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Experimental characterization of hopping dynamics in a multistable fiber laser

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We demonstrate experimental evidence of noise-induced attractor hopping in a multistable fiber laser. Multistate hopping dynamics displays complex statistical properties characterized by nontrivial scalings. When hopping is encountered between two states, the dynamics of the system is characterized by the $-3/2$ power law for the probability distribution of periodic windows versus their length, just as in the case of two-state on-off intermittency. A surprising noise saturation effect is found: average output noise in the hopping regime is almost independent of input noise. Such robustness of the system against external noise may be beneficial for some applications: for example, for communications with multistable systems or for designing noise-insensitive detectors.

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Many real complex systems demonstrate the coexistence of more than two attractors corresponding to the long-term behavior in phase space. In addition to a bistable system which displays the positive role of noise in the form of stochastic and coherence resonances [1], a system with multiple coexisting attractors subject to stochastic modulation can exhibit other interesting features, such as noise-enhanced multistability [2], noise-induced preference of attractor [3,4], noise-induced resonances [5], etc. Noise in such a multistable system provokes a competition between different attracting states; as the system seeks a regular motion in the neighborhood of one attractor, we can see it jumping among the different states [4,6]. This phenomenon, often called *attractor hopping* [7], is closely related to chaotic itinerancy [8], which has already been observed experimentally [9], showing the alternate motion between fully developed chaos and an ordered behavior. Chaotic itinerancy is often observed in high-dimensional systems such as globally coupled maps [10] and networks of neuronal oscillators [11]. The noise-induced attractor hopping is different from the low-dimensional ordered motion in chaotic itinerancy because the latter takes place between stable and unstable manifolds and therefore involves a saddle point. A particular case of attractor hopping in a bistable system, two-state on-off intermittency [12], has been observed experimentally in a laser [13]. Both feedback and nonfeedback techniques have been suggested to control such an intermittent behavior [14]. Recently, Kraut and Feudel [7] showed that in contrast to a bistable system, attractor hopping in a multistable system depends on the structure of the chaotic saddles separating the attractors.

In spite of a large number of theoretical papers devoted to noise-induced switches between multiple states, there has not been to our knowledge a previous experimental report on this phenomenon. In this Rapid Communication we provide what we think is the first experimental observation of noise-induced attractor hopping. This phenomenon manifests itself as *multistate intermittency* and requires the coexistence of multiple invariant subspaces. In a multistable system, relatively strong noise destabilizes the coexisting states and converts the multistable system into a metastable one. Multiple states that were stable without noise become unstable when noise is applied, giving birth to a new attractor: an intermittency state. As the noise amplitude is increased, the number of the coexisting attractors decreases, as they get involved in the hopping dynamics. It is in this sense that noise allows multistability control.

The experimental setup we used is similar to those already described by some of the present authors in previous papers [15–17]. The experiments are carried out with a Fabry-Perot-cavity 1560-nm erbium-doped fiber laser. The laser is subjected to the harmonic modulation of a diode pump 976-nm laser. Such a laser has various applications and is commonly used in many laboratories. This laser displays a very rich dynamics that has extensively been studied theoretically [17,18] and experimentally [15–17]. The 1.5-m laser cavity is formed by an active heavily doped erbium fiber of a 70-cm length and 2.7-μm core diameter and two fiber Bragg gratings with a 0.1-nm FWHM (full width on half-magnitude) bandwidth, having 91% and 95% reflectivities for the laser wavelength. In our experiments the diode current is fixed at $I=69$ mA corresponding to the pump power $P=19$ mW. The harmonic signal $A\sin(2\pi f_m t)$ (A and f_m being the amplitude and frequency of external modulation, respectively) from a signal generator and the additive Gaussian noise $N_{in}\xi$ (N_{in} and ξ being the external noise amplitude and a random generated number, respectively) from a noise generator are both applied simultaneously to the diode pump current.

