensory systems.

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[1] R. D. Luce, *Response Times* (Oxford University Press, New York, 1986).

[2] J. M. Cattell, Brain **8**, 512 (1886).

[3] D. E. Meyer, A. M. Osman, D. E. Irwin, and S. Yantis, Biol. Psychol. **26**, 3 (1988).

[4] D. L. Gilden, T. Thornton, and M. W. Mallon, Science **267**, 1837 (1995).

[5] C. M. Anderson and A. J. Mandell, in *Fractals of Brain, Fractals of Mind: in Search of a Symmetry Bond*, edited by E. A. Mac Cormac and M. I. Stamenov (John Benjamins Publishing Co., Amsterdam, 1996), p. 359.

[6] D. L. Gilden, Psychol. Sci. **8**, 296 (1997).

[7] D. L. Gilden, Psychol. Rev. **108**, 33 (2001).

[8] L. M. Ward, *Dynamical Cognitive Science* (The MIT Press, Cambridge, MA, 2002).

[9] E. J. Wagenmakers, S. Farrell, and R. Ratcliff, Psychon. Bull. Rev. **11**, 579 (2004).

[10] T. L. Thornton and D. L. Gilden, Psychon. Bull. Rev. **12**, 409 (2005).

[11] S. Farrell, E. J. Wagenmakers, and R. Ratcliff, Psychon. Bull. Rev. **13**, 737 (2006).

[12] C. T. Kello, B. C. Beltz, J. G. Holden, and G. C. Van Orden, J. Exp. Psychol. Gen. **136**, 551 (2007).

[13] R. Chocholle, Annee Psychol. **41**, 65 (1940).

[14] W. P. Banks, Percept. Psychophys. **13**, 321 (1973).

[15] P. Lennie, Vision Res. **21**, 815 (1981).

[16] P. Overbosch, R. Dewijk, T. J. R. Dejonge, and E. P. Koster, Physiol. Behav. **45**, 615 (1989).
[17] J. C. Baird, *Sensation and Judgement* (Lawrence Erlbaum Associates, Mahwah, NJ, 1997).
[18] C. Bonnet, M. C. Zamora, F. Buratti, and M. Guirao, Physiol. Behav. **66**, 549 (1999).
[19] H. Piéron, *The Sensations* (Yale University Press, New Haven, CT, 1952).
[20] K. H. Norwich, *Information, Sensation, and Perception* (Academic Press, San Diego, CA, 1993).
[21] N. Chater and G. D. A. Brown, Cognition **69**, B17 (1999).
[22] T. Stafford and K. N. Gurney, Psychon. Bull. Rev. **11**, 975 (2004).
[23] Y. F. Hsu, J. Math. Psychol. **49**, 450 (2005).
[24] J. Palmer, A. C. Huk, and M. N. Shadlen, J. Vision **5**, 376 (2005).
[25] M. J. Taylor, R. H. S. Carpenter, and A. J. Anderson, J. Physiol. (London) **573**, 741 (2006).
[26] B. B. Mandelbrot, *The Fractal Geometry of Nature* (W. H. Freeman Company, San Francisco, CA, 1982).
[27] T. Gisiger, Biol. Rev. Cambridge Philos. Soc. **76**, 161 (2001).
[28] D. Abbott, Chaos **11**, 526 (2001).
[29] M. E. J. Newman, Contemp. Phys. **46**, 323 (2005).
[30] P. Bak, *How Nature Works* (Springer-Verlag, New York, 1996).
[31] B. Lindner, J. Garcia-Ojalvo, A. Neiman, and L. Schimansky-Geier, Phys. Rep. **392**, 321 (2004).
[32] M. D. McDonnell, N. G. Stocks, C. E. M. Pearce, and D. Abbott, *Stochastic Resonance* (Cambridge University Press, New York, 2008).
[33] W. Singer, Neuron **24**, 49 (1999).
[34] F. Varela, J. P. Lachaux, E. Rodriguez, and J. Martinerie, Nat. Rev. Neurosci. **2**, 229 (2001).
[35] M. Copelli, A. C. Roque, R. F. Oliveira, and O. Kinouchi, Phys. Rev. E **65**, 060901(R) (2002).
[36] O. Kinouchi and M. Copelli, Nat. Phys. **2**, 348 (2006).
[37] H. Barlow, Network Comput. Neural Syst. **12**, 241 (2001).
[38] E. P. Simoncelli and B. A. Olshausen, Annu. Rev. Neurosci. **24**, 1193 (2001).
[39] K. H. Norwich and W. Wong, Math. Biosci. **125**, 83 (1995).
[40] K. H. Norwich and W. Wong, Percept. Psychophys. **59**, 929 (1997).
[41] P. L. Smith and R. Ratcliff, Trends Neurosci. **27**, 161 (2004).
[42] H. Haken, *Synergetics* (Springer-Verlag, Singapore, 1977).
[43] A. B. Carlson, *Communication Systems* (McGraw-Hill, New York, 1986).
[44] V. A. Billock and B. H. Tsou, J. Opt. Soc. Am. A **22**, 2289 (2005).
[45] V. Torre, J. F. Ashmore, T. D. Lamb, and A. Menini, J. Neurosci. **15**, 7757 (1995).
[46] S. S. Stevens, Science **170**, 1043 (1970).
[47] A. Felipe, M. J. Buades, and J. M. Artigas, Vision Res. **33**, 2461 (1993).
[48] I. J. Murray and S. Plainis, Vision Res. **43**, 2707 (2003).
[49] D. J. McKeefry, N. R. A. Parry, and I. J. Murray, Invest. Ophthalmol. Visual Sci. **44**, 2267 (2003).
[50] R. C. Aguirre, E. M. Colombo, and J. F. Barraza, J. Opt. Soc. Am. A **25**, 1790 (2008).
[51] K. I. Naka and W. A. H. Rushton, J. Physiol. (London) **185**, 587 (1966).
[52] D. G. Albrecht and D. B. Hamilton, J. Neurophysiol. **48**, 217 (1982).
[53] D. J. Heeger, Visual Neurosci. **9**, 181 (1992).
[54] T. Kasamatsu, U. Polat, M. W. Pettet, and A. M. Norcia, Exp. Brain Res. **138**, 163 (2001).
[55] I. M. Finn, N. J. Priebe, and D. Ferster, Neuron **54**, 137 (2007).
[56] J. D. Victor, E. M. Blessing, J. D. Forte, P. Buzas, and P. R. Martin, J. Physiol. (London) **579**, 29 (2007).
[57] P. C. Ivanov, L. A. N. Amaral, A. L. Goldberger, S. Havlin, M. G. Rosenblum, Z. R. Struzik, and H. E. Stanley, Nature (London) **399**, 461 (1999).
[58] M. Shimono, T. Owaki, K. Amano, K. Kitajo, and T. Takeda, Phys. Rev. E **75**, 051902 (2007).
[59] T. M. Makinen, L. A. Palinkas, D. L. Reeves, T. Paakkonen, H. Rintamaki, J. Leppaluoto, and J. Hassi, Physiol. Behav. **87**, 166 (2006).
[60] M. D. Fox, A. Z. Snyder, J. L. Vincent, and M. E. Raichle, Neuron **56**, 171 (2007).
[61] D. Claus, Muscle Nerve **13**, 1125 (1990).
[62] I. C. Bichao, D. Yager, and J. Meng, J. Opt. Soc. Am. A **12**, 2252 (1995).
[63] Y. G. Yu, R. Romero, and T. S. Lee, Phys. Rev. Lett. **94**, 108103 (2005).