Zero-temperature dynamics in the two-dimensional axial next-nearest-neighbor Ising model

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We investigate the dynamics of a two-dimensional axial next-nearest-neighbor Ising model following a quench to zero temperature. The Hamiltonian is given by $H=-J_0\Sigma_{i,j=1}^{L}S_{i,j}S_{i+1,j}-J_1\Sigma_{i,j=1}(S_{i,j}S_{i,j+1}-\kappa S_{i,j}S_{i,j+2})$. For $\kappa<1$, the system does not reach the equilibrium ground state but slowly evolves to a metastable state. For $\kappa>1$, the system shows a behavior similar to that of the two-dimensional ferromagnetic Ising model in the sense that it freezes to a striped state with a finite probability. The persistence probability shows algebraic decay here with an exponent $\theta=0.235\pm0.001$ while the dynamical exponent of growth $z=2.08\pm0.01$. For $\kappa=1$, the system belongs to a completely different dynamical class; it always evolves to the true ground state with the persistence and dynamical exponent having unique values. Much of the dynamical phenomena can be understood by studying the dynamics and distribution of the number of domain walls. We also compare the dynamical behavior to that of a Ising model in which both the nearest and next-nearest-neighbor interactions are ferromagnetic.

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I. INTRODUCTION

The dynamics of Ising models is a much studied phenomenon and has emerged as a rich field of present-day research. Models having identical static critical behavior may display different behavior when dynamic critical phenomena are considered [1]. An important dynamical feature commonly studied is the quenching phenomenon below the critical temperature. In a quenching process, the system has a disordered initial configuration corresponding to a high temperature and its temperature is suddenly dropped. This results in quite a few interesting phenomena like domain growth [2,3], persistence [4–8], etc.

In one dimension, a zero-temperature quench of the Ising model ultimately leads to the equilibrium configuration, i.e., all spins point up (or down). The average domain size D increases in time t as $D(t)\sim t^{1/z}$, where z is the dynamical exponent associated with the growth. As the system coarsens, the magnetization also grows in time as $m(t)\sim t^{1/2z}$. In two or higher dimensions, however, the system does not always reach equilibrium [8] although these scaling relations still hold good.

Apart from the domain growth phenomenon, another important dynamical behavior commonly studied is persistence. In the Ising model, in a zero-temperature quench, persistence is simply the probability that a spin has not flipped until time t and is given by $P(t)\sim t^{-\theta}$. θ is called the persistence exponent and is unrelated to any other known static or dynamic exponent.

Drastic changes in the dynamical behavior of the Ising model in the presence of a competing next-nearest-neighbor interaction have been observed earlier [9–11]. The one-dimensional axial next-nearest-neighbor Ising (ANNNI) model with L spins is described by the Hamiltonian

$$H=-J\sum_{i=1}^{L}(S_iS_{i+1}-\kappa S_iS_{i+2}). \qquad (1)$$

Here it was found that for $\kappa<1$, under a zero-temperature quench with single-spin-flip Glauber dynamics, the system does not reach its true ground state. (The ground state is ferromagnetic for $\kappa<0.5$, antiphase for $\kappa>0.5$, and highly degenerate at $\kappa=0.5$ [12].) On the contrary, after an initial short time, domain walls become fixed in number but remain mobile at all times, thereby making the persistence probability go to zero in a stretched exponential manner. For $\kappa>1$ on the other hand, although the system reaches the ground state at long times, the dynamical exponent and the persistence exponent are both different from those of the Ising model with only nearest-neighbor interactions [10].

The above observations, and the additional fact that even in the two-dimensional nearest-neighbor Ising model frozen-in striped states appear in a zero-temperature quench [8], suggest that the two-dimensional Ising model in the presence of competing interactions could show novel dynamical behavior. In the present work, we have introduced such an interaction (along one direction) in the two-dimensional Ising model, thus making it equivalent to the ANNNI model in two dimensions precisely. The Hamiltonian for the two-dimensional ANNNI model on an $L\times L$ lattice is given by

$$H=-J_0\sum_{i,j=1}^{L}S_{i,j}S_{i+1,j}-J_1\sum_{i,j=1}^{L}(S_{i,j}S_{i,j+1}-\kappa S_{i,j}S_{i,j+2}). \qquad (2)$$

Henceforth, we will assume the competing interaction to be along the x (horizontal) direction, while in the y (vertical) direction, there is only ferromagnetic interaction.

Although the thermal phase diagram of the two-dimensional ANNNI model is not known exactly, the ground state is known and simple. If one calculates the magnetization along the horizontal direction only, then for $\kappa<0.5$ there is ferromagnetic order and antiphase order for $\kappa>0.5$. Again, $\kappa=0.5$ is the fully frustrated point where the ground state is highly degenerate. On the other hand, there is always ferromagnetic order along the vertical direction. In Fig. 1, we show the ground state spin configurations along the x direction for different values of κ.

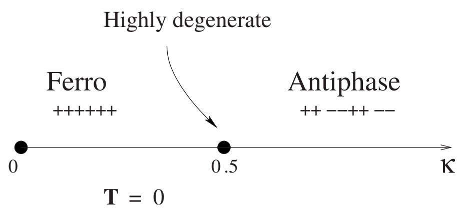

FIG. 1. Ground state (temperature $T=0$) spin configurations along the x direction for different values of κ. In the ferromagnetic phase, there is a twofold degeneracy and in the antiphase the degeneracy is fourfold. The ground state is infinitely degenerate at the fully frustrated point $\kappa=0.5$.

