Optimization versus randomness for car traffic regulation

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We focus on the differences among the analytical optimization of traffic flow on a road network, modeled by a fluid-dynamic approach, and a dynamic random one. In particular, two real urban networks are analyzed: Re di Roma Square, in Rome, and Via Parmenide crossing, in Salerno. With such two examples, it is possible to show that dynamic random algorithms are not the right choice for the improvement of traffic conditions.

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I. INTRODUCTION

In this paper, we focus our attention on urban traffic regulation through a random approach and an optimization algorithm for a fluid-dynamic model, introduced in [1]. The aim is to show that a dynamic random approach is absolutely not convenient in order to improve traffic conditions, as suggested in [2].

In particular, in [2], a discussion about benefits due to synchronization among a series of traffic lights using cellular automata is reported. The simulation of traffic flows is made through a different choice of signal period T and time delay δ. Moreover, it is shown that a correct synchronization gives some improvements only when the traffic density is low. When the traffic demand surpasses a given saturation value, synchronization is useless and also the use of a fixed T and a random delay δ, assigned to each traffic light, lets the throughput remain the same.

The principal aim of this paper is to use network models, as opposed to single roads, in order to verify the performances of traffic regulation algorithms. We show that an optimization approach outperforms a random one, even in heavy traffic conditions. Also, the use of random parameters for traffic regulation can lead to traffic conditions in which accidents are very frequent.

The mathematical modeling of vehicular traffic requires, first of all, the choice of the scale of representation. There are many examples of models at any scale, from the microscopic to the macroscopic through the kinetic. Each of them implies some technical approximations and suffers therefore from related drawbacks, either analytical or computational. In what follows, the attention is focused on a macroscopic model for car traffic.

Macroscopic models aim at describing the big picture without looking specifically at each single subject of the system; hence, they are computationally more efficient. Few partial differential equations, which can be solved numerically in a feasible time, are normally involved, and the global characteristics of the system are readily accessible. Nevertheless, now the modeling is, in a sense, less accurate than in the microscopic case, due to the fact that both approaches rely on the *continuum hypothesis*, clearly not physically satisfied by cars along a road. The number of vehicles should be large enough so that it makes sense to introduce concepts like macroscopic density, average speed, or kinetic distribution function as continuous functions of space and, in the latter case, also velocity. However, the continuum hypothesis can be profitably accepted as a technical approximation of physical reality, regarding macroscopic quantities globally as measures of traffic features and as tools to depict the spatial and temporal evolution of traffic waves.

Macroscopic modeling is based on the idea, originally proposed in the 1950s by Lighthill and Whitham and, independently, by Richards (LWR model; see [3,4]), that first-order differential equations, such as conservation laws, could describe the motion of cars along a road, provided a large-scale point of view is adopted so as to consider cars as small particles and their density as the main quantity to be looked at. This analogy remains nowadays in all macroscopic models of vehicular traffic, as terms like traffic pressure, traffic flow, and traffic waves demonstrate. To overcome the main limitations of the LWR model, various other approaches were considered; we refer to [5,6]. Such a modeling technique helps to describe some macroscopic phenomena such as shock-wave formation and propagation; see [7,8].

About 40 years separate the LWR model and the first models for car traffic networks. One macroscopic model for road networks was proposed in [9]; for a review, refer to [10]. Car traffic reconstruction through first-order models is discussed in [11]. Other important contributions for network models are [12–15].

Also optimization problems for road networks have been considered; see [1,16–19].

Here, two real cases study of urban networks are presented: the first one is Re di Roma Square, a large traffic circle inside the urban network of Rome, while the second one is Via Parmenide crossing, a little part of Salerno network, formed by two incoming roads and one outgoing road. There are some motivations which inspired the choice of these two case studies. Re di Roma Square is a part of Rome, which is highly interesting since it is characterized by some congestion phenomena, and it is very important to understand how to avoid them. Via Parmenide crossing, instead, unlike Re di Roma Square, is a small network, but its most important aspect is connected to queue formation due to a traffic light, which presents a cycle with a red phase too long.

The analysis of network performances for such two cases is made by two cost functionals that measure the average velocity and the average traveling time of cars, respectively. The results that arise from the study of these two types of networks are very interesting. In fact, simulations show that dynamic random algorithms and optimization approaches are very similar for Re di Roma Square and totally different for Via Parmenide crossing. The motivation of this behavior can be easily explained if we consider the statistical properties of dynamic random simulations. In short, a dynamic random simulation is similar to a particular case where all the parameters of the network are assumed equal to 0.5.

