Arbitrary bending of electromagnetic waves using realizable inhomogeneous and anisotropic materials

Wei Xiang Jiang, Tie Jun Cui,* Xiao Yang Zhou, Xin Mi Yang, and Qiang Cheng
State Key Laboratory of Millimeter Waves, Institute of Target Characteristics and Identification, Department of Radio Engineering, Southeast University, Nanjing 210096, People's Republic of China

(Received 12 May 2008; published 17 December 2008)

We propose an optical transformation to bend electromagnetic waves by designing proper inhomogeneous and anisotropic materials, which are hereinafter referred to as metamaterials (MTMs). When the waveguide bends are filled with MTMs, the incident waves will pass through the bends without any reflections (for full-parameter MTMs) or with very small reflections (for simplified-parameter MTMs). When MTMs are placed in air, the incident waves will be bent to any designed directions. We also discuss the wave bending using layered homogeneous uniaxial MTMs, which can be easily realized using artificial structures.

DOI: 10.1103/PhysRevE.78.066607 PACS number(s): 41.20.Jb, 42.25.Gy, 42.79.Dj

I. INTRODUCTION

In the last two years, much attention has been paid to the optical transformation, which employs the metric invariance property of Maxwell's equations [1,2]. Transformation optics opens up many possibilities and methods to control the electromagnetic (EM) fields and it can be employed to design reflectionless complex media, such as cloaks [2–13], concentrator [5,15], EM-wave rotator [19], and imaging devices [14,16]. The approach of optical transformation has also been utilized to improve the traditional EM devices [17,18].

Recently, Donderici and Teixeira proposed a derivation of new material blueprints for the reflectionless guidance of EM waves through waveguide bends [20]. More recently, the design of beam bends has been analyzed [21,22] while the paper is under review, in which the authors discussed the derivation of new material tensors at the "blueprint level." In this paper, we propose an optical transformation to bend EM waves to the desired directions inside a waveguide or in free space without any reflections (or with tiny reflections) by designing proper inhomogeneous and anisotropic materials. Since the relative permittivity and permeability of the materials may be less than one, they are hereinafter referred to as metamaterials (MTMs). The reduced-parameter MTMs proposed in this paper are uniaxial, and hence easy to realize using artificial structures.

II. GENERAL FORMULATIONS

We employ the finite embedded optical transformation to realize the wave bending. Consider a two-dimensional (2D) structure in the Cartesian coordinate system, as shown in Fig. 1. We assume the lengths of OA and OB to be a and b, respectively, and the height of the rectangle $ABCD$ to be h. We define a mapping which maps the rectangle $ABCD$ to an arch $ABC'D'$ as

$$x' = x\cos\theta y/h, \tag{1}$$

$$y' = x\sin\theta y/h, \tag{2}$$

$$z' = z, \tag{3}$$

where (x,y) is an arbitrary point in the original space $ABCD$, (x',y') is the corresponding point in the transformed space $ABC'D'$, and θ is the radian of the bending angle.

When $ABCD$ is transformed into $ABC'D'$, the electric permittivity and magnetic permeability tensors of the medium in the transformed space are calculated by [2]

$$\overline{\varepsilon'} = \frac{\bar{\Lambda}\bar{\varepsilon}\bar{\Lambda}^T}{\det(\bar{\Lambda})}, \quad \overline{\mu'} = \frac{\bar{\Lambda}\bar{\mu}\bar{\Lambda}^T}{\det(\bar{\Lambda})}, \tag{4}$$

where $\bar{\varepsilon}$ and $\bar{\mu}$ are the constitutive parameter tensors in the original space. If the original space is free space, then $\bar{\varepsilon} = \bar{\mathbf{I}}\varepsilon_0$ and $\bar{\mu}=\bar{\mathbf{I}}\mu_0$. Hence, the relative permittivity and permeability tensors of the medium in the transformed space are expressed as

$$\overline{\varepsilon_r'^{ij}} = \overline{\mu_r'^{ij}} = \frac{1}{kr'}g^{ij}, \tag{5}$$