In Sec. II, we have given a list of the quantities calculated. In Sec. III, we discuss the dynamic behavior in detail. In order to compare the results with those of a model without competition, we have also studied the dynamical features of a two-dimensional Ising model with ferromagnetic next-nearest-neighbor interaction, i.e., the model given by Eq. (2) in which $\kappa<0$. These results are also presented in Sec. III. Discussions and concluding statements are made in the last section.

II. QUANTITIES CALCULATED

We have estimated the following quantities in the present work.

(1) The persistence probability $P(t)$: As already mentioned, this is the probability that a spin does not flip until time t. In case the persistence probability shows a power law form, $P(t)\sim t^{-\theta}$, one can use the finite-size-scaling relation [13]

$$P(t,L) \sim t^{-\theta} f(L/t^{1/z}). \tag{3}$$

For finite systems, the persistence probability saturates at a value $L^{-\alpha}$ at large times. Therefore, for $x \ll 1$, $f(x)\sim x^{-\alpha}$ with $\alpha=z\theta$. For large x, $f(x)$ is a constant.

It has been shown that the exponent α is related to the fractal dimension of the fractal formed by the persistent spins [13]. Here we obtain an estimate of α using the above analysis.

(2) Number of domain walls N_D: Taking a single strip of L spins at a time, one can calculate the number of domain walls for each strip and determine the average. In the $L\times L$ lattice, we consider the fraction $f_D=N_D/L$ and study the behavior of f_D as a function of time. One can take strips along both the x and y directions (see Fig. 2 where the calculation of f_D in simple cases has been illustrated). As the system is anisotropic, it is expected that the two measures f_{D_x} along the x direction and f_{D_y} along the y direction will show different dynamical behavior in general. The domain size D increases as $t^{1/z}$ as already mentioned and it has been observed earlier that the dynamic exponent occurring in coarsening dynamics is the same as that occurring in the finite-size scaling of $P(t)$ [Eq. (3)] [13]. Although we do not calculate the domain sizes, the average number of domain walls per strip is shown to follow a dynamics given by the same exponent z, at least for $\kappa>1$.

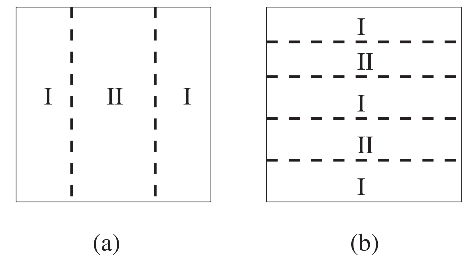

FIG. 2. Schematic pictures of configurations with flat interfaces separating domains of type I and II: (a) when the interface lies parallel to the y axis, we have nonzero f_{D_x} ($=2/L$ in this particular case) and (b) with interfaces parallel to the x axis we have nonzero f_{D_y} ($=4/L$ here).

(3) Distribution $P(f_D)$ [or $P(N_D)$] of the fraction (or number) of domain walls at steady state; this is also done for both x and y directions.

(4) Distribution $P(m)$ of the total magnetization at steady state for $\kappa \leqslant 0$ only.

We have taken lattices of size $L\times L$ with $L=40$, 100, 200, and 300 to study the persistence behavior and dynamics of the domain walls of the system and have averaged over at least 50 configurations for each size. For estimating the distribution N_D we have averaged over a much larger number of configurations (typically 4000) and restricted to system sizes 40×40, 60×60, 80×80, and 100×100. Periodic boundary conditions have been used in both x and y directions. $J_0=J_1=1$ has been used in the numerical simulations.

III. DETAILED DYNAMICAL BEHAVIOR

Before going into the details of the dynamical behavior, let us discuss the stability of simple configurations or structures of spins which will help us in appreciating the fact that the dynamical behavior is strongly dependent on κ.

A. Stability of simple structures

An important question that arises in dynamics is the stability of spin configurations—it may happen that configurations which do not correspond to the global minimum of energy still remain stable dynamically. This has been termed "dynamic frustration" [14] earlier. A known example is of course the striped state occurring in the two- or higher-dimensional Ising models, which is stable but not a configuration that has minimum energy.

In the ANNNI model, the stability of the configurations depend very much on the value of κ. A previous analysis for the one-dimensional ANNNI model has shown that $\kappa=1$ is a special point, above and below which the dynamical behavior changes completely because of the stability of certain structures in the system.

Let us consider the simple configuration of a single up spin in a sea of down spins. Obviously, it will be unstable as long as $\kappa<2$. For $\kappa>2$, although this spin is stable, all the neighboring spins are unstable. However, for $\kappa<2$, only the up spin is unstable and the dynamics will stop once it flips.

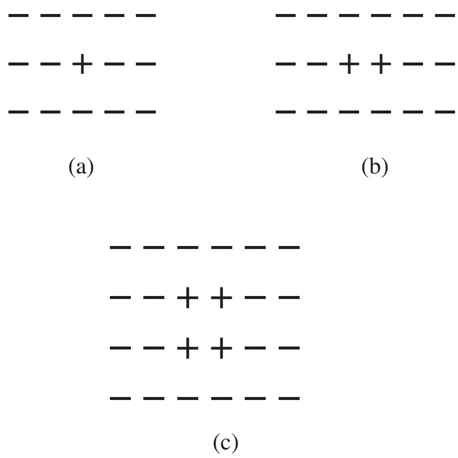

FIG. 3. Analysis of stability of simple structures. (a) Single up spin in sea of down spins; here for $\kappa < 2$ all the spins except the up spin is stable. (b) Two up spins in a sea of down spins; all spins except the two up spins are stable for $\kappa < 1$. (c) A 2×2 structure of up spins; here all the spins are stable for $\kappa < 1$ while neighboring spins are not (see text for details).