When we consider networks with a great number of nodes (Re di Roma Square), the time average of optimal parameters can approach 0.5, and this justifies the similarities among dynamic random simulations and optimal ones. Hence, to discriminate between them, it is necessary to introduce the stop-and-go wave (SGW) functional (see [1,20]), which measures the variation velocity, cause of car accidents.

For simple networks with one junction, like Via Parmenide crossing, we have a totally different behavior from Re di Roma Square. In this case, the network topology imposes the adoption of only one traffic parameter, whose analytical optimization (see [1]) gives a solution far from 0.5; hence, the dynamic random simulation and optimal ones cannot be similar.

This paper is organized as follows. We introduce the model for car traffic on an urban network and the optimization of such a model in Sec. II. Then, Sec. III contains simulations related to Re di Roma Square and Via Parmenide crossing, with consequent discussions about dynamic random and optimization algorithms in Sec. IV. The paper ends with conclusions in Sec. V.

II. MODEL FOR ROAD NETWORKS AND OPTIMIZATION

In order to model car traffic networks, we consider the formulation based on conservation laws. Lighthill and Whitham [3] and Richards [4], for a single road, proposed the equation

$$\partial_t \rho + \partial_x f(\rho) = 0, \tag{1}$$

where $\rho = \rho(t,x) \in [0, \rho_{\max}]$, $(t,x) \in \mathbb{R}^2$, is the density of cars, $\rho_{\max}$ is the maximal density of cars, $f(\rho) = \rho v(\rho)$ is the flux, and $v(\rho)$ is the average velocity. Recently, this approach was extended to networks [9,10].

A network is described by the couple $(\mathcal{I}, \mathcal{J})$, where $\mathcal{I} = \{I_i : i = 1, \ldots, N\}$ represents the set of roads, while $\mathcal{J}$ is the collection of junctions, which connect roads to each other. The evolution is determined by Eq. (1) on each line I_i and by Riemann solvers RS_J at each node (see [9]), based on rights of way and traffic distribution parameters p and α. Roughly speaking, for each initial datum constant on each road, RS_J assigns a solution formed by a single wave on each road (see [10]). Here, we consider p and α as controls.

In particular, for road junctions J with n incoming roads and m outgoing roads, Riemann solvers RS_J (see [9,10]) are based on the following rules.

(A) There are some fixed coefficients, which represent the preferences of the drivers. These coefficients indicate the distribution of the traffic from the incoming roads to the outgoing ones, and thus can be collected in a traffic distribution matrix:

$$A = \{\alpha_{ji}\}_{j=n+1,\ldots,n+m, i=1,\ldots,n} \in \mathbb{R}^{m\times n},$$

such that

$$0 < \alpha_{ji} < 1, \quad \sum_{j=n+1}^{n+m} \alpha_{ji} = 1,$$

for $i = 1, \ldots, n$ and $j = n+1, \ldots, n+m$, where α_{ji} is the percentage of drivers who, arriving from the ith incoming road, take the jth outgoing road.

If we refer to junctions with one incoming road $(n=1)$, a, and two outgoing roads $(m=2)$, b and c, respectively, then matrix A reduces to the column vector

$$\begin{pmatrix} \alpha \\ 1-\alpha \end{pmatrix},$$

where α $(1-\alpha)$ represents the probability that drivers could go to the outgoing road b (c), from the incoming road a.

(B) Respecting (A), drivers behave so as to maximize the flux through the junction J.

If $n > m$, a yielding rule is needed. In particular, for junctions with two incoming roads $(n=2)$, a and b, and one outgoing road $(m=1)$, c, such a rule can be stated as follows.

(C) Assume that not all cars can enter the road c and let Q be the amount that can do it. Then, pQ cars come from the road a and $(1-p)Q$ cars from the road b.

Notice that p can be thought as a right-of-way parameter.

In order to measure the efficiency of the network, we consider two cost functionals, measuring the velocity at which cars travel through the network and the time taken by cars to travel on the network.

Since the model considers macroscopic quantities, we can estimate the averages integrating over time and space the average velocity and the reciprocal of average velocity, respectively. If ρ_i indicates the density on road i, we thus define the following:

$$J_1(t) = \sum_i \int_{I_i} v(\rho_i(t,x))dx,$$

$$J_2(t) = \sum_i \int_{I_i} \frac{1}{v(\rho_i(t,x))}dx.$$

We consider a fixed temporal interval $[0,T]$ for some $T > 0$.

