# Electromagnetic transparency by coated spheres with radial anisotropy

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We establish an account of electromagnetic scattering by coated spheres with radial dielectric and magnetic anisotropy. Within full-wave scattering theory, we show that the total scattering cross section $Q_s$ is strongly dependent on both the dielectric anisotropy and magnetic anisotropy. As a consequence, by a suitable adjustment of the radius ratio, one may make the anisotropic coated particle nearly transparent or invisible. In the quasistatic case, we take one step forward to derive the effective permittivity and permeability for the coated particle, and the near-zero scattering radius ratio can be well described within effective medium theory. To one's interest, the introduction of radial anisotropy is helpful to achieve better transparency quality such as a much smaller $Q_s$ and wider range of near-zero scattering ratio. Moreover, when the coated particle is anisotropic, the position of the near-zero scattering radius ratio can be tunable, resulting in a tunable electromagnetic cloaking.

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## I. INTRODUCTION

The creation of an electromagnetic cloak of invisibility has received much attention in recent years because of its potential applications in nanotechnology and engineering. For instance, planes and weapons with cloaks may be invisible to radar, which is very important for military purposes. To achieve "invisibility" or "low observability" for an object in electromagnetic waves, various methods or schemes were put forward such as the coordinate transformation [1–3], tunneling light transmittance [4,5], partial resonance [6,7], and zero-scattering mechanism in the dipolar limit [8]. Later, Cai *et al.* proposed the design of a nonmagnetic cylindrical cloak operating at optical frequencies based on a coordinate transformation [9,10]. The optical cloak is of great potential interest and brings us one step closer to the ultimate illusion of optical invisibility.

In addition, based on Mie scattering theory [11,12], the use of coating materials with metamaterials or plasmonic materials can drastically reduce the total scattering section of spherical or cylindrical objects, and hence make the objects "invisible" or "transparent" [13]. Since the realization of transparency relies on the nonresonant mechanism, it is almost invariant with the change of the shape, geometrical, and electromagnetic properties of the cloaked object [14]. Further investigation on cloaking and transparency was made for more realistic systems such as collections of particles with metamaterial and plasmonic covers [15], multilayered spheres, coated spheroids, and two-phase random mixtures [16]. In the quasistatic case, the transparency condition, under which the total scattering section of the composite particles is zero, was derived based on "neutral inclusion" idea [16]. Moreover, in metal and dielectric microspheres, with the proper design of the metal and dielectric shell, the dispersion spectra of the system can be tailored to make the forward-scattering cross section suppressed, resulting in plasmon-assisted transparency [17]. More recently, achieving transparency and maximizing scattering with metamaterial-coated conducting cylinders has been given [18].

In this paper, in order to achieve better transparency or invisibility, we would like to consider coated spheres with radial anisotropy in physical properties including both permittivity and permeability tensors. To one's interest, here the anisotropic tensors are assumed to be radially anisotropic; i.e., they are diagonal in spherical coordinates with values $\epsilon_r$ ($\mu_r$) in the radial direction and $\epsilon_t$ ($\mu_t$) in the tangential directions. Actually, such kind of anisotropy was indeed found in phospholipid vesicle systems [19,20] and in cell membranes containing mobile charges [21,22]. Furthermore, the radial anisotropy can be easily established from a problem of graphitic multishells [23], spherically stratified medium [24]. In the quasistatic limit, the third-order nonlinear optical susceptibility in graded mixtures [25] and the second and third-harmonic generations for a suspensions of coated particles [26] were investigated, and it was found that the choice of radial anisotropy plays a role in determining the magnitude of nonlinearity enhancement and resonant frequencies [25,26].

Motivated by the recent progress in an analytical demonstration of perfect invisibility for Pendry's cloak [27], the interactions of electromagnetic waves with the coated sphere of radial anisotropies in both electric and magnetic parameters have been established based on full-wave electromagnetic scattering theory [11]. Incidentally, scattering by solid particles of radial anisotropy was investigated analytically and numerically for parametric studies by the concept anisotropy ratio [28]. More recently, peculiar light scattering and the role of anisotropy in plasmonic resonances were studied [29], and dyadic Green's functions for arbitrarily mulitlayered radially anisotropic spheres were established [30].

In addition to the analytical establishment of Debye potentials and the scattering coefficients, we aim at the effects

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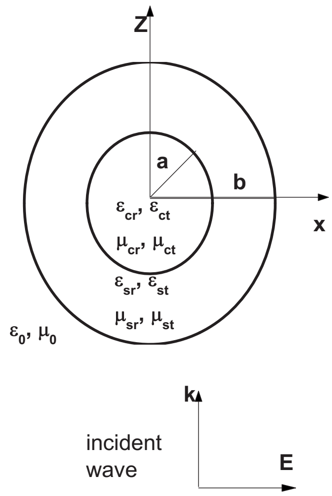


FIG. 1. Geometry of scattering of a plane wave by a coated sphere with permittivity and permeability tensors.

of anisotropic parameters in the core and/or the shell on the reduction of the total scattering section, so as to make the objects nearly "transparent" or "invisible." In the quasistatic limit, we present an effective medium theory that simultaneously determines the effective permittivity and permeability of coated particles with radial anisotropy. As a consequence, the approximate transparency conditions can be derived as a first step to design the reduction of total scattering section of coated particles whose dimensions are comparable with the wavelength of operation.

We turn now to the body of the paper. We derive the wave equations for the coated sphere with dielectric and magnetic anisotropies in both the core and the shell in Sec. II. In Sec. III, from the boundary conditions, Mie scattering coefficients are determined and the far-field solution is given. An effective medium theory for radially anisotropic magnetodielectric coated spheres is proposed and the near-transparency condition is derived in the quasistatic limit in Sec. IV. In Sec. V, numerical results are shown. The paper ends with a discussion and conclusion in Sec. VI.

## II. EQUATIONS FOR DEBYE POTENTIALS AND FIELDS

We consider electromagnetic scattering of a plane wave by a coated spherical particle with radial anisotropy when the polarized wave with unit amplitude $\mathbf{E}_i=\mathbf{e}_x e^{ik_0 z}$ is incident upon it (see Fig. 1), where $k_0 \equiv \omega\sqrt{\epsilon_0\mu_0}=\omega/c$ and $\epsilon_0$ and $\mu_0$ are the permittivity and permeability for a vacuum. The coated particle is composed of a core of radius $a$ and permittivity (permeability) tensors $\overleftrightarrow{\epsilon}_c$ ($\overleftrightarrow{\mu}_c$) and a shell of radius $b$ and tensors $\overleftrightarrow{\epsilon}_s$ ($\overleftrightarrow{\mu}_s$). Here we assume that the core and the shell are a kind of rotationally uniaxial material characterized by radial anisotropy,

$$\overleftrightarrow{\epsilon}_i=\begin{pmatrix}\epsilon_{ir} & 0 & 0\\ 0 & \varepsilon_{it} & 0\\ 0 & 0 & \epsilon_{it}\end{pmatrix},\quad \overleftrightarrow{\mu}_i=\begin{pmatrix}\mu_{ir} & 0 & 0\\ 0 & \mu_{it} & 0\\ 0 & 0 & \mu_{it}\end{pmatrix}, \tag{1}$$

where $\epsilon_{ir}$ ($\mu_{ir}$) and $\epsilon_{it}$ ($\mu_{it}$) stand for the permittivity (permeability) elements corresponding to the electric- (magnetic-) field vector normal to and tangent to the local optical axis for $i=c,s$. For a harmonic electromagnetic wave (i.e., $\mathbf{E}\sim e^{-i\omega t}$), with Maxwell equations, the time-independent parts of the local electric and magnetic fields are written as

$$\nabla\times\mathbf{H}=-i\omega\overleftrightarrow{\epsilon}_i\mathbf{E}, \tag{2}$$

$$\nabla\times\mathbf{E}=i\omega\overleftrightarrow{\mu}_i\mathbf{H}. \tag{3}$$

In spherical polar coordinates, Eqs. (1) and (2) become [31–33]

$$\frac{1}{r^2\sin\theta}\left(\frac{\partial(rH_\phi\sin\theta)}{\partial\theta}-\frac{\partial(rH_\theta)}{\partial\phi}\right)=-i\omega\epsilon_{ir}E_r,$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial H_r}{\partial\phi}-\frac{\partial(rH_\phi\sin\theta)}{\partial r}\right)=-i\omega\epsilon_{it}E_\theta,$$

$$\frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r}-\frac{\partial H_r}{\partial\theta}\right)=-i\omega\epsilon_{it}E_\phi \tag{4}$$

and

$$\frac{1}{r^2\sin\theta}\left(\frac{\partial(rE_\phi\sin\theta)}{\partial\theta}-\frac{\partial(rE_\theta)}{\partial\phi}\right)=-i\omega\mu_{ir}H_r,$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial E_r}{\partial\phi}-\frac{\partial(rE_\phi\sin\theta)}{\partial r}\right)=-i\omega\mu_{it}H_\theta,$$

$$\frac{1}{r}\left(\frac{\partial(rE_\theta)}{\partial r}-\frac{\partial E_r}{\partial\theta}\right)=-i\omega\mu_{it}H_\phi. \tag{5}$$