When $\kappa = 2$ the spin may or may not flip, i.e., the dynamics is stochastic.

Next we consider a domain of two up spins in a sea of down spins. These two may be oriented along either the horizontal or vertical direction. These spins will be stable for $\kappa > 1$ only while all the neighboring spins are unstable. For $\kappa < 1$, all spins except the up spins are stable. When $\kappa = 1$, the dynamics is again stochastic.

A two by two structure of up spins in a sea of down spins on the other hand will be stable for any value of $\kappa > 0$. But the neighboring spins along the vertical direction will be unstable for $\kappa \geqslant 1$. This shows that for $\kappa < 1$, one can expect that the dynamics will affect the minimum number of spins and therefore the dynamics will be slowest here. A picture of the structures described above are shown in Fig. 3.

One can take more complicated structures but the analysis of these simple ones is sufficient to expect that there will be different dynamical behavior in the regions $\kappa < 1$, $\kappa = 1$, $\kappa > 1$, $\kappa = 2$, and $\kappa > 2$. However, we find that as far as persistence behavior is concerned, there are only three regions with different behavior: $\kappa < 1$, $\kappa = 1$, and $\kappa > 1$. On the other hand, when the distribution of the number of domain walls in the steady state is considered, the three regions $1 < \kappa < 2$, $\kappa = 2$, and $\kappa > 2$ have clearly distinct behavior.

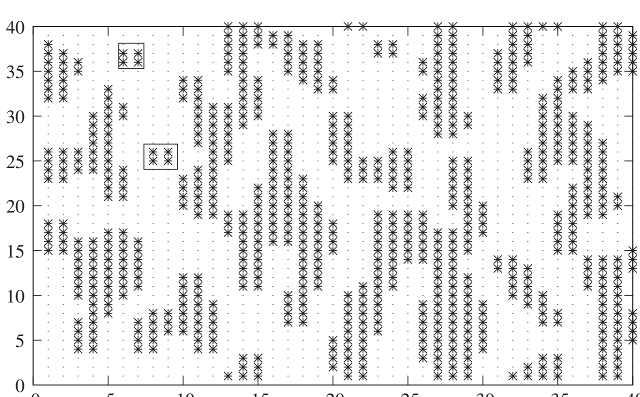

FIG. 4. Snapshot of a 40×40 system at time $t = 10$ for $\kappa < 1$. A few simplest quasifrozen structures are highlighted.

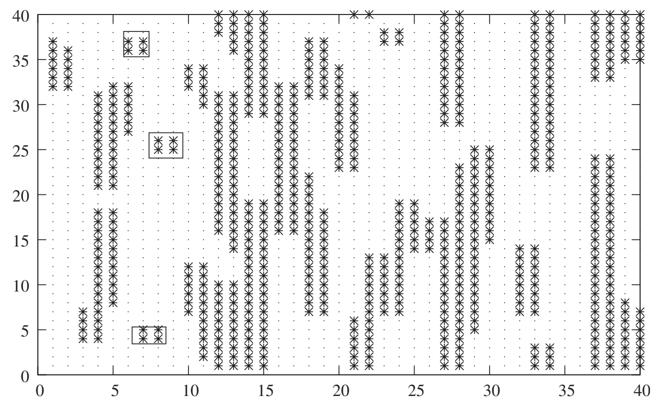

FIG. 5. Same as Fig. 4 with $t = 100$.

B. $0 < \kappa < 1$

We find that, as in [10], in the region $0 < \kappa < 1$, the system has identical dynamical behavior for all κ. Also, as in the one-dimensional case, here the system does not go to its equilibrium ground state. However, the dynamics continues for a long time, albeit very slowly for the reasons mentioned above. In Figs. 4–7, we show snapshots of the system at different times for a typical quench to zero temperature. As already mentioned, here domains of sizes 1 and 2 will vanish very fast, and certain structures, the smallest of which is a 2×2 domain of up or down spins in a sea of oppositely oriented spins can survive, until very long times. These structures we call quasifrozen, as the spins inside these structures (together with the neighborhood spins) are locally stable; they can be disturbed only when the effect of a spin flip occurring at a distance propagates to its vicinity, which usually takes a long time.

The pictures at the later stages also show that the system tends to attain a configuration in which the domains have straight vertical edges; it can be easily checked that structures with kinks are not stable. We find a tendency to form

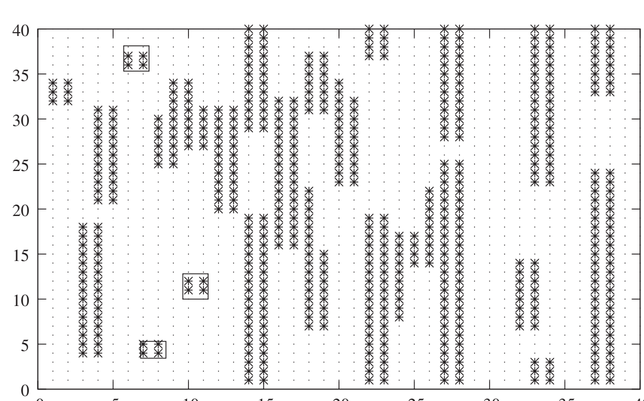

FIG. 6. Same as Fig. 4 with $t = 500$. One of the 2×2 structures has melted while another one has formed. The ladderlike structures which have formed are perfectly stable.