Without external modulation ($A=0$) and in the absence of external noise ($N_{in}=0$), the fiber laser power exhibits small-amplitude oscillations (1%–2% of the magnitude of the steady-state power) with an average broadband frequency $f_0=30$ kHz of its relaxation oscillations. Figure 1 shows the laser dynamical state diagram in the parameter space of the modulation frequency and the external noise amplitude. One can see how more and more periodic orbits are involved in the hopping dynamics within a wider parameter range, as the noise amplitude is increased. Moreover, higher periodic orbits not observed prior to noise introduction appear in the hopping dynamics. This is in contrast to earlier theoretical prediction that higher-periodic orbits disappear with increasing noise strength [4]. The opposite effect was also previously predicted: attractors with a small basin gain more initial conditions, and the basins of many of them become even larger as compared with no noise basin [3,19]. Experimen-

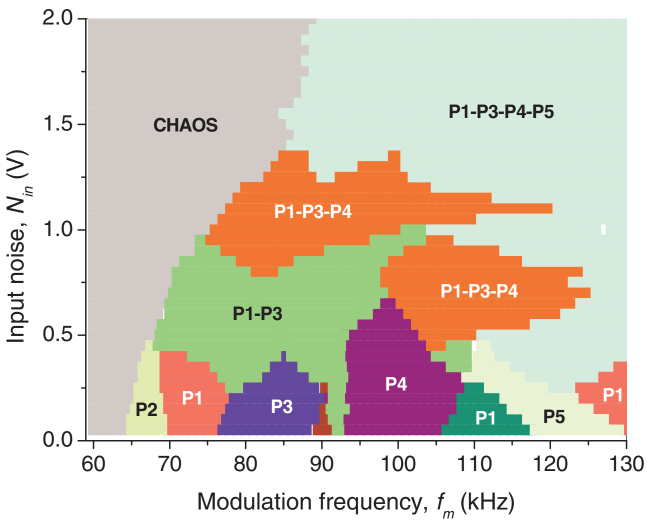

FIG. 1. (Color) Fiber laser state diagram in the modulation frequency and noise amplitude parameter space. Different colors stand for different periodic and intermittent states.

tally it is not yet possible to vary all initial conditions in order to give a final statement of whether an attractor with small basin existed without noise or a new periodic orbit was induced by noise.

In this work we are interested in a high-frequency region where the laser exhibits multistability [15]. In Fig. 2 we plot the time series and their corresponding power spectra for three coexisting attractors. As seen in Fig. 2(f), the frequency for $P4$ is not exactly $f_m/4$ because of nonlinear interaction with f_0 [20]. Switching on and off the signal generator, the laser initial conditions are changed and the corresponding coexisting attractors can be found. At a relatively low noise amplitude, the fiber laser is in a periodic state determined by the initial conditions. For instance, if the laser starts with the initial conditions corresponding to $P3$, it will remain in this state for an infinitely long time showing noisy oscillations with frequency $f_m/3$ [Fig. 2(b)]. As the input noise amplitude is increased, the ground level for each attractor also increases until a certain noise threshold is reached and the laser starts jumping back and forth from $P3$ oscillations to $P1$, as shown in Fig. 3(a). In a multistate intermittent regime, the trajectory visits more than two periodic states [Figs. 3(b) and 3(c)]. The number of periodic states among which the laser jumps increases with the noise amplitude. For example, the period-5 orbit not existent for low noise appears only when $N_{in} > 1.3$ V [Fig. 3(c)].

With the aim of studying characteristic properties of hopping dynamics in mind, we address the following question: how does output noise depend on input noise? Figure 4(a) shows, for different coexisting attractors, the dependence of the average output noise N_{out} taken at the modulation frequency f_m of the power spectrum on the input noise amplitude N_{in}. For each coexisting state ($P1$, $P3$, and $P4$), the output noise spectral component can be approximated by a linear dependence on the input noise amplitude [solid lines in Fig. 4(a)] with the slope increasing as does the orbit's period; the larger the period, the higher the slope. When the input noise amplitude is increased above 0.2 V, $P3$ and $P1$ melt

