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Phase synchronization in mutually coupled chaotic diode lasers

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Semiconductor lasers with optical feedback have chaotically pulsating output behavior. When two similar chaotic lasers are optically coupled, they can become synchronized in their optical fluctuations. Here we show that the synchronization is not only in the amplitude and in the timing of the pulses but that the short pulses are also phase coherent with each other. This is true even when the lasers are separated by distances much larger than their coherence length.

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The synchronization of the optical phase of two mutually coupled chaotic diode lasers is experimentally examined under isochronal and achronal conditions. We show that the emergence of full or partial correlations in the chaotic laser intensities is accompanied by similar optical phase correlations. The chaotic lasers are thus coherent with each other either instantaneously (isochronal) or with a time delay between them equal to an integer multiple of the optical propagation time between the lasers (achronal).

Two similar mutually coupled semiconductor lasers, each with self-feedback in addition to their mutual coupling, have been shown recently [1–4] to exhibit zero-lag (isochronal) synchronization in their chaotic intensity output. The chaotically fluctuating intensity pattern of each laser has been experimentally examined with greater than 100 ps resolution [5] and has been shown to consist of short random intensity spikes, emitted by each laser. The spike trains emitted by the lasers are nearly identical with zero time delay between them, in spite of the fact that the two lasers can be physically separated by an arbitrarily large distance. Such systems are of great interest in the general fundamental study of coupled dynamics as well as in such diverse fields as neural networks [6], cryptography [7–9] and secure optical communications [10–13].

If the two similar coupled lasers lack self-feedback, a configuration known as face-to-face, chaotic fluctuations are again observed. Synchronization between the lasers is observed in this configuration as well, but in this case the synchronization is achronal, that is delayed by the propagation time of the coupling light between the lasers. The achronal synchronization mode can be of the leader-laggard type, in which one of the lasers always precedes the other in time, or a mode where the leader position is taken randomly by each of the lasers with each laser taking an equal share of leader and laggard positions [14,15]. The simplest configuration of course is two lasers which are unidirectionally coupled in which case the receiving laser is injection locked; the receiving laser copies the time-dependent intensity of the transmitting laser. If the transmitting laser happens to be chaotic (due to self-feedback, for example) the receiving laser will copy the intensity fluctuations with a delay corresponding to the light propagation time.

A natural question arises as to whether the two intensity wise, isochronally or achronally synchronized sources are also phase synchronized? On the one hand, one would naturally expect that the intensity spike emitted by a laser at a specific time and with a specific amplitude is determined by the precise time varying phase and phase history in the laser cavity. Thus if two lasers emit synchronized intensity spikes their instantaneous phases would also be synchronized and they should be coherent with each other. On the other hand, the phase in a semiconductor lasers varies greatly on very short time scales and it is possible that the ″long″ intensity spike emitted by the laser represents some time average of this rapidly varying phase. This is especially the case where the optical distance between a pair of mutually coupled lasers is much larger than the solitary laser coherence length. In this case one might expect that only some average phase of the two synchronized lasers is required to be the same and the two lasers would be instantaneously phase incoherent.

The phase coherence, or lack of it, is even less intuitive in a face-to-face, achronal or anticipated synchronization configuration. For such configurations the intensity correlation between the laser pulse trains is not perfect, and only a partial overlap is observed in the time shifted correlation of the intensities of the lasers, while for isochronal synchronization, the unshifted intensity correlation based on numerical calculations as well as experiment is near perfect. Thus for achronal synchronization with only a partial intensity correlation, which occurs for long time shifts, it is not obvious whether the partial time shifted intensity correlation necessarily implies a partial time shifted phase correlation. Similar questions of phase synchronization have been addressed in a solid state laser array system [16].