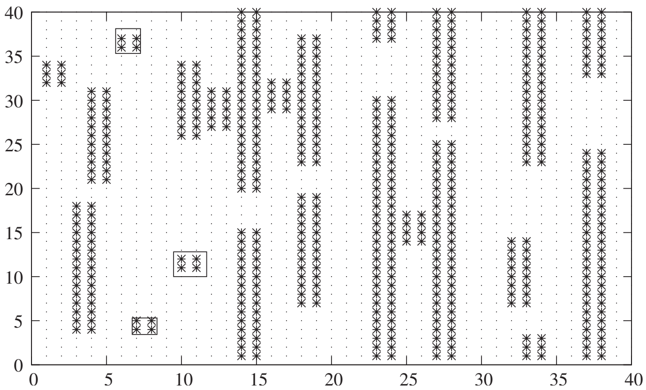

FIG. 7. Same as Fig. 4 with $t=75\,000$. This snapshot is taken after a very long time to show that the system has undergone nominal changes compared to the length of the time interval. The whole configuration now consists of ladders and the dynamics stops once the system reaches such a state.

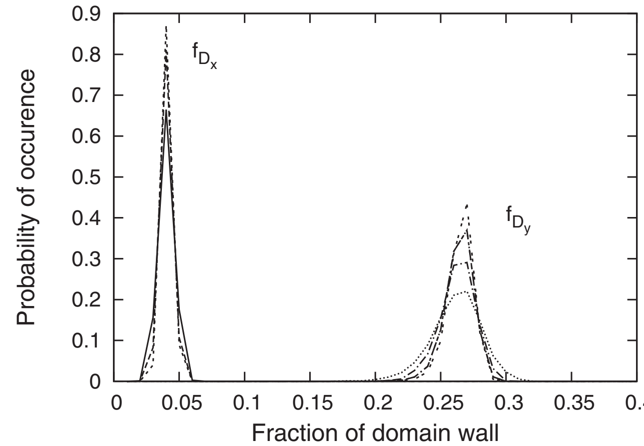

FIG. 9. Steady state distributions of fraction of domain walls at $\kappa < 1$ for different system sizes. The distributions become narrower as the system size is increased.

strips of width 2 ("ladders") along the vertical direction—this is due to the second-neighbor interaction; however, these strips do not span the entire lattice in general. The domain structure is obviously not symmetric, e.g., ladders along the horizontal direction will not form stable structures. The dynamics stops once the entire lattice is spanned by only ladders of height $\mathcal{N} \leq L$.

The persistence probability for $\kappa < 1$ shows a very slow decay with time which can be approximated by $1/\log(t)$ for an appreciable range of time. At later times, it approaches a saturation value in an even slower manner. The slow dynamics of the system accounts for this slow decay.

The fractions of domain walls, f_{D_x} and f_{D_y}, along the x and y directions show remarkable difference as functions of time. While that in the x direction saturates quite fast, in the y direction, it shows a gradual decay until very long times (see Fig. 8). This indicates that the dynamics essentially keeps the number of domains unchanged along the x direction while that in the other direction changes slowly in time. The behavior of f_{D_x} is similar to what happens in one dimension. In fact, the average number of domain walls N_{D_x} at large times is also very close to that obtained for the ANNNI chain; it is about $0.27L$. However, in contrast to the one-dimensional case where the domain walls remain mobile, here the mobility of the domain walls is impeded by the

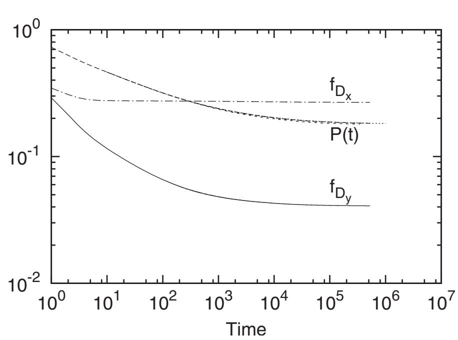

FIG. 8. Persistence $P(t)$ and average number of domain walls per site, f_D, are shown for $\kappa < 1$.

presence of the ferromagnetic interaction along the vertical direction causing a kind of pinning of the domain walls.

The distribution of the fraction of domain walls in the steady state shown in Fig. 9 also reveals some important features. The distributions for f_{D_x} and f_{D_y} are both quite narrow with the most probable values being $f_{D_x} \simeq 0.27$ and $f_{D_y} \simeq 0.04$ (these values are very close to the average values). With increase in system size, the distributions tend to become narrower, indicating that they approach a δ-function-like behavior in the thermodynamic limit.

C. $\kappa > 1$

It was already observed that $\kappa = 1$ is the value at which the dynamical behavior of the ANNNI model changes drastically in one dimension. In two dimensions, this is also true; however, we find that the additional ferromagnetic interaction along the vertical direction is able to affect the dynamics to a large extent. Again, as in the one-dimensional case, we have different dynamical behavior for $\kappa = 1$ and $\kappa > 1$. In this section we discuss the behavior for $\kappa > 1$ while the $\kappa = 1$ case is discussed in the next section.

The persistence probability follows a power law decay with $\theta = 0.235 \pm 0.001$ for all $\kappa > 1$, while the finite-size-scaling analysis made according to (3) suggests a z value 2.08 ± 0.01. This is checked for different values of κ ($\kappa = 1.3, 1.5, 2.0, 20, 100$) and the values of θ and z have negligible variations with κ which do not show any systematics. Hence we conclude that the exponents are independent of κ for $\kappa > 1$. A typical behavior of the raw data as well as the data collapse is shown in Fig. 10.