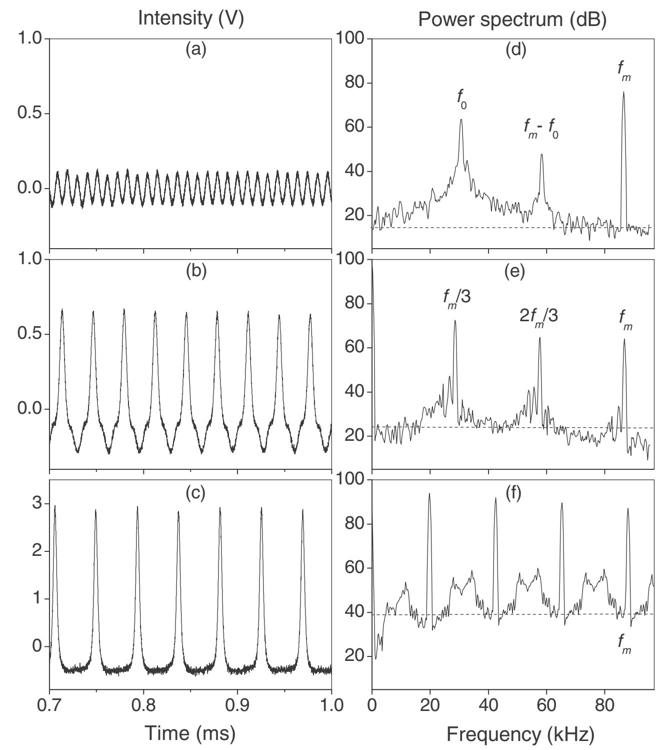

FIG. 2. (a)–(c) Time series and (d)–(f) power spectra of the laser intensity for coexisting (a),(d) period-1 ($P1$), (b),(e) period-3 ($P3$), and (c),(f) period-4 ($P4$) regimes when an external noise N_{in} $=150$ mV is applied. This noise is relatively small, so that no hopping dynamics is observed. For each dynamical regime, the power spectrum displays a different level of frequency-dependent output noise (ground level) indicated by the horizontal dashed lines for f_m=87 kHz. Note the difference in the intensity scale in (c).

into the intermittent $P3$-$P1$ attractor, apparently keeping the $P3$ regime slope for noise dependence. When noise again increases above 0.7 V, the laser starts jumping between three periodic states ($P3$-$P1$-$P4$), showing that the output-input noise dependence is practically lost. The same diminute slope remains when $P5$ takes part in the hopping dynamics ($P3$-$P1$-$P4$-$P5$). It is only for very strong input noise that all periodic states are mixed and the output noise increases again with the $P1$ attractor's slope. The output noise given in volts can be approximated by $N_{out}(\mathrm{V}) \sim \exp(\lambda_i N_{in})$, where λ_i is the scaling exponent of the i-periodic orbit. We find λ_1 $=1.54$, $\lambda_3=4.52$, and $\lambda_4=15.89$, whereas for the hopping attractor $\lambda_{3\text{-}1\text{-}4}=0.20$; i.e., it is smaller than λ_1 by a factor of 7.7. The physical mechanisms of the *noise-saturation effect* observed in the experiment are not yet clear. The phenomenon itself is very interesting and as such requires further investigation. One could look for a possible explanation in the noise energy distribution over the increasing number of jumps between higher-periodic orbits, so that the average noise level may remain almost unchanged.

Similar to other intermittent regimes, the hopping dynamics can be characterized by particular scaling laws. To reveal the physical mechanisms responsible for these scaling relations, we use a statistical approach which is often employed in the study of noisy systems. We are now interested in the nonzero transition probability of every periodic orbit to ev-