We shall solve Eqs. (4) and (5) together with boundary conditions including the continuities of $E_\theta$, $E_\phi$, $H_\theta$, and $H_\phi$. Actually, the solution of the above equations can be regarded as a superposition of two linearly independent fields such as $(E_{TM},H_{TM})$ and $(E_{TE},H_{TE})$, possessing the properties $E_{r,TM}=E_r$, $H_{r,TM}=0$ for transverse magnetic (TM) waves and $E_{r,TE}=0$, $H_{r,TE}=H_r$ for transverse electric (TE) ones [32,34]. Then the Debye potential for the TM case $\Phi_{TM}$ can be defined as [31]

$$E_{\theta,TM}=\frac{1}{r}\frac{\partial^2(r\Phi_{TM})}{\partial r\partial\theta},\quad E_{\phi,TM}=\frac{1}{r\sin\theta}\frac{\partial^2(r\Phi_{TM})}{\partial r\partial\phi},$$

$$H_{\theta,TM}=-\frac{i\omega\epsilon_{it}}{r\sin\theta}\frac{\partial(r\Phi_{TM})}{\partial\phi},\quad H_{\phi,TM}=\frac{i\omega\epsilon_{it}}{r}\frac{\partial(r\Phi_{TM})}{\partial\theta}. \tag{6}$$

By means of Eqs. (2)–(6), it can be shown that $\Phi_{TM}$ satisfies

$$\frac{\epsilon_{ir}}{\epsilon_{it}}\frac{1}{r}\frac{\partial^2(r\Phi_{TM})}{\partial r^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi_{TM}}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi_{TM}}{\partial\phi^2}+\omega^2\epsilon_{ir}\mu_{it}\Phi_{TM}=0. \tag{7}$$

The Debye potential for the TE case $\Phi_{TM}$ has a similar form as Eq. (7), i.e.,

$$\frac{\mu_{ir}}{\mu_{it}}\frac{1}{r}\frac{\partial^2(r\Phi_{TE})}{\partial r^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi_{TE}}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi_{TE}}{\partial\phi^2}+\omega^2\mu_{ir}\epsilon_{it}\Phi_{TE}=0. \tag{8}$$

By separation of variables, the solutions of Eqs. (7) and (8) are

$$r\Phi_{TM} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}[c_l^{TM}\psi_{v_{i1}^l}(k_{it}r) + d_l^{TM}\chi_{v_{i1}^l}(k_{it}r)]P_l^{(m)}(\cos\theta) \times [a_m^{TM}\cos(m\phi) + b_m^{TM}\sin(m\phi)], \quad (9)$$

$$r\Phi_{TE} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}[c_l^{TE}\psi_{v_{i2}^l}(k_{it}r) + d_l^{TM}\chi_{v_{i2}^l}(k_{it}r)]P_l^{(m)}(\cos\theta) \times [a_m^{TE}\cos(m\phi) + b_m^{TE}\sin(m\phi)],$$

where $a$, $b$, $c$, and $d$ are coefficients whose values are determined by the relevant boundary conditions, $P_l^{(m)}(\cos\theta)$ are the associated Legendre polynomials, $k_{it}=\omega\sqrt{\epsilon_{it}\mu_{it}}$,

$$v_{i1}^l = \sqrt{l(l+1)\frac{\epsilon_{it}}{\epsilon_{ir}} + \frac{1}{4}} - \frac{1}{2}, \quad v_{i2}^l = \sqrt{l(l+1)\frac{\mu_{it}}{\mu_{ir}} + \frac{1}{4}} - \frac{1}{2}, \quad (10)$$

and $\psi_{v_i}$ and $\chi_{v_i}$ are the Ricatti-Bessel functions defined by

$$\psi_v(x) = \sqrt{\frac{\pi x}{2}}J_{v+1/2}(x), \quad \chi_v(x) = -\sqrt{\frac{\pi x}{2}}N_{v+1/2}(x), \quad (11)$$

where $J_{v+1/2}(x)$ and $N_{v+1/2}(x)$ are the Bessel functions and Neumann functions. Once the potentials for the TM and TE cases are given, the complete solution of the fields can be written in the form by adding the two fields,

$$E_r = \frac{\partial^2(r\Phi_{TM})}{\partial r^2} + \omega^2\epsilon_{it}\mu_{it}r\Phi_{TM},$$

$$E_\theta = \frac{1}{r}\frac{\partial^2(r\Phi_{TM})}{\partial r\partial\theta} + \frac{\mathrm{i}\omega\mu_{it}}{r\sin\theta}\frac{\partial(r\Phi_{TE})}{\partial\phi},$$

$$E_\phi = \frac{1}{r\sin\theta}\frac{\partial^2(r\Phi_{TM})}{\partial r\partial\phi} - \frac{\mathrm{i}\omega\mu_{it}}{r}\frac{\partial(r\Phi_{TE})}{\partial\theta}$$

$$H_r = \frac{\partial^2(r\Phi_{TE})}{\partial r^2} + \omega^2\epsilon_{it}\mu_{it}r\Phi_{TE},$$

$$H_\theta = -\frac{\mathrm{i}\omega\epsilon_{it}}{r\sin\theta}\frac{\partial(r\Phi_{TM})}{\partial\phi} + \frac{1}{r}\frac{\partial^2(r\Phi_{TE})}{\partial r\partial\theta},$$

$$H_\phi = \frac{\mathrm{i}\omega\epsilon_{it}}{r}\frac{\partial(r\Phi_{TM})}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial^2(r\Phi_{TE})}{\partial r\partial\phi}. \quad (12)$$

To this point, we have derived the equations for Debye potentials and the electric and magnetic vectors in terms of Debye potentials. In what follows, we shall apply the formulas to the coated sphere with radial anisotropy in both the permittivity and permeability tensors.

## III. MIE COEFFICIENTS AND SCATTERING CROSS SECTION

When the incident plane polarized wave, propagating along the positive $z$ axis, has its electric vector of unit amplitude vibrating parallel to the $x$ axis, it is characterized by [31,35]

$$\mathbf{E}^{in} = \mathbf{e}_x e^{ik_0 r\cos\theta}, \quad \mathbf{H}^{in} = \mathbf{e}_y\sqrt{\frac{\epsilon_0}{\mu_0}}e^{ik_0 r\cos\theta}. \quad (13)$$

Correspondingly, the Debye potentials for incident fields can be expressed as

$$r\Phi_{TM}^{in} = \frac{1}{k_0^2}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}\psi_l(k_0 r)P_l^{(1)}(\cos\theta)\cos\phi,$$

$$r\Phi_{TE}^{in} = \frac{1}{k_0^2\sqrt{\mu_0/\epsilon_0}}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}\psi_l(k_0 r)P_l^{(1)}(\cos\theta)\sin\phi, \quad (14)$$

while for the scattering wave, they should be written as

$$r\Phi_{TM}^{sc} = -\frac{1}{k_0^2}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}A_l^{TM}\zeta_l(k_0 r)P_l^{(1)}(\cos\theta)\cos\phi,$$

$$r\Phi_{TE}^{sc} = -\frac{1}{k_0^2\sqrt{\mu_0/\epsilon_0}}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}A_l^{TE}\zeta_l(k_0 r)P_l^{(1)}(\cos\theta)\sin\phi, \quad (15)$$

where $\zeta_l(x) \equiv \psi_l(x) - \mathrm{i}\chi_l(x) = \sqrt{\pi x/2}H_{l+1/2}^1(x)$, with the first-kind Hankel functions $H_{l+1/2}^1(x)$.