For the regulation of traffic, we want to maximize J_1 and minimize J_2. A systematic presentation of the optimization algorithms and obtained results for such cost functionals is given in [1].

For a more precise description of traffic conditions, it is useful to define a third cost functional. It is known that the number of car accidents increases as the difference in the velocities of single drivers does. We follow the approach of [1,20], considering the SGW functional, namely

$$\zeta = \int_0^T \int_{\cup I_i} |Dv(\rho)| dt\, dx,$$

which gives estimates on the security of drivers, who travel on the network, because it measures the velocity variation.

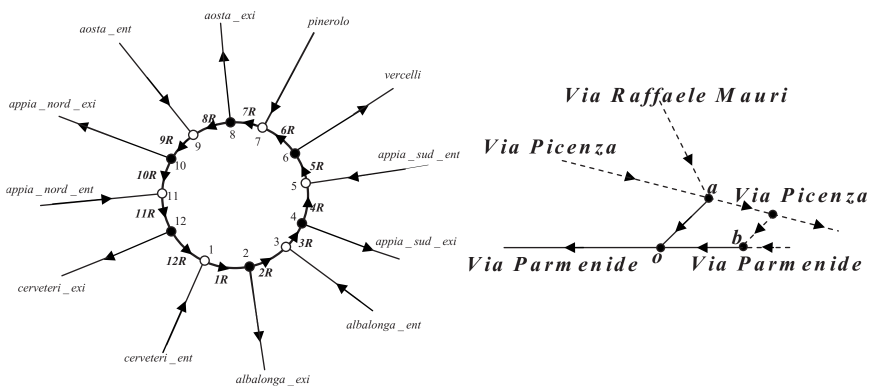

FIG. 1. Topology of Re di Roma Square (left) and Via Parmenide crossing in Salerno (right).

III. SIMULATION CASES

This section is devoted to the presentation of two different studies of real urban networks.

First, we focus on Re di Roma square, a large traffic circle in Rome, formed by junctions having two incoming and one outgoing road (2×1 junctions) and junctions with one incoming and two outgoing roads (1×2 junctions). In this case, the traffic distribution coefficients at 1×2 junctions are in reality completely determined by road capacities (and the characteristics of the nearby portion of the Rome urban network), and only right-of-way parameters for the 2×1 junctions can be chosen for the optimization. In Fig. 1 (left), we report the topology of Re di Roma Square, where junctions of 2×1 type (1, 3, 5, 7, 9, 11) are in white, while junctions of 1×2 type (2, 4, 6, 8, 10, 12) are in black.

Then, we consider a small area of the Salerno (Italy) urban network. In particular, in Fig. 1 (right) the portion of the interested area is depicted. We focus on the crossing indicated by o, consisting of two incoming roads and one outgoing road. The incoming road from point a to point o (a portion of Via Mauri, which we can call road a-o) is very short and connects Via Picenza to Via Parmenide. The incoming road from point b to point o (which we can call road b-o) is a part of Via Parmenide. Crossing o is ruled by a traffic light, with a cycle of 2 min (120 s), where the phase of green is 15 s for drivers, who travel on the road a-o. It is evident that such a situation leads to very high traffic densities on the road a-o as the short duration of the green phase does not always allow the absorption of queues. From a probabilistic point of view, we can say that road b-o has a right-of-way parameter that is

$$p = \frac{105}{120} = 0.875,$$

while road a-o has a right-of-way parameter

$$q = 1 - p = 1 - 0.875 = 0.125.$$

Our aim is to study this particular crossing in order to understand how it is possible to improve the conditions of traffic in presence of a traffic light.

For the first and second cases that we have described, the evolution of the traffic behavior is simulated in a time interval $[0,T]$, where $T=30$ min for a flux function $f(\rho)=\rho(1-\rho)$, considering $\rho_{max}=1$ and $v(\rho)=1-\rho$. As for the initial conditions on the roads of the network, we assume that, at the starting instant of simulation ($t=0$), all roads are empty. Moreover, for Re di Roma Square, we assume boundary condition 0.3 or 0.75 for roads with end points not infinite. In order to simulate Via Parmenide crossing, we consider a boundary datum 0.8 for roads that enter the junction o and a boundary condition 0.3 for the outgoing road.

We study three simulation cases: (a) right-of-way parameters, which optimize the cost functionals J_1 and J_2 (*optimal case*); (b) fixed right-of-way parameters (*fixed case*); it means that the right-of-way parameter is the same for each junction ($p=0.2$ for Re di Roma Square; for Via Parmenide crossing, we assume that the fixed case is given the real situation ruled by the traffic light, which is to say that $p=0.875$); (c) dynamic random parameters (*dynamic random case*), the right-of-way parameters for junctions of 2×1 type change randomly at every step of the simulation process.