The dynamics of the average fraction of domain walls along the horizontal direction, f_{D_x}, again shows a fast saturation while that in the y direction has a power law decay with an exponent $\simeq 0.48$ (Fig. 11). This exponent is also independent of κ. As mentioned in Sec. II, we find that there is a good agreement of the value of this exponent with that of $1/z$ obtained from the finite-size-scaling behavior of $P(t)$, implying that the average domain size D is inversely proportional to f_{D_y}. This is quite remarkable, as the fraction of

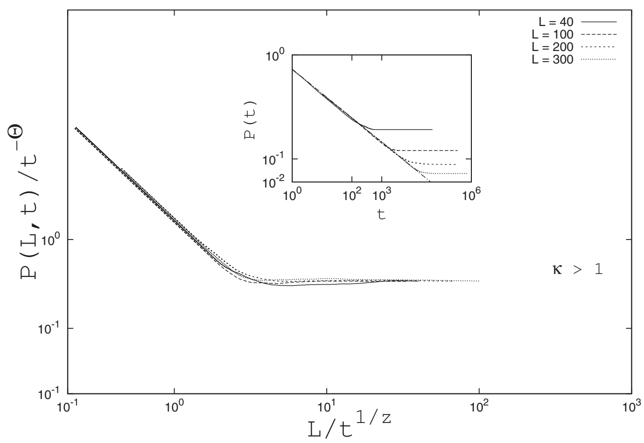

FIG. 10. Collapse of scaled persistence data versus scaled time using $\theta=0.235$ and $z=2.08$ for different system sizes for $\kappa>1$. Inset shows the unscaled data.

domain walls calculated in this manner is not exactly equivalent to the inverse of domain sizes in a two-dimensional lattice; the fact that f_{D_x} remains constant may be the reason behind the good agreement (essentially the two-dimensional behavior is getting captured along the dimension where the number of domain walls shows significant change in time).

Although the persistence and dynamic exponents are κ independent, we find that the distribution of the number of domain walls has some nontrivial κ dependence.

Although the system, for all $\kappa>1$, evolves to a state with antiphase order along the horizontal direction, the ferromagnetic order along vertical chains is in some cases separated by one or more domain walls. A typical snapshot is shown in Fig. 12 displaying that one essentially gets a striped state here as in the two-dimensional Ising model.

Interfaces that occur parallel to the y axis, separating two regions of antiphase and keeping the ferromagnetic ordering along the vertical direction intact, are extremely rare, the probability vanishing for larger sizes. Quantitatively, this means we should get $f_{D_x}=0.5$ at long times, which is confirmed by the data (Fig. 11). Hence in the following our discussions on the striped state will always imply flat horizontal interfaces, i.e., antiphase ordering along each horizontal row, but the ordering can be of different types (e.g., a $++--++--\cdots$ type and a $--++--++\cdots$ type, which one can call a "shifted" antiphase ordering with respect to the first type).

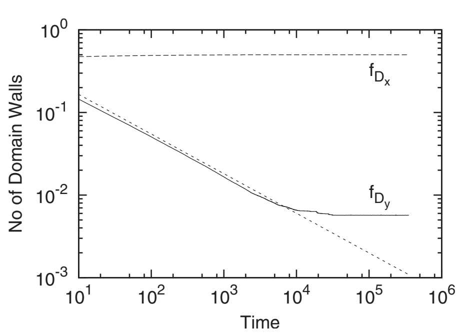

FIG. 11. Decay of the fraction of domain walls with time at $\kappa>1$ along horizontal and vertical directions. The dashed line has slope equal to 0.48.

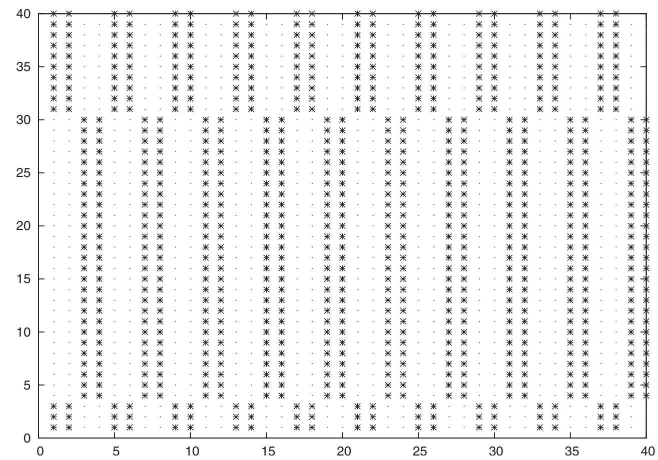

FIG. 12. Typical snapshot of a steady state configuration for $\kappa>1$ with flat horizontal interfaces separating two regions of antiphase ordering (see text).

It is of interest to investigate whether these striped states survive in infinite systems. To study this, we consider the distribution of the number of domain walls rather than the fraction for different system sizes. The probability that there are no domain walls, or a perfect ferromagnetic phase along the vertical direction, turns out to be weakly dependent on the system sizes but having different values for different ranges of values of κ. For $1<\kappa<2$, it is $\simeq 0.632$, and for $\kappa=2.0$, it is $\simeq 0.544$, while for any higher value of κ, this probability is about 0.445. Thus it increases for κ although not in a continuous manner and, as in the two-dimensional case, we find that there is indeed a finite probability to get a striped state.

While we look at the full distribution of the number of domain walls at steady state (Fig. 13), we find that there are dominant peaks at $N_{D_y}=0$ (corresponding to the unstriped state) and at $N_{D_y}=2$ (which means there are two interfaces).