For the core and the shell, the Debye potentials are described by Eq. (9). However, due to orthogonality between $P_l^{(1)}(x)$ and $P_l^{(m)}(x)$ for $m \neq 1$, only the term with $m=1$ survives. As a result, for the shell, the Debye potentials are

$$r\Phi_{TM}^{s} = -\frac{1}{k_{st}^2}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}[D_l^{TM}\psi_{v_{s1}^l}(k_{st}r) + E_l^{TM}\chi_{v_{s1}^l}(k_{st}r)]P_l^{(1)} \times(\cos\theta)\cos\phi,$$

$$r\Phi_{TE}^{s} = -\frac{1}{k_{st}^2\sqrt{\mu_0/\epsilon_0}}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}[D_l^{TE}\psi_{v_{s2}^l}(k_{st}r) + E_l^{TE}\chi_{v_{s2}^l}(k_{st}r)]P_l^{(1)}(\cos\theta)\sin\phi, \quad (16)$$

while for the core, they are given by

$$r\Phi_{TM}^{c} = -\frac{1}{k_{ct}^2}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}F_l^{TM}\psi_{v_{c1}^l}(k_{ct}r)P_l^{(1)}(\cos\theta)\cos\phi,$$

$$r\Phi_{TE}^{c} = -\frac{1}{k_{ct}^2\sqrt{\mu_0/\epsilon_0}}\sum_{l=1}^{\infty} i^{l-1}\frac{2l+1}{l(l+1)}F_l^{TE}\psi_{v_{c2}^l}(k_{ct}r)P_l^{(1)} \times(\cos\theta)\sin\phi. \quad (17)$$

To derive the scattering coefficients $A_l^{TE}$ and $A_l^{TM}$, the boundary conditions [which can be derived from Eq. (12)]

must be applied. For the present model, they are

$$\epsilon_{ct} r \Phi^{c}_{TM} = \epsilon_{st} r \Phi^{s}_{TM}, \quad \mu_{ct} r \Phi^{c}_{TE} = \mu_{st} r \Phi^{s}_{TE},$$

$$\frac{\partial(r\Phi^{c}_{TM})}{\partial r} = \frac{\partial(r\Phi^{s}_{TM})}{\partial r}, \quad \frac{\partial(r\Phi^{c}_{TE})}{\partial r} = \frac{\partial(r\Phi^{s}_{TE})}{\partial r}, \quad \text{at } r=a, \tag{18}$$

and

$$\epsilon_{st} r \Phi^{s}_{TM} = \epsilon_0 r(\Phi^{in}_{TM} + \Phi^{sc}_{TM}), \quad \mu_{st} r \Phi^{s}_{TE} = \mu_0 r(\Phi^{in}_{TE} + \phi^{sc}_{TE}),$$

$$\frac{\partial(r\Phi^{s}_{TM})}{\partial r} = \frac{\partial(r\Phi^{in}_{TM} + r\Phi^{sc}_{TM})}{\partial r},$$

$$\frac{\partial(r\Phi^{s}_{TE})}{\partial r} = \frac{\partial(r\Phi^{in}_{TE} + r\Phi^{sc}_{TE})}{\partial r}, \quad \text{at } r=b. \tag{19}$$

Substituting Eqs. (14)–(17) into Eqs. (18) and (19) yields

$$A_l^{TM} = \frac{\begin{vmatrix} \mu_{st}\psi_l(k_0 b) & \mu_0 \psi_{v_{s1}^l}(k_{st}b) & \mu_0 \chi_{v_{s1}^l}(k_{st}b) & 0 \\ k_{st}\psi_l'(k_0 b) & k_0 \psi'_{v_{s1}^l}(k_{st}b) & k_0 \chi'_{v_{s1}^l}(k_{st}b) & 0 \\ 0 & \mu_{ct}\psi_{v_{s1}^l}(k_{st}a) & \mu_{ct}\chi_{v_{s1}^l}(k_{st}a) & -\mu_{st}\psi_{v_{c1}^l}(k_{ct}a) \\ 0 & k_{ct}\psi'_{v_{s1}^l}(k_{st}a) & k_{ct}\chi'_{v_{s1}^l}(k_{st}a) & -k_{st}\psi'_{v_{c1}^l}(k_{ct}a) \end{vmatrix}}{\begin{vmatrix} \mu_{st}\zeta_l(k_0 b) & \mu_0 \psi_{v_{s1}^l}(k_{st}b) & \mu_0 \chi_{v_{s1}^l}(k_{st}b) & 0 \\ k_{st}\zeta_l'(k_0 b) & k_0 \psi'_{v_{s1}^l}(k_{st}b) & k_0 \chi'_{v_{s1}^l}(k_{st}b) & 0 \\ 0 & \mu_{ct}\psi_{v_{s1}^l}(k_{st}a) & \mu_{ct}\chi_{v_{s1}^l}(k_{st}a) & -\mu_{st}\psi_{v_{c1}^l}(k_{ct}a) \\ 0 & k_{ct}\psi'_{v_{s1}^l}(k_{st}a) & k_{ct}\chi'_{v_{s1}^l}(k_{st}a) & -k_{st}\psi'_{v_{c1}^l}(k_{ct}a) \end{vmatrix}}, \tag{20}$$

$$A_l^{TE} = \frac{\begin{vmatrix} \epsilon_{st}\psi_l(k_0 b) & \epsilon_0 \psi_{v_{s2}^l}(k_{st}b) & \epsilon_0 \chi_{v_{s2}^l}(k_{st}b) & 0 \\ k_{st}\psi_l'(k_0 b) & k_0 \psi'_{v_{s2}^l}(k_{st}b) & k_0 \chi'_{v_{s2}^l}(k_{st}b) & 0 \\ 0 & \epsilon_{ct}\psi_{v_{s2}^l}(k_{st}a) & \epsilon_{ct}\chi_{v_{s2}^l}(k_{st}a) & -\epsilon_{st}\psi_{v_{c2}^l}(k_{ct}a) \\ 0 & k_{ct}\psi'_{v_{s2}^l}(k_{st}a) & k_{ct}\chi'_{v_{s2}^l}(k_{st}a) & -k_{st}\psi'_{v_{c2}^l}(k_{ct}a) \end{vmatrix}}{\begin{vmatrix} \epsilon_{st}\zeta_l(k_0 b) & \epsilon_0 \psi_{v_{s2}^l}(k_{st}b) & \epsilon_0 \chi_{v_{s2}^l}(k_{st}b) & 0 \\ k_{st}\zeta_l'(k_0 b) & k_0 \psi'_{v_{s2}^l}(k_{st}b) & k_0 \chi'_{v_{s2}^l}(k_{st}b) & 0 \\ 0 & \epsilon_{ct}\psi_{v_{s2}^l}(k_{st}a) & \epsilon_{ct}\chi_{v_{s2}^l}(k_{st}a) & -\epsilon_{st}\psi_{v_{c2}^l}(k_{ct}a) \\ 0 & k_{ct}\psi'_{v_{s2}^l}(k_{st}a) & k_{ct}\chi'_{v_{s2}^l}(k_{st}a) & -k_{st}\psi'_{v_{c2}^l}(k_{ct}a) \end{vmatrix}}, \tag{21}$$

where the primes on $\psi$, $\chi$, and $\zeta$ denote differentiations with respect to the arguments.