Figure 2 shows some simulation results of Re di Roma Square for the temporal behavior of the cost functionals J_2,

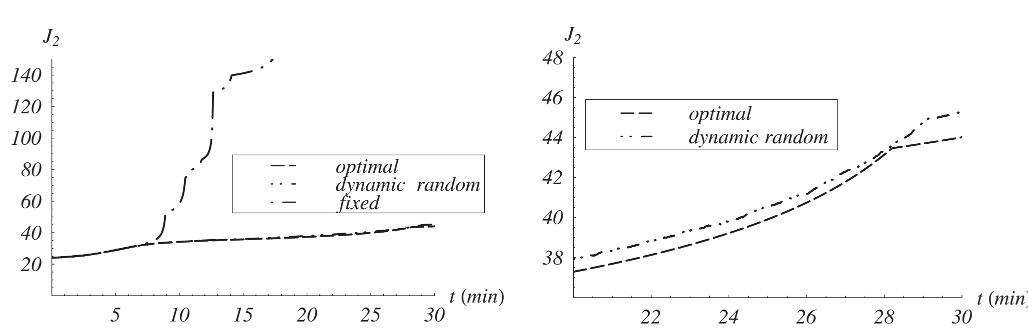

FIG. 2. J_2 with boundary conditions equal to 0.3 (left) and close-up around the optimal and dynamic random cases (right).

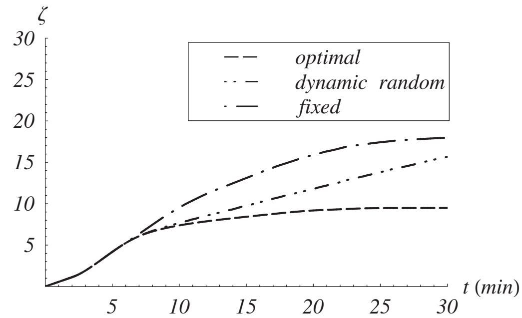

FIG. 3. Behavior of the SGW functional ζ in the case of boundary conditions equal to 0.75.

computed on the whole network. We can note that the fixed configuration is worse than the optimal one. The performances of the optimal and dynamic random cases are very similar. We could ask if we can avoid optimizing the network and operate in dynamic random conditions, although one could think that the conditions of traffic are heavily chaotic.

Figure 3 shows the SGW functional ζ, which indicates that the security on the roads is greater in the optimal case than in the dynamic random configuration. Observe that the optimal case for ζ is simulated according to the optimization algorithm for the cost functionals J_1 and J_2 (and not for ζ itself).

To complete the discussion on the security issue, we refer to [20].

Now, we present the simulation results for Via Parmenide crossing. First of all, notice Figs. 4 and 5, where we report the functional J_1. It is evident that the optimal case is higher than the fixed simulation (which corresponds to the real case $p=0.875$, obtained by the cycle of the traffic light for the junction o); hence, actually, Via Parmenide in Salerno does not follow a traffic optimization policy. Some traffic engineers have observed the traffic behavior for Via Parmenide, and they have seen that there are some intervals of time in which some cars are stopped by the traffic light, while other roads are completely empty. This situation means that the cycle of the traffic light is too long. A solution could be to reduce the cycle or substitute the traffic light with a stop sign.

Let us focus on the performances of the dynamic random simulation. It is obvious that such a simulation does not match the optimal case and, moreover, it is possible to see that its behavior is very similar to a fixed simulation with $p=0.5$. Notice that $p=0.5$ represents the minimum for J_1. In fact, for the flux function $f(\rho)=\rho(1-\rho)$ (see [1]), we have

$$J_1(p)=\chi-\frac{1}{2}\sqrt{1-4cp}-\frac{1}{2}\sqrt{1-4c(1-p)},$$

where χ and $c>0$ are constant, not depending on p. As

$$\frac{dJ_1(p)}{dp}\geqslant 0 \Rightarrow p\in[0.5,1],$$

we conclude that $J_1(p)$ is decreasing in $[0,0.5]$ and increasing in $[0.5,1]$. Hence, $p=0.5$ is the minimum for J_1.

Thus, the dynamic random choice corresponds to the worst case.