in which $r'=\sqrt{x'^2+y'^2}$, $k=\theta/h$, and

$$g^{ij} = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{12} & g_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

is the metric tensor of the coordinate transformation with

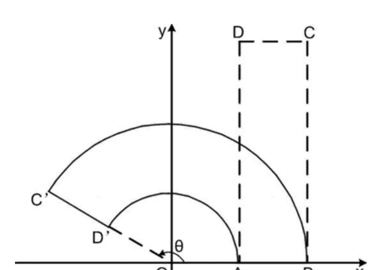

FIG. 1. The 2D wave bending structure in the Cartesian coordinate system. The lengths of OA, OB, and BC are a, b, and h, respectively.

*tjcui@seu.edu.cn

$$g_{11} = \frac{x'^2}{r'^2} + k^2 y'^2, \tag{7}$$

$$g_{12} = x' y' \left(\frac{1}{r'^2} - k^2 \right), \tag{8}$$

$$g_{22} = \frac{y'^2}{r'^2} + k^2 x'^2. \tag{9}$$

For the sake of simplicity, we drop the primes in above equations, and the medium parameters are written in the direct notation as

$$\varepsilon_{xx} = \mu_{xx} = \frac{1}{kr} \left(\frac{x^2}{r^2} + k^2 y^2 \right), \tag{10}$$

$$\varepsilon_{xy} = \mu_{xy} = \frac{xy}{kr} \left(\frac{1}{r^2} - k^2 \right), \tag{11}$$

$$\varepsilon_{yx} = \varepsilon_{xy} = \mu_{yx} = \mu_{xy}, \tag{12}$$

$$\varepsilon_{yy} = \mu_{yy} = \frac{1}{kr} \left(\frac{y^2}{r^2} + k^2 x^2 \right), \tag{13}$$

$$\varepsilon_{zz} = \mu_{zz} = \frac{1}{kr}, \tag{14}$$

where $r = \sqrt{x^2 + y^2}$.

Equations (10)–(14) provide full expressions of the permittivity and permeability tensors for the wave-bending region in the Cartesian coordinate. In order to simplify the complexity, we transform the tensors from the Cartesian coordinates to the cylindrical coordinates. The identical transformation to z component and the standard transformation to x and y components yield

$$\varepsilon_r = \mu_r = \varepsilon_z = \mu_z = 1/(kr), \tag{15}$$

$$\varepsilon_\phi = \mu_\phi = kr. \tag{16}$$

In this paper, we will focus on the TM plane-wave incidence, in which the magnetic fields are polarized along the z axis. In such a case, only μ_z, ε_r, and ε_ϕ are required in Eqs. (15) and (16). The dispersion relations of the transformation medium remain unchanged as long as the products of $\mu_z \varepsilon_r$ and $\mu_z \varepsilon_\phi$ are kept the same in above equations. One advantageous choice is to select $\mu_z = 1$ so that $\varepsilon_r = 1/(k^2 r^2)$ and $\varepsilon_\phi = 1$ due to the practical reason. This reduced-parameter medium provides the same wave trajectory inside the wave-bending region [3,11], and there will be a very small reflection because of the impedance mismatch [8].

Noticing that $k = \theta / h$, the simplified medium parameters can be then written as

$$\mu_z = 1, \quad \varepsilon_r = h^2/(\theta r)^2, \quad \varepsilon_\phi = 1. \tag{17}$$

In this set of parameters, only ε_r is spatially inhomogeneous and h is a free variable, which makes the wave bending structure much easier realize. Since the relative permittivity and permeability may be less than one, in this paper, the materials defined in Eqs. (11)–(16) and Eq. (17) are referred to as MTMs. Clearly, the full-parameter equations (11)–(16) are difficult to realize in practice, while the simplified-parameter equation (17) is easy to realize using artificial structures.