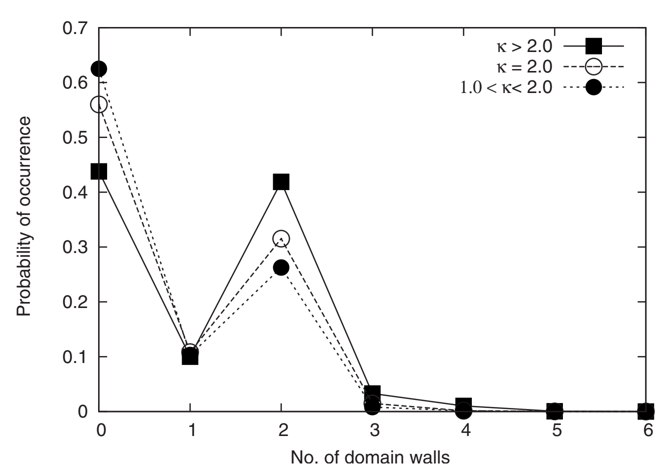

FIG. 13. Normalized steady state distributions of number of domain walls for different $\kappa>1$, showing that striped states occur with higher probability as κ increases. The lines are guides to the eye.

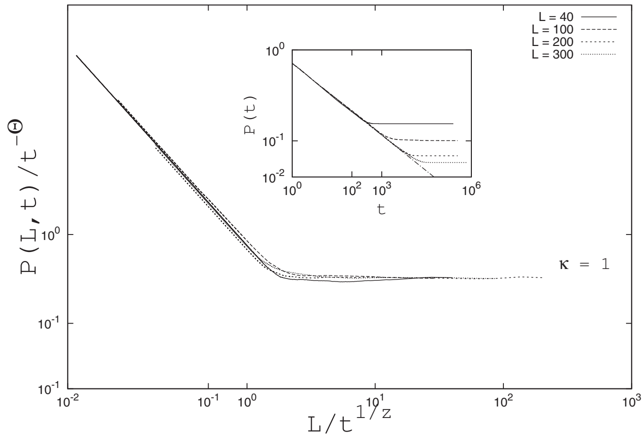

FIG. 14. Collapse of scaled persistence data versus scaled time using θ=0.263 and z=1.84 for different system sizes at κ=1. Inset shows the unscaled data.

However, we find that the distribution shows that there could be odd values of N_{D_y} as well. This is because the antiphase has a fourfold degeneracy and a "shifted" ordering can occur in several ways such that odd values of N_{D_y} are possible. In any case, the number of interfaces never exceeds N_{D_y}=6 for the system sizes considered.

D. κ=1

Here we find that the persistence probability follows a power law decay with θ=0.263±0.001. The finite-size-scaling analysis suggests a z value 1.84±0.01 (Fig. 14).

We have again studied the dynamics of f_{D_x} and f_{D_y}; the former shows a fast saturation at 0.5 while the latter shows a rapid decay to zero after an initial power law behavior with an exponent ≈0.515 (Fig. 15). This value, unlike in the case κ>1, does not show very good agreement with $1/z$ obtained from the finite-size-scaling analysis. We will get back to this point in the next section.

The results for f_{D_x} and f_{D_y} imply that the system reaches a perfect antiphase configuration as there are no interfaces left in the system with f_{D_x}=0.5 and f_{D_y}=0 at later times.

E. κ≤0.0

In order to make a comparison with the purely ferromagnetic case, we have also studied the Hamiltonian (2) with

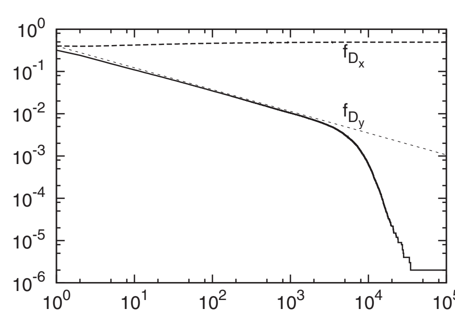

FIG. 15. Decay of the fraction of domain walls with time at κ=1 along horizontal and vertical directions. The dashed line has slope equal to 0.515.

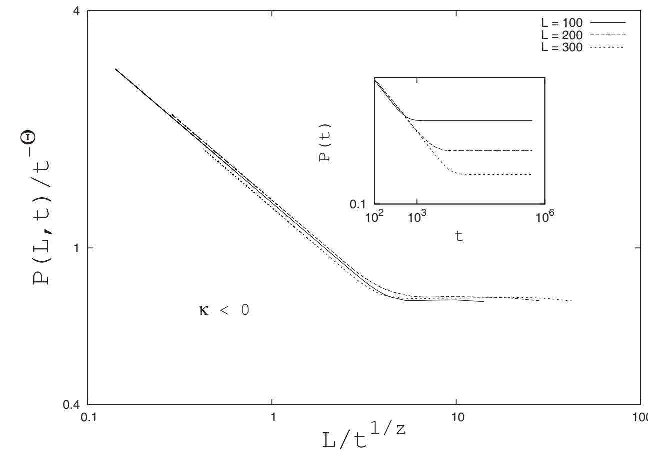

FIG. 16. Collapse of scaled persistence data versus scaled time using θ=0.20 and z=2.0 for different system sizes for κ<−1. Inset shows the unscaled data.

negative values of κ which essentially corresponds to the two-dimensional Ising model with anisotropic next-nearest-neighbor ferromagnetic interaction.