The full-wave total scattering cross section of the coated particles is defined by

$$Q_s = \frac{2\pi}{k_0^2} \sum_{l=1}^{\infty} (2l+1)(|A_l^{TM}|^2 + |A_l^{TE}|^2). \tag{22}$$

In addition, the scattering (the far field) of linearly polarized light by the coated particle can be well described by the two basic scattering amplitudes [31,35]

$$S_1(\theta) = \sum_{l=1}^{\infty} \frac{2l+1}{l(l+1)} \left[ A_l^{TM} \frac{P_l^{(1)}(\cos\theta)}{\sin\theta} + A_l^{TE} \frac{dP_l^{(1)}(\cos\theta)}{d\theta} \right],$$

$$S_2(\theta) = \sum_{l=1}^{\infty} \frac{2l+1}{l(l+1)} \left[ A_l^{TE} \frac{P_l^{(1)}(\cos\theta)}{\sin\theta} + A_l^{TM} \frac{dP_l^{(1)}(\cos\theta)}{d\theta} \right]. \tag{23}$$

## IV. EFFECTIVE MEDIUM THEORY IN THE QUASISTATIC LIMIT

In this section, we aim at deriving the effective permittivity and permeability for the coated particles in long-

-wavelength and low-frequency limit. Note that the effective permittivity and permeability are isotropic for radial anisotropy [36]. In the long-wavelength limit $k_0 b \ll 1$, higher-order moments proportional to $(k_0 b)^{2l+1}$ are expected to be quite small, and we may keep only the dipole terms $l=1$. In this sense, the dipole field coefficient for the TM case is

$$A_1^{TM} = \frac{\begin{vmatrix} \mu_{st}\psi_1(k_0 b) & \mu_0 \psi_{v_{s1}^1}(k_{st} b) \\ k_{st}\psi_1'(k_0 b) & k_0 \psi_{v_{s1}^1}'(k_{st} b) \end{vmatrix} P_1 - \begin{vmatrix} \mu_{st}\psi_1(k_0 b) & \mu_0 \chi_{v_{s1}^1}(k_{st} b) \\ k_{st}\psi_1'(k_0 b) & k_0 \chi_{v_{s1}^1}'(k_{st} b) \end{vmatrix} P_2}{\begin{vmatrix} \mu_{st}\zeta_1(k_0 b) & \mu_0 \psi_{v_{s1}^1}(k_{st} b) \\ k_{st}\zeta_1'(k_0 b) & k_0 \psi_{v_{s1}^1}'(k_{st} b) \end{vmatrix} P_1 - \begin{vmatrix} \mu_{st}\zeta_1(k_0 b) & \mu_0 \chi_{v_{s1}^1}(k_{st} b) \\ k_{st}\zeta_1'(k_0 b) & k_0 \chi_{v_{s1}^1}'(k_{st} b) \end{vmatrix} P_2}, \tag{24}$$

with

$$P_1 = \mu_{st} k_{ct} \psi_{v_{c1}^1}(k_{ct} a) \chi_{v_{s1}^1}'(k_{st} a) - \mu_{ct} k_{st} \chi_{v_{s1}^1}(k_{st} a) \psi_{v_{c1}^1}'(k_{ct} a),$$

$$P_2 = \mu_{st} k_{ct} \psi_{v_{c1}^1}(k_{ct} a) \psi_{v_{s1}^1}'(k_{st} a) - \mu_{ct} k_{st} \psi_{v_{s1}^1}(k_{st} a) \psi_{v_{c1}^1}'(k_{ct} a). \tag{25}$$

Incidentally, the dipole coefficient for the TE case $A_1^{TE}$ can be obtained from Eqs. (24) and (25) by replacements of $\epsilon \to \mu$ and $v_{i1} \to v_{i2}$, respectively. In the long-wavelength ($k_0 b \ll 1$) and low-frequency ($k_{it} b \ll 1$) limits, since the arguments for the functions $\psi$, $\chi$, and $\zeta$ are small, the leading terms for these functions can only be retained, that is,

$$\psi_n(x) = \frac{\sqrt{\pi x/2}}{\Gamma(n+3/2)} \left(\frac{x}{2}\right)^{n+1/2}, \quad \chi_n(x) = \frac{\sqrt{\pi x/2}\,\Gamma(n+1/2)}{\pi} \left(\frac{2}{x}\right)^{n+1/2} \quad \text{as } x \to 0, \tag{26}$$

where $\Gamma(\cdots)$ is the Euler gamma function.

Substituting Eq. (26) into Eqs. (24) and (25) and those for $A_1^{TE}$ leads to

$$A_1^{TM} = C \frac{(\epsilon_{sr} v_{s1}^1 - \epsilon_0)[\epsilon_{cr} v_{c1}^1 + \epsilon_{sr}(1+v_{s1}^1)] + \left(\frac{a}{b}\right)^{(2v_{s1}^1+1)} [\epsilon_0 + (1+v_{s1}^1)\epsilon_{sr}](\epsilon_{cr} v_{c1}^1 - \epsilon_{sr} v_{s1}^1)}{(\epsilon_{sr} v_{s1}^1 + 2\epsilon_0)[\epsilon_{cr} v_{c1}^1 + \epsilon_{sr}(1+v_{s1}^1)] + \left(\frac{a}{b}\right)^{(2v_{s1}^1+1)} [2\epsilon_0 - (1+v_{s1}^1)\epsilon_{sr}](\epsilon_{sr} v_{s1}^1 - \epsilon_{cr} v_{c1}^1)},$$

$$A_1^{TE} = C \frac{(\mu_{sr} v_{s2}^1 - \mu_0)[\mu_{cr} v_{c2}^1 + \mu_{sr}(1+v_{s2}^1)] + \left(\frac{a}{b}\right)^{(2v_{s2}^1+1)} [\mu_0 + (1+v_{s2}^1)\mu_{sr}](\mu_{cr} v_{c2}^1 - \mu_{sr} v_{s2}^1)}{(\mu_{sr} v_{s2}^1 + 2\mu_0)[\mu_{cr} v_{c2}^1 + \mu_{sr}(1+v_{s2}^1)] + \left(\frac{a}{b}\right)^{(2v_{s2}^1+1)} [2\mu_0 - (1+v_{s2}^1)\mu_{sr}](\mu_{sr} v_{s2}^1 - \mu_{cr} v_{c2}^1)}, \tag{27}$$

with $C=\frac{2}{3}\mathrm{i}(k_0 b)^3$.

To search for the effective responses for coated particles, one always assumes that the coated particles of radial anisotropy are embedded in an effective medium with isotropic effective permittivity $\epsilon_{eff}$ and permeability $\mu_{eff}$. In this sense, they can be determined by the condition that both $A_1^{TM}$ and $A_1^{TE}$ vanish, if $\epsilon_0$ and $\mu_0$ are replaced by $\epsilon_{eff}$ and $\mu_{eff}$ [37]. As a result, we have

$$\epsilon_{eff} = \frac{\epsilon_{sr} v_{s1}^1 [\epsilon_{cr} v_{c1}^1 + (1+v_{s1}^1)\epsilon_{sr}] + (a/b)^{(2v_{s1}^1+1)} (1+v_{s1}^1)\epsilon_{sr}(\epsilon_{cr} v_{c1}^1 - \epsilon_{sr} v_{s1}^1)}{\epsilon_{cr} v_{c1}^1 + (1+v_{s1}^1)\epsilon_{sr} - (a/b)^{(2v_{s1}^1+1)} (\epsilon_{cr} v_{c1}^1 - \epsilon_{sr} v_{s1}^1)}, \tag{28}$$

$$\mu_{eff} = \frac{\mu_{sr} v_{s2}^1 [\mu_{cr} v_{c2}^1 + (1+v_{s2}^1)\mu_{sr}] + (a/b)^{(2v_{s2}^1+1)} (1+v_{s2}^1)\mu_{sr}(\mu_{cr} v_{c2}^1 - \mu_{sr} v_{s2}^1)}{\mu_{cr} v_{c2}^1 + (1+v_{s2}^1)\mu_{sr} - (a/b)^{(2v_{s2}^1+1)} (\mu_{cr} v_{c2}^1 - \mu_{sr} v_{s2}^1)}. \tag{29}$$