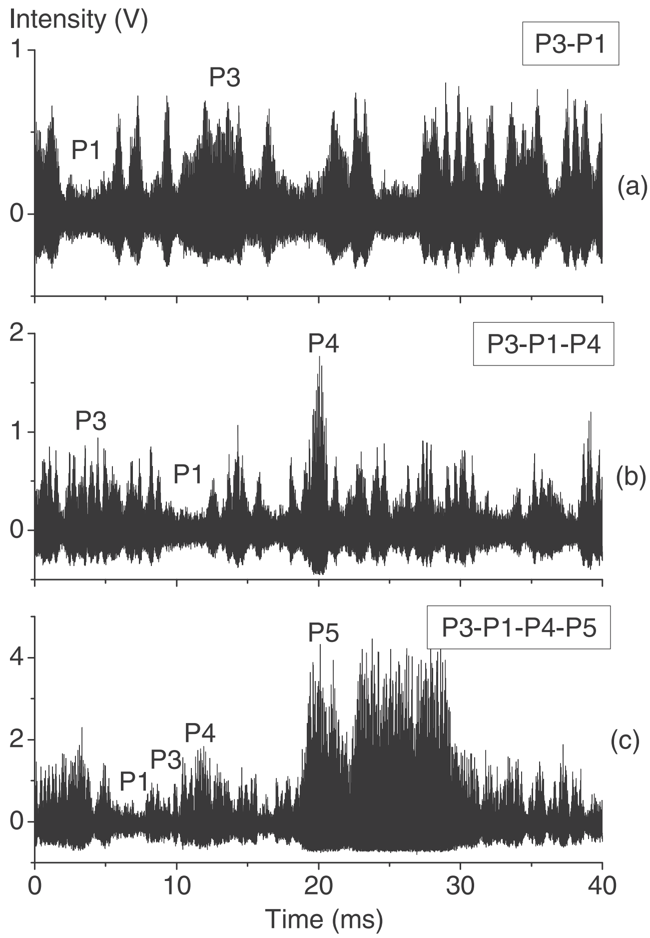

FIG. 3. Time series in hopping dynamics involving (a) two periodic orbits ($P3$ and $P1$) at N_{in}=0.5 V, (b) three periodic orbits ($P3$, $P1$, and $P4$) at N_{in}=0.9 V, and (c) four periodic orbits ($P3$, $P1$, $P4$, and $P5$) at N_{in}=1.5 V. Note the difference in the intensity scales.

ery other periodic orbit via a transient on a chaotic saddle—i.e., the probability of the trajectory visiting each one of the coexisting periodic states. Figure 4(b) shows this probability P_i as a function of the external noise amplitude. In hopping dynamics, the probability of visiting the $P3$ orbit, P_3, decays exponentially as N_{in} is increased [bold line in Fig. 4(b)]. The best fit yields the value −0.36 for this characteristic exponent. In the contrary, the probability of visiting $P1$, P_1, grows as N_{in} is larger and becomes equal to P_3 at N_{in} =0.75 V. It is exactly at this noise value that the output noise curve in Fig. 4(a) changes its slope.

Using the same approach as Poon and Grebogi [6], we qualify order and randomness by encoding dynamics into symbolic sequences of n elements in which the trajectory visits the different attracting sets by crossing the chaotic sets in the boundaries. We assign a symbol s_i for every periodic orbit i that appears in the hopping dynamics. In our case, we need up to four symbols in the alphabet, $s_i=1,2,3,4$. The complexity of the symbolic string among the attracting sets can be estimated using the Shannon entropy, in analogy to the Kolmogorov-Sinai entropy [21]:

$$h = \lim_{n\to\infty} \frac{H_n}{n} = \lim_{n\to\infty} \frac{1}{n}\left(-\sum_{|S|=n} p(S)\ln p(S)\right), \qquad (1)$$

where $S=s_1,s_2,\ldots,s_n$ denotes a finite symbol sequence, $p(S)$ is the probability of S, and H_n is the block entropy of block length n. Figure 4(c) shows the entropy versus input noise estimated from the experimental data using Eq. (1). Every periodic sequence yields a value of 0. The entropy increases rapidly at the bifurcation point when a new regime appears in hopping dynamics, and then it is almost constant. The nontrivial time-scaling appearance in the noisy laser is the consequence of the complex interplay between the coherent and random structures. The horizontal plateau in Fig. 4(c)