κ=0 corresponds to the pure two-dimensional Ising model for which the numerically calculated value of θ ≃0.22 is verified. We find a result when κ is allowed to assume negative values: the persistence exponent θ has a value ≃0.20 for $|\kappa|>1$ while for $0<|\kappa|\leq 1$, the value of θ has an apparent dependence on κ, varying between 0.22 to 0.20. However, it is difficult to numerically confirm the nature of the dependence in such a range and we have refrained from doing it. At least for $|\kappa|\geq 1$, the persistence exponent is definitely different from that of at κ=0. The growth exponent z, however, appears to be constant and ≃2.0 for all values of κ≤0. A data collapse for large negative κ is shown in Fig. 16 using θ=0.20 and z=2.0.

The effect of the anisotropy shows up clearly in the behavior of f_{D_x} and f_{D_y} as functions of time (Fig. 17). For κ=0, they have identical behavior, both reaching a finite saturation value showing that there may be interfaces generated

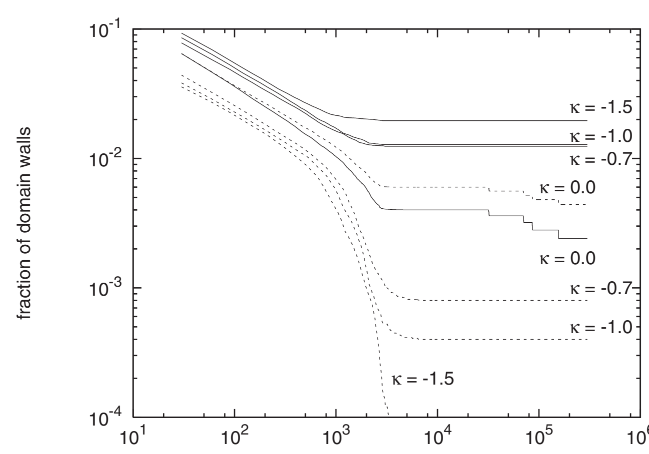

FIG. 17. Decay of the fraction of domain walls with time at κ≤0 along horizontal (f_{D_x}, shown by dotted lines) and vertical (f_{D_y}, shown by solid lines) directions.

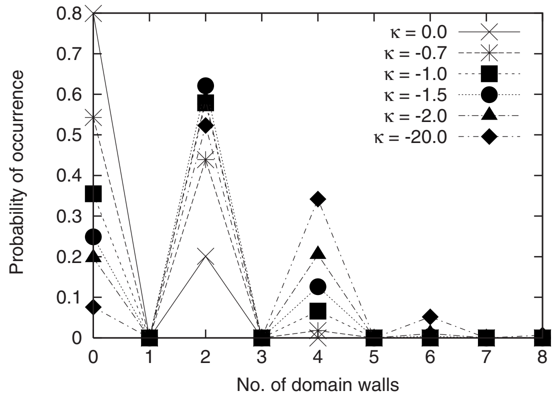

FIG. 18. Normalized steady state distributions of number of domain walls for different $\kappa \leqslant 0$ showing that striped states occur with higher probability as $|\kappa|$ increases. The lines are guides to the eye.

in either of the directions (corresponding to the striped states which are known to occur here). As the absolute value of κ is increased, f_{D_x} shows a fast decay to zero while f_{D_y} attains a constant value. The saturation value attained by f_{D_y} increases markedly with $|\kappa|$ while for f_{D_x} the decay to zero becomes faster. One can conduct a stability analysis for striped states to show that such states become unstable when the interfaces are vertical and κ increases beyond 1, leading to the result $f_{D_x} \to 0$. Extracting the z value from the variations of f_{D_x} or f_{D_y} is not very simple here as the quantities do not show smooth power law behavior over a sufficient interval of time.

The fact that f_{D_y} and/or f_{D_x} reach a finite saturation value indicates that striped states occur here as well. The behavior of f_{D_x} and f_{D_y} suggests that, in contrast to the isotropic case where interfaces can appear either horizontally or vertically, here the interfaces appear dominantly along the x direction as κ is increased. Thus the normalized distribution of the number of domain walls along y is shown in Fig. 18. We find that as κ is increased in magnitude, more and more interfaces appear. However, the number of interfaces is always consistent with the fact that interfaces occur between ferromagnetic domains of all up and all down spins.

Last, in this section, we discuss the behavior of the magnetization, which is the order parameter in a ferromagnetic system. As striped states are formed, the magnetization will assume values less than unity. The probability of configurations with magnetization equal to unity shows a stepped behavior, with values changing at $|\kappa|=1$ and 2 and assuming constant values at $1<|\kappa|<2$ and above $|\kappa|=2$ (Fig. 19).

IV. DISCUSSION AND CONCLUSIONS

We have investigated some dynamical features of the ANNNI model in two dimensions following a quench to zero temperature. We have obtained the result that the dynamics is very much dependent on the value of κ, the ratio of the antiferromagnetic interaction to the ferromagnetic interaction along one direction. This is similar to the dynamics of the one-dimensional model studied earlier, but here we have more intricate features, e.g., that of the occurrence of quasifrozen-in structures for $\kappa<1$, where the persistence probability shows a very slow decay with time. The persistence probability is algebraic for $\kappa \geqslant 1$, but exactly at $\kappa=1$ the exponents θ and z are different from those at $\kappa>1$. The exponents for $\kappa>1$ are in fact very close to those of the two-dimensional Ising model with nearest-neighbor ferromagnetic interaction. (This was not at all true for the one-dimensional ANNNI chain, where the persistence exponent at $\kappa>1$ was found to be appreciably different from that of the one-dimensional Ising chain with nearest-neighbor ferromagnetic interaction.) This shows that the ferromagnetic interaction along the vertical direction is able to negate the effect of the antiferromagnetic interaction to a great extent. This is apparently a counterintuitive phenomenon, $\kappa=0$ and $\kappa>1$ having very similar dynamic behavior while at the intermediate values the dynamics is qualitatively and quantitatively different. As far as dynamics is concerned, the ANNNI model in two dimensions cannot therefore be treated perturbatively.