Now, the coated sphere particle is regarded as a uniform sphere in the long-wavelength and low-frequency limit, and the normalized scattering cross section may be simplified as [31]

$$\frac{Q_s}{\lambda_0^2} = \frac{64\pi^4}{3}\left(\frac{b}{\lambda_0}\right)^6\left(\left|\frac{\epsilon_{eff}-\epsilon_0}{\epsilon_{eff}+2\epsilon_0}\right|^2 + \left|\frac{\mu_{eff}-\mu_0}{\mu_{eff}+2\mu_0}\right|^2\right), \tag{30}$$

where $\lambda_0=2\pi/k_0$. It is evident that for $\epsilon_{eff}=\epsilon_0$ and $\mu_{eff}=\mu_0$, $Q_s$ is zero, making the coated particles invisible or transparent to an outside observer. However, for magnetodielectric coated particles, the conditions $\epsilon_{eff}=\epsilon_0$ and $\mu_{eff}=\mu_0$ cannot be satisfied simultaneously for a given $a/b$. Consequently, the coated particle may be nearly invisible if $Q_s$ achieves a minimum for certain $a/b$. The condition for near transparency is determined by the relation

$$\frac{(\epsilon_{eff}-\epsilon_0)\epsilon_0}{(\epsilon_{eff}+2\epsilon_0)^3}\frac{d\epsilon_{eff}}{d(a/b)} + \frac{(\mu_{eff}-\mu_0)\mu_0}{(\mu_{eff}+2\mu_0)^3}\frac{d\mu_{eff}}{d(a/b)} = 0. \tag{31}$$

For nonmagnetic (or pure magnetic) coated particles, $\mu_{eff}$ ($\epsilon_{eff}$) is equal to $\mu_0$ ($\epsilon_0$). In this connection, to make the coated particle (nearly) transparent, from Eq. (31) one yields $\epsilon_{eff}=\epsilon_0$ ($\mu_{eff}=\mu_0$) corresponding to

$$\frac{a}{b} = \left\{\frac{(\epsilon_{sr}v_{s1}^1-\epsilon_0)[\epsilon_{cr}v_{c1}^1+\epsilon_{sr}(v_{s1}^1+1)]}{(\epsilon_{sr}v_{s1}^1-\epsilon_{cr}v_{c1}^1)[\epsilon_0+\epsilon_{sr}(1+v_{s1}^1)]}\right\}^{1/(2v_{s1}^1+1)} \tag{32}$$

for nonmagnetic particles and

$$\frac{a}{b} = \left\{\frac{(\mu_{sr}v_{s2}^1-\mu_0)[\mu_{cr}v_{c2}^1+\mu_{sr}(v_{s2}^1+1)]}{(\mu_{sr}v_{s2}^1-\mu_{cr}v_{c2}^1)[\mu_0+\mu_{sr}(1+v_{s2}^1)]}\right\}^{1/(2v_{s2}^1+1)} \tag{33}$$

for magnetic materials. It is evident that for isotropic coated particles, Eq. (32) is exactly the same as the transparency condition derived by Alu and Engheta [13] and Zhou and Hu [16], respectively.

## V. NUMERICAL RESULTS

In what follows, we perform numerical calculations for the normalized scattering section $Q_s/\lambda_0^2$ with Eq. (22) (valid for general full-wave scattering), and Eq. (30) (valid for the quasistatic limit including long-wavelength and low-frequency limits).

In Fig. 2, $Q_s$ is shown for the coated dielectric sphere with plasmonic shell and the core of radial dielectric anisotropy as a function of the the radius ratio $a/b$ for various particles sizes. We find that for small particle sizes such as $b=0.01\lambda_0$ [see Fig. 2(b)], Eq. (30) yields the same results as the Mie full-wave scattering theory, Eq. (22), as expected. The scattering cross section is almost zero, indicating the transparency or "invisibility" of the particles, when $a/b$ takes some values, which can be exactly determined by the relation that $\epsilon_{eff}=\epsilon_0$ [see Fig. 2(a)]. In addition, for small $a/b$, one can get the effective permittivity $\epsilon_{eff}$ of coated particles to be $-2\epsilon_0$. As a result, strong resonant behavior takes place as predicted from Eq. (30). For large particles in Figs. 2(c) and 2(d), the number of multipolar terms contributing to the scattering increases rapidly, and hence one cannot resort to effective medium theory. In this situation, we still find that $Q_s$ exhibits a significant reduction at small radius ratio, in

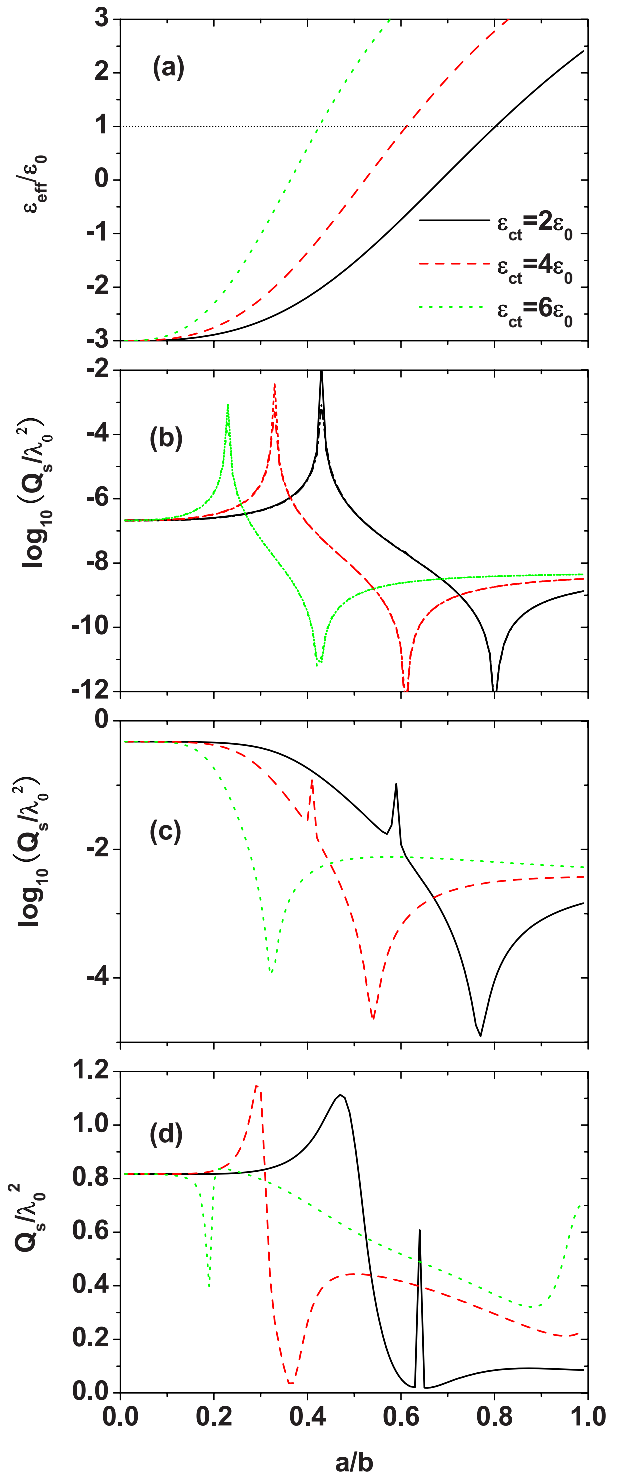


FIG. 2. (Color online) (a) The effective permittivity $\epsilon_{eff}$ of the coated dielectric particle versus radius ratio $a/b$ for $\epsilon_{ct}=2\epsilon_0$ (black solid line), $4\epsilon_0$ (red dashed line), and $6\epsilon_0$ (green dotted line), (b) normalized scattering section $Q_s/\lambda_0^2$ of the coated particle for the full-wave case with $b=0.01\lambda_0$ for $\epsilon_{ct}=2\epsilon_0$ (black solid line), $4\epsilon_0$ (red dashed line), and $6\epsilon_0$ (green dotted line) and for the effective medium for $\epsilon_{ct}=2\epsilon_0$ (black dash-dotted line), $4\epsilon_0$ (red short dashed line), and $6\epsilon_0$ (green short dotted line), and (c) and (d) $Q_s/\lambda_0^2$ for the full-wave case with $b=0.1\lambda_0$ and $b=0.2\lambda_0$ for $\epsilon_{ct}=2\epsilon_0$ (black solid line), $4\epsilon_0$ (red dashed line), and $6\epsilon_0$ (green dotted line). Other parameters are $\epsilon_{cr}=4\epsilon_0$, $\epsilon_{sr}=\epsilon_{st}=-3\epsilon_0$, and $\mu_{cr}=\mu_{ct}=\mu_{sr}=\mu_{st}=\mu_0$.