IV. RANDOMNESS vs OPTIMAL CHOICE

From what we have seen, it is necessary to make a discussion that unifies the performances of the dynamic random simulation for the two case studies (Re di Roma Square and Via Parmenide). We could ask why dynamic random simulations are so dissimilar for such analyzed networks.

The answer can be found through examination of the statistical properties of dynamic random coefficients. Suppose that we have a network with N junctions. It is necessary to consider the optimization of right-of-way parameters for junctions with a number of incoming roads greater than the number of outgoing ones. In other cases, we consider distribution coefficients, which are usually fixed, as they depend on the paths of drivers, and so (with the exception of particular cases) no optimization algorithm is used for such coefficients.

Consider a junction i of the network, for which we have to consider a right-of-way parameter p_i. In a dynamic random simulation, the right-of-way parameters are chosen in a random way for each step of the simulation process, supposed to have M steps. Then, the parameter p_i changes M times, and its average value is

$$\frac{1}{M}\sum_{i=1}^{M}p_i.$$

Then, p_i is a random variable with uniform distribution in the real interval $(0,1)$. If M is very large, then by the strong law of large numbers, we obtain that

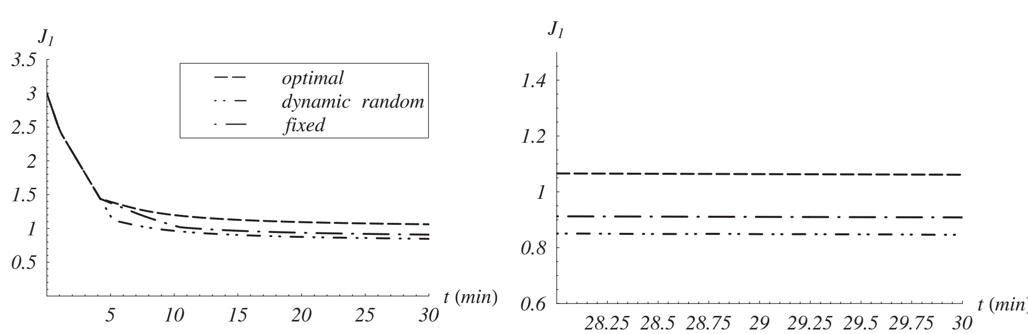

FIG. 4. Behavior of J_1 for Via Parmenide crossing (left) and close-up around the simulation cases (right).

FIG. 5. Behavior of J_1 for Via Parmenide crossing among the dynamic random simulation (double-dot-dashed line) and the fixed simulation with $p=0.5$ (solid line).

$$\lim_{M\to\infty}\frac{1}{M}\sum_{i=1}^{M}p_i \to E[p_i]=0.5,$$

and hence it is possible to affirm that the average right of way parameters tends to 0.5. In other words, a dynamic random simulation of a network approximates a particular case, where the junction i has a fixed right-of-way parameter $p_i=0.5$.

Now, let us focus on the two cases study. For Re di Roma Square, there are six junctions with two incoming roads and one outgoing road, for which it is necessary an accurate choice of right-of-way parameters. With the optimal algorithm, such parameters are computed to be very different from 0.5. However, it is possible to show that the average value of right-of-way parameters is very close to 0.5, and this justifies the similar behavior among dynamic random simulations and optimal ones. For Via Parmenide, the optimal solution is far from $p=0.5$. Now, the dynamic random simulation cannot match the optimal case.

In the case of Re di Roma Square, the adoption of the SGW functional and the analysis of densities (described in detail in [1]) are necessary because, if we consider networks with many junctions, the great variety of values of traffic flows could let the dynamic random simulation be similar to the optimal one.

For Via Parmenide crossing, unlike Re di Roma Square, the optimal solution, from the mathematical definition of the cost functionals, is never equal to $p=0.5$ (see [1]); hence, the dynamic random approach must fail, as confirmed by simulations.

Finally, we can conclude that dynamic random simulations are absolutely not convenient for the optimization of urban networks, both in the case of networks with many junctions (like Re di Roma Square) and for networks with only one junction (like Via Parmenide crossing).

V. CONCLUSIONS

In this paper, we consider the problem of car traffic regulation through the analysis of two different approaches: a random approach, as described in [2], and the optimization of traffic parameters, as in [1]. Some cost functionals that measure average velocity and average traveling time of cars are introduced. Then, two real cases of urban networks—Re di Roma Square, in Roma, and Via Parmenide crossing, in Salerno—are considered. Although such two cases are completely different from each other, it is possible to affirm that random approaches are not useful to improve the traffic behavior.

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