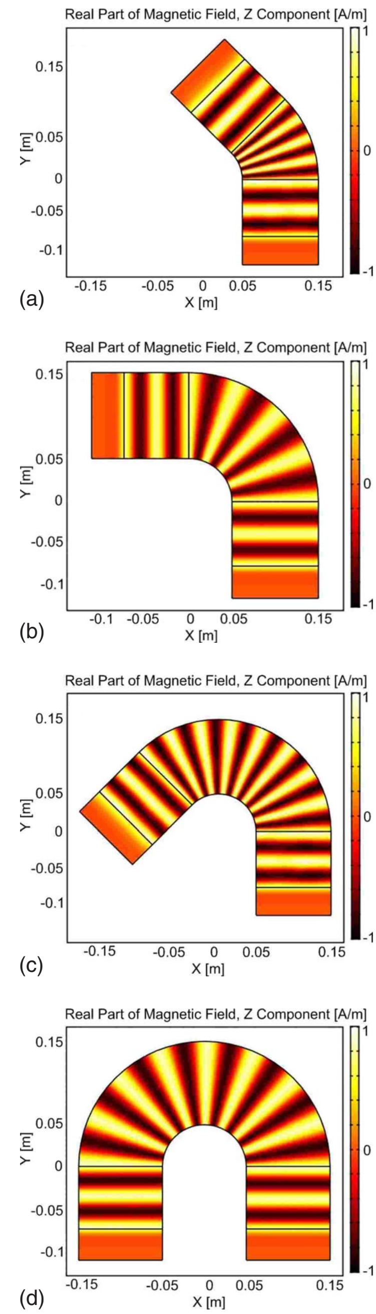

FIG. 2. (Color online) The distributions of magnetic fields (z components) inside the waveguide bends. (a) $\theta = \pi/4$ and $h = 4\lambda_0$. (b) $\theta = \pi/2$ and $h = 4\lambda_0$. (c) $\theta = 3\pi/4$ and $h = 8\lambda_0$. (d) $\theta = \pi$ and $h = 8\lambda_0$.

III. NUMERICAL SIMULATION RESULTS

In order to demonstrate the feasibility of the wave-bending structure, we make accurate numerical simulations

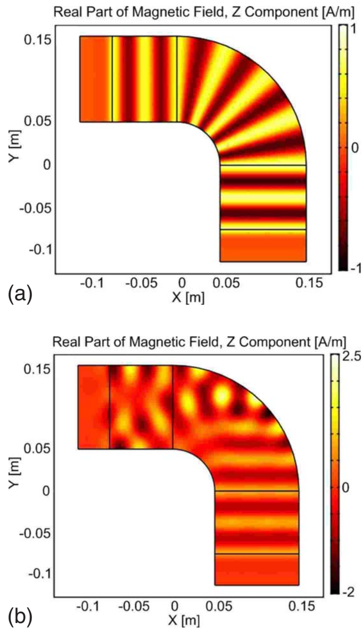

FIG. 3. (Color online) The distributions of magnetic fields (z components) inside the waveguide bends. (a) With lossy MTM. (b) Without MTM.

for the proposed structure based on the finite element method, where the perfectly matched layers are set as the absorbing boundary conditions in the two ends of the structure. TM-polarized plane waves are incident from the bottom with the working frequency of 8 GHz, for which the free-space wavelength is $\lambda_0=37.5$ mm. We remark that if the electromagnetic parameters were independent of the frequency, then the performance of wave-bending structure would not vary significantly versus frequency, because the transformation media parameters depend only on the geometry of the problem. However, all known metamaterials are frequency dispersive, and hence the working bandwidth of wave-bending structure will be limited.

First we consider the case when the wave-bending structure is bounded by perfectly electrical conducting (PEC) sheets, which simulates a waveguide bend. As we know, an empty waveguide bend will have significant reflections to incoming waves and destroy the field patterns. Filled with the ideal wave-bending structure composed of the inhomogeneous and anisotropic medium given by Eqs. (15) and (16), we expect that all incoming waves will be guided along the structure without any reflections.