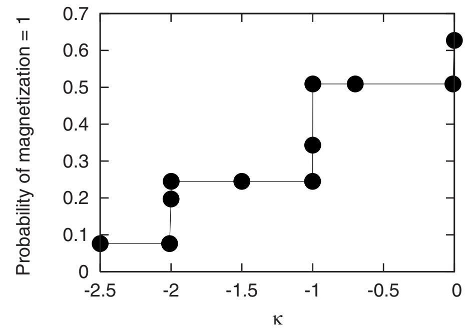

FIG. 19. Probability that the magnetization takes a steady state value equal to unity against κ when $\kappa \leqslant 0$.

Although the values of θ and z are individually quite close for $\kappa=0$ and $\kappa>1$, the products $z\theta=\alpha$ are quite different. For $\kappa=0$, $\alpha \simeq 0.44$ while for $\kappa>1$, it is 0.486 ± 0.002. This shows that the spatial correlations of the persistent spins are quite different for the two and one can safely say that the dynamical classes for $\kappa=0$ and $\kappa>1$ are not the same. $\kappa=1$ is the special point where the dynamic behavior changes radically. Here there appears to be some ambiguity regarding the value of z; estimating α from the finite-size-scaling analysis gives $\alpha \approx 0.484 \pm 0.005$, while on using the z value from the domain dynamics, the estimate is approximately equal to 0.51. However, the dynamics of the domain sizes may not be very accurately reflected by the dynamics of f_{D_y} in which case $\alpha \approx 0.48$ is a more reliable result. Thus we find that, although the values of θ and z are quite different for $\kappa=1$ and $\kappa>1$, the α values are close.

We would like to add here that when there is a power law decay of a quantity related to the domain dynamics, it is highly unlikely that it will be accompanied by an exponent which is different from the growth exponent. Thus, even though we get slightly different values of z for $\kappa=1$ from the two analyses, it is more likely that this is an artifact of the numerical simulations.

Another feature present in the two-dimensional Ising model is the finite probability with which it ends up in a striped state. The same happens for $\kappa>1$, but here the probabilities are quite different and also dependent on κ. We find that there is a significant role of the point $\kappa=2$ here as this probability has different values at $\kappa=2$, $\kappa>2$, and $\kappa<2$.

Comparison of the ANNNI dynamics with that of the ferromagnetic anisotropic Ising model shows some interesting features. In the latter, one gets a different value of persistence exponent for $\kappa<-1$ while in the former a different value is obtained for $\kappa\geqslant 1$. The values (except for $\kappa=1$) are in fact very close to those of the two-dimensional Ising model, but simulations done for identical system sizes averaged over the same number of initial configurations are able to confirm the difference. The qualitative behavior of the domain dynamics is again strongly κ dependent when κ is negative. Another point to note is that the probability that the system evolves to a pure state is κ dependent in both the ANNNI model and the Ising model. In both cases, in fact, this probability decreases in a steplike manner with increasing magnitude of κ. We also find the interesting result that, while the distributions of the number of domain walls can have nonzero values at odd values of N_D in the ANNNI model because of the fourfold degeneracy of the antiphase, for the Ising model, odd values of N_D are not permissible as the ferromagnetic phase is twofold degenerate.

Finally, we comment on the fact that, although the dynamical behavior, as far as domains are concerned, reflects the inherent anisotropy of the system (in both the ferromagnetic and antiferromagnetic models), the persistence probability is unaffected by it. In order to verify this, we estimated $P(t)$ along an isolated chain of spins along the x and y directions separately and found that the two estimates gave identical results for all values of κ.

In conclusion, it is found that, except for the region $0<|\kappa|<1$, the dynamical behavior of the Hamiltonian (2) is remarkably similar for negative and positive κ; the persistence and growth exponents are only marginally affected compared to the values of the two-dimensional Ising case ($\kappa=0$) and the domain distributions have similar nature. However, the region $0<\kappa<1$ is extraordinary, where algebraic decay of persistence is absent. There is dynamic frustration as the system gets locked in a metastable state consisting of ladderlike domains and the dynamics is very slow because of the presence of quasifrozen structures. There is in fact dynamic frustration at other κ values also in the sense that, except for $\kappa=1$, the system has a tendency to get locked in a striped state. However, even in that case, the algebraic decay of the persistence probability is observed. Thus algebraic decay of persistence probability seems to be valid only when the metastable state is a striped state. Although there is no dynamic frustration at $\kappa=1$ in the sense that it always evolves to a state with perfect antiphase structure, it happens to be a very special point where the persistence exponent and growth exponents are unique and appreciably different from those of the $\kappa=0$ case.

In this paper, the behavior of the two-dimensional ANNNI model under a zero temperature has been discussed; the dynamics at finite temperature can be in fact quite different. At finite temperatures, the spin flipping probabilities are stochastic, and the dynamical frustration may be overcome by the thermal fluctuations. It has been observed earlier [14] that, in a thermal annealing scheme of the one-dimensional ANNNI model, the $\kappa=0.5$ point becomes significant. A similar effect can occur for the two-dimensional case as well. The definition of persistence being quite different at finite temperatures [15], it is also not easy to guess its behavior (for either the one- or two-dimensional model) simply from the results of the zero-temperature quench. Indeed, the behavior of the ANNNI model under a finite-temperature quench is an open problem which could be addressed in the future.

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