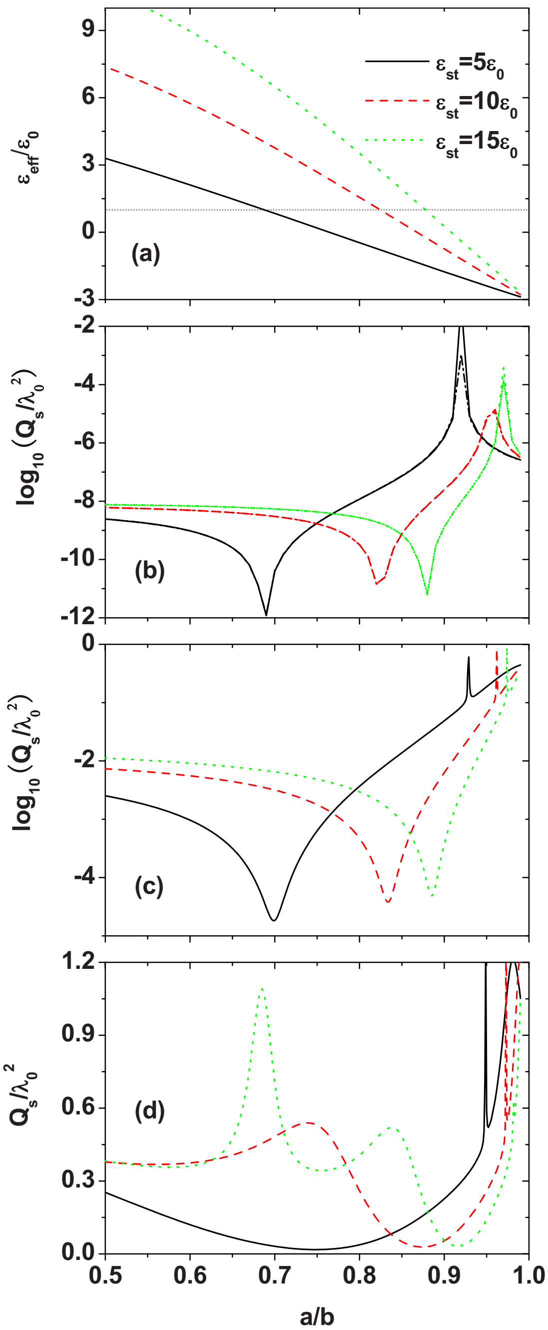


FIG. 3. (Color online) Similar as Fig. 2, but for the radial dielectric anisotropy in the shell. Other parameters are $\epsilon_{cr}=\epsilon_{ct}=-3\epsilon_0$, $\epsilon_{sr}=10\epsilon_0$, and $\mu_{cr}=\mu_{ct}=\mu_{sr}=\mu_{st}=\mu_0$.

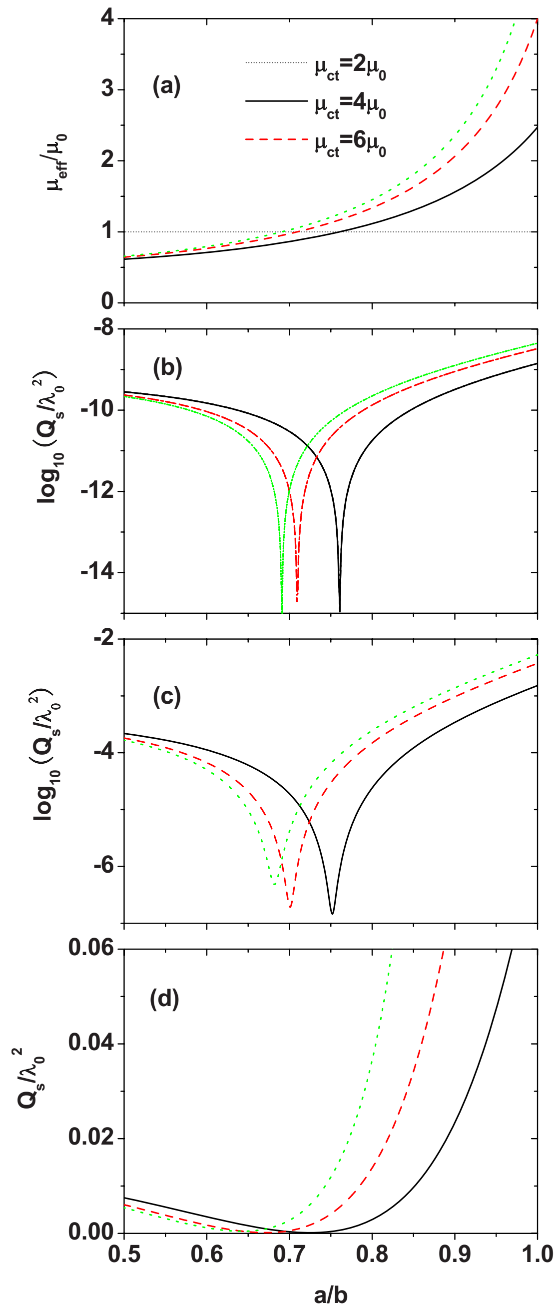


FIG. 4. (Color online) Same as Fig. 2, but for coated magnetic particles with radial magnetic anisotropy in the core. Other parameters are $\mu_{cr}=4\mu_0$, $\mu_{sr}=\mu_{st}=0.5\mu_0$, and $\epsilon_{cr}=\epsilon_{ct}=\epsilon_{sr}=\epsilon_{st}=\epsilon_0$.

comparison with the quasistatic case. To one's interest, when the dielectric anisotropy is taken into account, decreasing $\epsilon_{ct}$ may result in a much lower scattering section (near zero scattering section) and thereby better transparency at the cost of large size of plasmonic shell. For instance, for small dielectric anisotropy $\epsilon_{ct}=2$, one achieves a smaller scattering section than the one for the isotropic case, $\epsilon_{ct}=4$. One further notes that from Fig. 2(d), close to the "near-zero-scattering" ratio, a sharp peak exists, resulting from the resonant phenomenon of $A_3^{TM}$ (not shown here).

In Fig. 3, we apply the transparency phenomenon to a plasmonic particle with a coating shell of radial dielectric anisotropy. In the quasistatic limit, for an isotropic shell $\epsilon_{sr}=\epsilon_{st}=10$, one would expect that the coated particle is transparent for $a/b\approx 0.825$, at which $\epsilon_{eff}=\epsilon_0$, as shown in Fig. 3(a). Good agreement is again found between full-wave theory and effective medium theory. Incidentally, since the core is plasmonic, the resonance takes place in the thin shell limit $a/b\to 1$. A more interesting phenomenon is that through the suitable adjustment of dielectric anisotropy in the shell, one can achieve a much small scattering section and tune the near-zero scattering radius ratio. In detail, the minimum of $Q_s$ for the anisotropic case with $\epsilon_{st}=5$, which occurs at $a/b\approx 0.69$, is one order smaller than the one for the

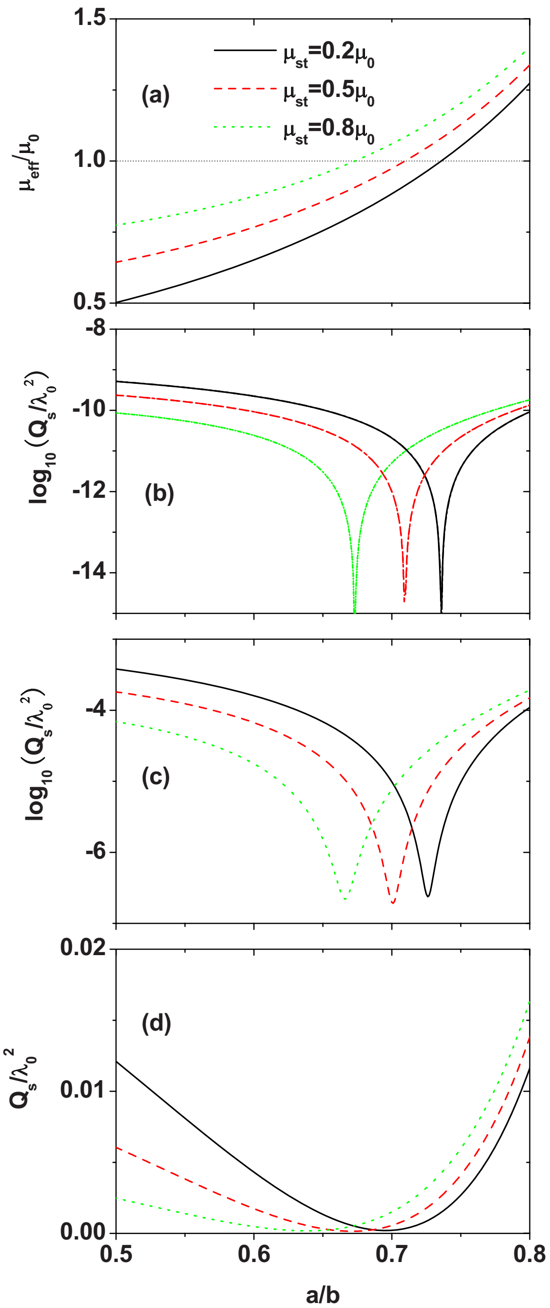


FIG. 5. (Color online) Same as Fig. 4, but for coated magnetic particles with radial magnetic anisotropy in the shell. Parameters are $\mu_{cr}=\mu_{ct}=4\mu_0$, $\mu_{sr}=0.5\mu_0$, and $\epsilon_{cr}=\epsilon_{ct}=\epsilon_{sr}=\epsilon_{st}=\epsilon_0$.