Figure 2 illustrates the magnetic-field distributions inside the waveguide bends with different bending angles and different lengths h: (a) $\theta=\pi/2$ and $h=4\lambda_0$; (b) $\theta=\pi/2$ and $h=4\lambda_0$; (c) $\theta=3\pi/4$ and $h=8\lambda_0$; and (d) $\theta=\pi$ and $h=8\lambda_0$, in which $a=0.05$ m and $b=0.15$ m are fixed. From Fig. 2, it is clear that the incident waves travel through the waveguide bends reflectionlessly and keep their original field patterns in all cases.

As a comparison, Figure 3 demonstrates the magnetic-field distributions inside the waveguide bend with and without the designed MTM, in which all parameters are the same as those in Fig. 2(b) except that an electric and a magnetic

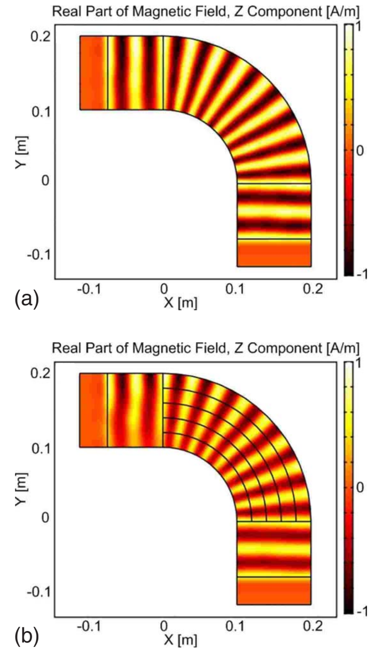

FIG. 4. (Color online) The distributions of magnetic fields (z components) inside the waveguide bends. (a) With the simplified-parameter inhomogeneous MTM. (b) With five-layer homogeneous MTMs.

loss tangent of 0.01 have been added in MTM. Obviously, the strength of magnetic field becomes weakened because the lossy material absorbs some of the forward traveling-wave power. The loss degrades the wave-bending performance in a straightforward way when the incident waves propagate through the bend, but the field patterns remain the same. No significant differences are observed when the loss tangent reduces to 0.001 (not shown). In the empty wave-

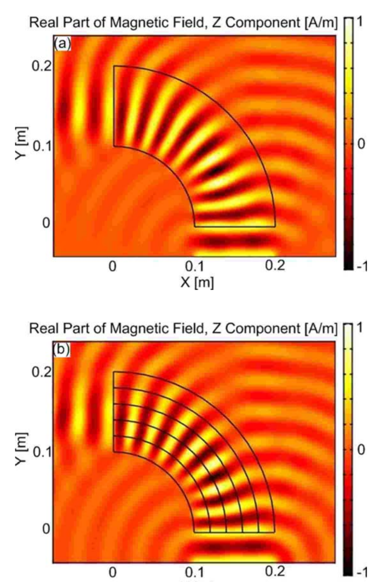

FIG. 5. (Color online) The distributions of magnetic fields (z components) inside the spatial wave-bending structures using simplified-parameter MTMs. (a) The case of inhomogeneous MTM. (b) The case of five-layered homogeneous MTMs.

guide bend, however, significant reflections are observed and the field patterns are completely destroyed.

Inhomogeneous and anisotropic MTMs with complex parameter tensors are difficult to realize in practice. We then consider the simplified-parameter MTM given by Eq. (17). Figure 4(a) shows the corresponding magnetic-field distribution, in which $a=0.1$ m, $b=0.2$ m, $h=8\lambda_0$, and $\theta=\pi/2$. In such a case, ε_r varies from 0.95 to 3.77, which can be easily realized using the artificial MTM. From Fig. 4(a), a very good transmission of EM waves has been observed. Furthermore, the realistic fabrication makes the artificial MTM be discrete values. Hence we approximate the inhomogeneous MTM by five-layered homogeneous MTMs, whose values of ε_r are 3.02, 2.16, 1.62, 1.26, and 1.01, respectively. The corresponding simulation results are shown in Fig. 4(b). Although there is a small distortion, the wave patterns are well kept after traveling through the bend.