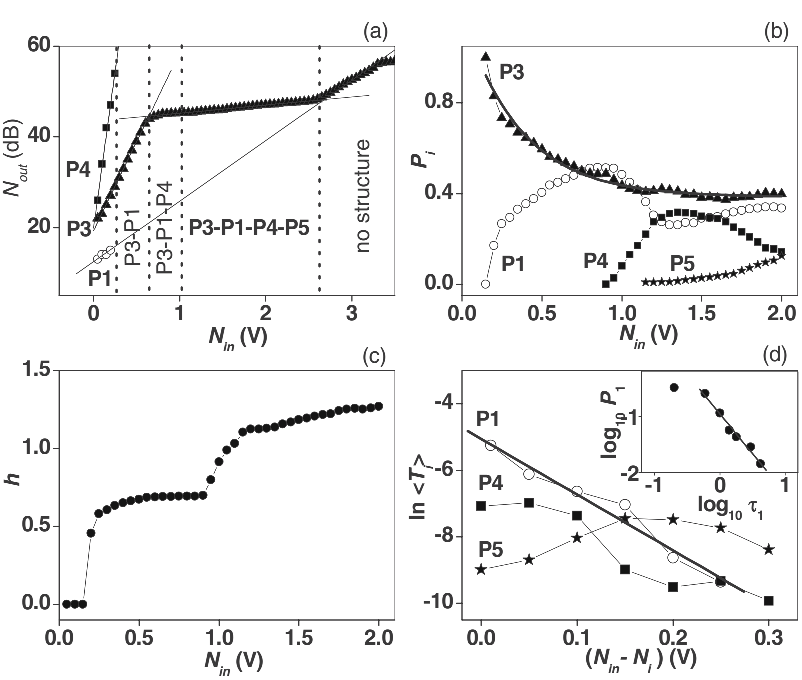

FIG. 4. (a) Average output noise versus input noise for three coexisting attractors and intermittency regimes. The dotted lines show the boundaries between different regimes and the solid lines are linear fits of the slopes. The three attractors coexistence is observed only for relatively low external noise (N_{in}<0.2 V). The noise saturation effect is clearly seen in the middle part of the figure. (b) Probability of visiting different attracting sets calculated by summing the duration of periodic windows in ten time series for every noise value. The bold line is the exponential decay fit for the period-3 orbit. (c) The Shannon entropy of the symbol sequence as a function of noise. (d) Mean escape times $\langle T_i \rangle$ for different attracting sets as a function of the noise amplitude excess over critical noise N_i at the onset of intermittency for the period-i state. $N_{1,4,5}$ =190, 800, 1150 mV. The inset in Fig. 4(d) shows the scaling of the probability distribution for period-1 windows inside the intermittent $P3$-$P1$ regime showing the −3/2 power law (straight line) for N_{in}=200 mV.

indicates the existence of a certain coherent structure in the set of all possible symbol sequences.

The bifurcation responsible for the jumps between different periodic states can be considered as a kind of crisis, and hence the process can be characterized by scaling laws for the characteristic lifetimes [22]. In the cases of noise-induced crisis [23] or on-off intermittency [24] where only two attracting sets participate in the intermittent dynamics and hence there is only one critical parameter responsible for the onset of intermittency, the noise intensity itself is usually used for scaling with the noise level. Instead, in multistate intermittency there exists different critical noise amplitude N_i for every i-periodic orbit. The period-i orbit does not appear in the hopping dynamics when $N_{in} < N_i$. Therefore, to be able to compare the slopes for different orbits, we estimate from the experimental data the mean escape time $\langle T_i \rangle$ the trajectory takes to leave the neighborhood of the period-i attracting set as a function of the noise amplitude excess over its critical value, $N_{in} - N_i$. Figure 4(d) shows these dependences for different attracting sets. For $P1$ the mean time $\langle T_1 \rangle$ can be well fitted by the exponential decay with noise (bold line); however, for other periods, the dynamics is very different—for instance, $\langle T_5 \rangle$ increases with noise and has a maximum at $N_{in} - N_5 \approx 200$ mV. Another interesting quantity of hopping dynamics to investigate is the probability distribution $P(\tau)$ for the length τ of the periodic windows sequence. For the $P1$ windows in the two-state intermittency regime between $P3$ and $P1$, we find that $P(\tau)$ obeys the universal scaling law for on-off (or two-state on-off) intermittency [24] to be $P(\tau_1) \sim \tau_1^{-3/2}$ [inset in Fig. 4(d)].

In conclusion, we have characterized experimentally noise-induced hopping dynamics in a multistable diode-pumped erbium-doped fiber laser with coexisting periodic attractors. Under additive noise applied to the diode pump current, the laser displays hopping dynamics. The hopping between two periodic states is characterized by the $-3/2$ power law for the probability distribution of laminar phase versus laminar length near the onset of intermittency, typical of a two-state on-off intermittency. When the noise amplitude is increased, the number of periodic orbits involved in the hopping dynamics goes up. The average lengths of laminar phases during which the trajectory is in the neighborhood of a particular periodic state varies irregularly depending on the noise amplitude. The character of this dependence is determined by a particular state. The laminar phase can either decrease or increase as the noise amplitude increases, resulting in a surprising noise saturation effect, when the output noise of the intermittent state is almost independent of the input noise. Such robustness of the system against external noise could be useful for some applications: for example, in communications with multistable systems or for designing noise-insensitive detectors.

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