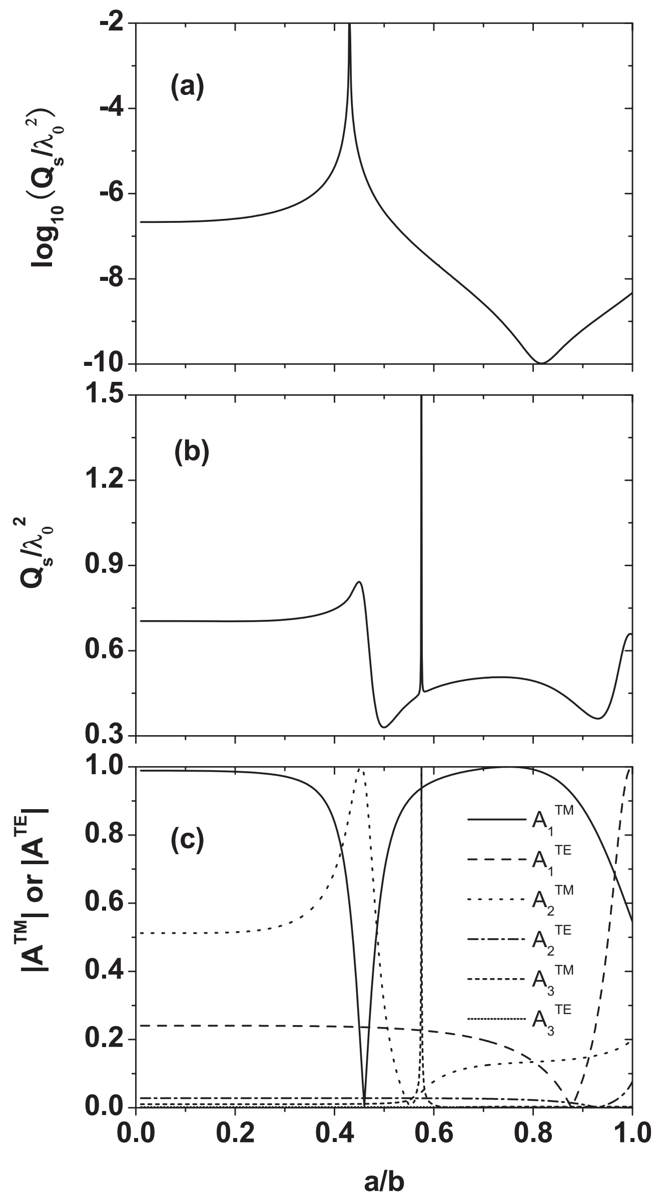


FIG. 6. $Q_s/\lambda_0^2$ for (a) $b=0.01\lambda_0$ and (b) $b=0.2\lambda_0$, and (c) contributions of the several scattering coefficients such as $A_1^{TM}$ (solid line), $A_1^{TE}$ (dashed line), $A_2^{TM}$ (dotted line), $A_2^{TE}$ (dash-dotted line), $A_3^{TM}$ (short dashed line), and $A_3^{TE}$ (short dotted line) versus $a/b$ for the magnetodielectric coated particle with both radially dielectric and magnetic anisotropy. Parameters are $\epsilon_{cr}=4\epsilon_0$, $\epsilon_{ct}=2\epsilon_0$, $\mu_{cr}=\mu_{ct}=\mu_0$, $\epsilon_{sr}=\epsilon_{st}=-3\epsilon_0$, $\mu_{sr}=0.2\mu_0$, and $\mu_{st}=0.5\mu_0$.

isotropic case with $\epsilon_{st}=10$ at $a/b=0.825$, thus resulting in much better transparent behavior. Therefore, to get better transparency, we require the permittivity in the radial direction to be larger than the one in the transverse direction. This should be in contrast with that in Ref. [3], in which the permittivity in the radial direction is smaller than the tangential one. Actually, the latter is based on the coordinate transformation technique, while our work is based on the dipole-canceling mechanism [13]. As for large coated particles, more resonant peaks or bands are predicted due to high-order resonant modes [see Fig. 3(d)]. Here we emphasize that since the transparency phenomenon does not result from the resonant effect, one would expect a relatively broad range for the radius ratio, around which the scattering section is almost zero. By decreasing the shell anisotropy $\epsilon_{st}$, the "near-zero-scattering" ratio band becomes much broader, accompanied with much less scattering. As a result, the adjustment of shell anisotropy may be helpful to improve the transparency quality.

In Figs. 4 and 5, only permeability has radial anisotropy. It is observed that in the quasistatic limit [see Figs. 4(b) and 5(b)], the scattering section can be well described by effective medium theory and the transparency phenomenon takes place at the radius ratio, corresponding to the one at which

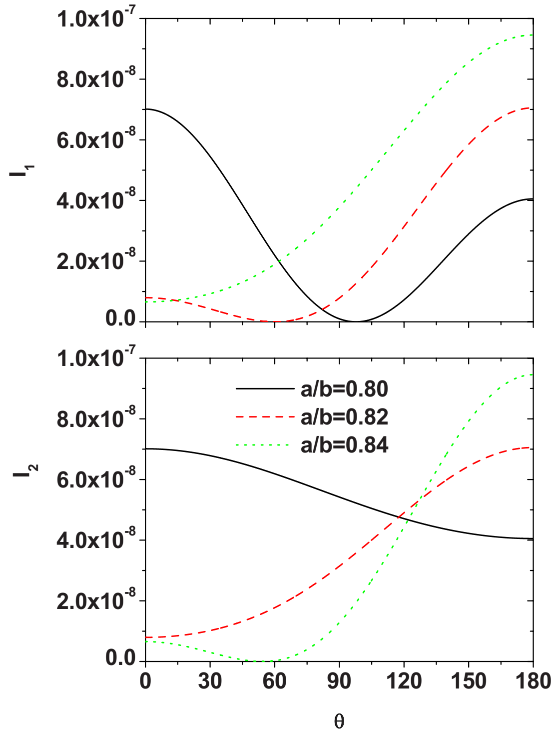


FIG. 7. (Color online) Normalized differential cross sections $I_1=|S_1(\theta)|^2/(k_0^2\pi b^2)$ and $I_2=|S_2(\theta)|^2/(k_0^2\pi b^2)$ as a function of the scattering angle $\theta$ for $b=0.01\lambda_0$ and $a/b=0.80$ (black solid line), 0.82 (red dashed line), and 0.84 (green dotted line). Other parameters are the same as in Fig. 6.

the effective permeability is taken as $\mu_0$ instead of $\epsilon_0$ [see Figs. 4(a) and 5(a)]. Due to the choice of the positive permeability for the core and the shell, no resonance can be excited. As a result, there are no sharp peaks for these cases. On the other hand, for large particle size [see Figs. 4(c) and 5(c)], a near-zero scattering section, making the particle invisible or transparent, is still found in a wide range of radius ratio $a/b$.