Next we consider the case when the wave-bending structure is placed in the air. In such a case, the structure can bend the spatial waves to any designed directions, which has been verified by using the full-parameter MTMs with different bending angles. Here, we only show the simulation results of wave bending using the simplified-parameter MTMs. Figures 5(a) and 5(b) illustrate the magnetic-field distributions of the spatial wave-bending structures made of inhomogeneous MTM and layered homogeneous MTMs, respectively, in which all material parameters are the same as those in Fig. 4. Obviously, the realistic wave-bending structure guides the incoming plane waves to the desired direction, as shown in Fig. 5(b).

IV. CONCLUSIONS

We have proposed a wave-bending structure using the optical transformation, which is composed of simple and realizable MTMs. When such MTMs are filled inside metallic waveguide bends, the incident waves will be guided through the bends with very small reflections. When such MTMs are placed in the air, the incident waves will be bent to any designed directions. We also discussed the discretization effect of MTMs for real applications, which shows good performance in the wave bending.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation of China under Grants No. 60871016, No. 60671015, No. 60601002, and No. 60621002, in part by the Natural Science Foundation of Jiangsu Province under Grant No. BK2008031, in part by the National Basic Research Program (973) of China under Grant No. 2004CB719802, and in part by the 111 Project under Grant No. 111-2-05.

[1] A. J. Ward and J. B. Pendry, J. Mod. Opt. **43**, 773 (1996).
[2] J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
[3] U. Leonhardt, Science **312**, 1777 (2006).
[4] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
[5] M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, Photonics Nanostruct. Fundam. Appl. **6**, 87 (2008).
[6] S. A. Cummer, B. I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, Phys. Rev. E **74**, 036621 (2006).
[7] D. Schurig, J. B. Pendry, and D. R. Smith, Opt. Express **14**, 9794 (2006).
[8] W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, Nat. Photonics **1**, 224 (2007).
[9] G. W. Milton, M. Briane, and J. R. Willis, New J. Phys. **8**, 248 (2006).
[10] D. H. Kwon and D. H. Werner, Appl. Phys. Lett. **92**, 013505 (2008).
[11] Y. Huang, Y. Feng, and T. Jiang, Opt. Express **15**, 11133 (2007).
[12] W. X. Jiang, T. J. Cui, G. X. Yu, X. Q. Lin, Q. Cheng, and J. Y. Chin, J. Phys. D **41**, 085504 (2008).
[13] M. Rahm, S. A. Cummer, D. Schurig, J. B. Pendry, and D. R. Smith, Phys. Rev. Lett. **100**, 063903 (2008).
[14] A. V. Kildishev and V. M. Shalaev, Opt. Lett. **33**, 43 (2008).
[15] W. X. Jiang, T. J. Cui, Q. Cheng, J. Y. Chin, X. M. Yang, R. Liu, and D. R. Smith, Appl. Phys. Lett. **92**, 264101 (2008).
[16] M. Tsang and D. Psaltis, Phys. Rev. B **77**, 035122 (2008).
[17] F. Kong, B. Wu, J. A. Kong, J. Huangfu, S. Xi, and H. Chen, Appl. Phys. Lett. **91**, 253509 (2007).
[18] O. Ozgun and M. Kuzuoglu, IEEE Microw. Wirel. Compon. Lett. **17**, 754 (2007).
[19] H. Y. Chen and C. T. Chan, Appl. Phys. Lett. **90**, 241105 (2007).
[20] B. Donderici and F. L. Teixeira, IEEE Microw. Wirel. Compon. Lett. **18**, 233 (2008).
[21] M. Rahm, D. A. Roberts, J. B. Pendry and D. R. Smith, Opt. Express **16**, 11555 (2008).
[22] J. Huangfu, S. Xi, F. Kong, J. Zhang, H. Chen, D. Wang, B.-I. Wu, L. Ran, and J. A. Kong, J. Appl. Phys. **104**, 014502 (2008).