In Fig. 6, we examine the case for the magnetodielecric coated particle in which the permittivity and permeability are radially anisotropic. In the quasistatic limit, both the full-wave expression, Eq. (22), and effective medium theory, Eq. (30), predict that there exist one enhancement peak and one near-transparent position, characterized by a nearly zero $Q_s$. Since no magnetic resonance occurs, the resonant peak is due to the electric resonance. However, magnetic and electric spectra contribute to the electromagnetic transparency of the coated particle at $a/b=0.82$, which should be determined by Eq. (31), rather than a simple formula $\epsilon_{eff}=\epsilon_0$ or $\mu_{eff}=\mu_0$. For large coated particles, a high-scattering resonant peak appears due to the $A_3^{TM}$ term as shown in Fig. 6(c). In addition, the minimum of $Q_s$ is not zero, but with an appreciable scattering. This mainly results from scattering terms such as $A_1^{TE}$ and $A_2^{TM}$. Therefore, one may expect to tune the anisotropic parameters to decrease the magnitudes of $A_1^{TE}$ and $A_2^{TM}$, so as to realize the near-transparency condition in this region.

Compared with the isotropic coated sphere [13], the anisotropic coated sphere has introduced more physical parameters for us to achieve transparency. For large particles, higher-order scattering coefficients $A_l^{TM}$ and $A_l^{TM}$ may tend to zero by our suitable adjustment of these anisotropic physical parameters.

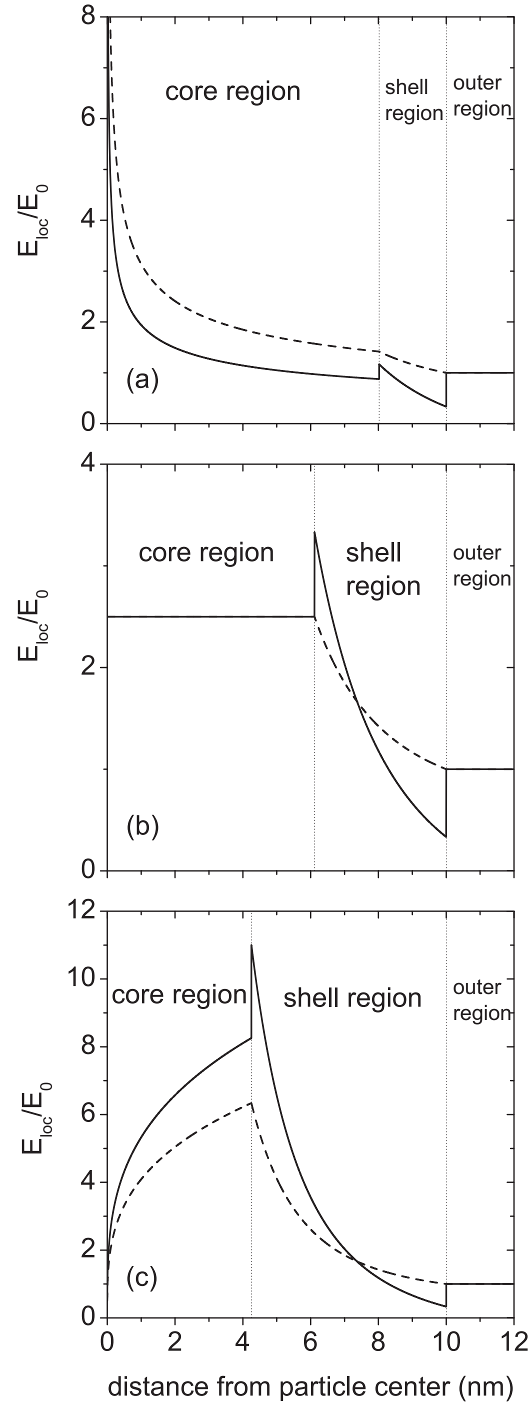


FIG. 8. Spatial dependence of the the electric-field ratio in and around coated particle at transparency condition for (a) $\epsilon_{ct}=2\epsilon_0$, (b) $\epsilon_{ct}=4\epsilon_0$, and (c) $\epsilon_{ct}=6\epsilon_0$. The solid curve for the field parallel to the applied electric field polarization direction $\mathbf{E}_0$, and the dotted line for the one perpendicular to $\mathbf{E}_0$. Other parameters are the same as in Fig. 2.

Next, we would like to aim at the normalized differential scattering sections $I_1=|S_1(\theta)|^2/(k_0^2\pi b^2)$ (the scattering pattern in the $yz$ plane) and $I_2=|S_2(\theta)|^2/(k_0^2\pi b^2)$ (the scattering pattern in the $xz$ plane) in Fig. 7. These parameters are very important because they can be used to calculate the various experimentally observable scattering variables [19]. In the quasistatic limit, as we have shown, the coated particles possess a near-zero scattering section for $a/b\approx 0.82$ and thus are transparent. However, as to the discussion of the differential scattering section, such a choice is not always perfect. For instance, from Fig. 7(a), we find that the scattered power for $a/b=0.82$ (at which $Q_s\approx 0$) is larger than the one for $a/b=0.80$ in the backward direction and the one for $a/b=0.84$ in the forward direction. A similar discussion was performed by Zhou and Hu for acoustic wave transparency [38]. However, our model is quite different from Pendry's cloak, in which zero backscattering is always found even when a type of loss is introduced [27].

In the end, electric field distributions for the dielectric coated particle with radial anisotropy in the core is shown in Fig. 8. For simplicity, the quasistatic case is studied. For isotropic case [see Fig. 8(b)], it is found that the local field is uniform in the core. However, the introduction of the dielectric anisotropy leads to large fluctuations in the local field, which may be useful for the enhancement of optical nonlinearity. Here, the principal observation from these curves is that the field outside the particles is nothing but the applied field, which proves that the scattered fields are indeed canceled. Therefore, the cloaking mechanism here is distinguished from Pendry's idea [1], where the incident fields cannot penetrate into the core and the fields in the core are always zero.

## VI. CONCLUSION AND DISCUSSION

In this paper, we have established electromagnetic scattering theory by coated particles of radial electric and magnetic anisotropies. Effects of anisotropic physical parameters in both the core and the shell on the total scattering section are systematically investigated. Based on full-wave scattering theory, we show that by a suitable adjustment of the radius ratio, one may make the coated particle nearly transparent or invisible. In the quasistatic case, the effective medium concept is valid, and we derive effective permittivity and permeability for the coated particle. The near-zero scattering radius ratio can be well described within effective medium theory. It shows that the introduction of radial anisotropy may be helpful to achieve much better transparency such as much lower $Q_s$ and wider near-zero scattering ratio, and to adjust the position of the radius ratio, exhibiting a tunable electromagnetic transparency.

Here we would like to add a few comments. The key to realize the electromagnetic transparency of the coated particles lies in nearly zero value of the numerator of the scattering coefficients. On the other hand, if one needs large scattering, the anomalous plasmonic resonance should be induced. In this connection, the strong electromagnetic resonance in a large collection of coated particles may create the negative permeability. As a consequence, one may realize double-negative metamaterials with coated nonmagnetic spheres of radially dielectric anisotropy [39]. Therefore, it is of interest to develop effective medium theory for coated particles of radial anisotropy beyond the quasistatic limit and to investigate the effect of anisotropic parameters on the resonant behavior of the effective permittivity and permeability.

Due to the reduction of both backscattering and forward scattering, two-dimensional cylindrical cloaks were realized in experiment [3]. Accordingly, full-wave, finite-element numerical simulations for cylindrical invisibility cloaks were done [40], and mirage effect whereby the source seems to radiate from a shifted location, were observed [41]. Theoretically, Zhang *et al.* [42] and Ruan *et al.* [43] investigated the electromagnetic response of cylindrical invisibility cloaks within the framework of electromagnetic wave scattering theory. In this regard, our work can be generalized to two-dimensional cylindrical invisibility cloaks with radial anisotropy without any difficulty. In the quasistatic case, for coated cylinders with radially dielectric anisotropy, partial resonance conditions are derived as $s\epsilon_{sr}+\epsilon_0=0$ and $s\epsilon_{sr}+\epsilon_c=0$ with $s=\sqrt{\epsilon_{st}/\epsilon_{sr}}$ and $\epsilon_c$ the permittivity of the isotropic core [44]. When the partial resonance is satisfied, the cloaking may be proved for finite collections of polarizable line dipoles that lie within a specific distance from a coated cylinder with radial anisotropy [7].

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