In this paper we address this question by directly measuring the phase coherence for two isochronally synchronized diode lasers, as well as for two lasers in a face-to-face configuration. We show that intensity correlation between two lasers, whether achronally or isochronally synchronized, also implies a corresponding phase correlation and coherence between the two laser outputs. The intensity correlation is measured by, $\rho(\Delta t)$ [1]:

$$\rho(\Delta t)=\frac{\Sigma_i(I_A^i-\langle I_A^i\rangle)(I_B^{i+\Delta t}-\langle I_B^{i+\Delta t}\rangle)}{\sqrt{\Sigma_i(I_A^i-\langle I_A^i\rangle)^2\Sigma_i(I_B^{i+\Delta t}-\langle I_B^{i+\Delta t}\rangle)^2}}, \tag{1}$$

where I_A and I_B are the time-dependent intensities of lasers A and B, respectively, and $\Box_i$ stands for an average over a given window size. When isochronal synchronization is es-

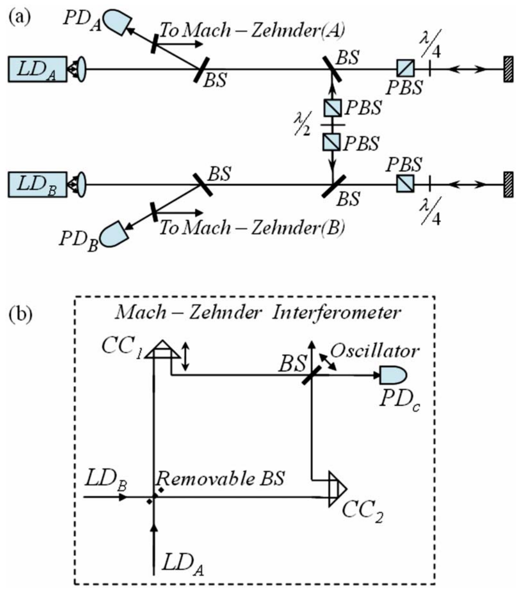

FIG. 1. (Color online) Schematic diagram of the experimental setup. (a) Two mutually coupled lasers in isochronal configuration. (b) Mach-Zehnder interferometer setup for visibility measurements. LD, semiconductor laser diode; BS, beam splitter; PBS, polarizing beam splitter; PD, photodetector; CC, corner cube.

tablished, $\rho(\Delta t=0)$, has a maximum value of 1. Additional high correlations are also found at time delays corresponding to integer, n, multiples of the optical transit time between the two lasers, $n\tau$, with the value of ρ decreasing as n increases. For τ in the range of 20 ns, we can typically observe intensity correlation up to $n\sim 10$, implying that the lasers lose all correlation with their previous history on a time scale of $\sim$200 ns. For achronal synchronization, $\rho(\Delta t=0)=0$, and the highest correlation peaks are obtained at a delay time $\pm\tau$.

The phase correlation of the two lasers is measured via interfering the two laser outputs and obtaining a fringe pattern, whose contrast corresponds to the degree of phase coherence. The contrast or visibility, $V(\Delta t)$, is obtained from the interfering fringe pattern

$$V(\Delta t) \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}, \qquad (2)$$

where $I_{\max}$ is the intensity at the peak of the brightest fringe, $I_{\min}$ is the intensity in the darkest valley between fringes, and Δt is varied by delaying one laser output with respect to the other inside the interferometer. For perfectly phase coherent beams the fringe contrast is optimum and $V=1$ while for phase incoherent beams, no fringes are formed and $V=0$.

The experimental setup consists of two similar Fabry-Perot semiconductor lasers A and B, shown in Fig. 1(a), emitting near 655 nm wavelength. Both lasers are subject to delayed self-feedback with a propagation loop time τ_d and mutual coupling with loop time τ_c, so that each laser receives feedback from two delayed signals, one from self-coupling and one from mutual coupling. Isochronal synchronization is achieved by setting the loop times equal, $\tau_d=\tau_c=\tau$, and for a range of values of the self-feedback, κ, and the mutual coupling, σ [3]. κ and σ are adjusted by using quarter-wave plates and a half-wave plate in combination with polarizing beam splitters, as indicated in Fig. 1. For the experiments reported in this paper τ was chosen to be 14.125 ns. Under these conditions, when the two lasers were tuned to operate at nearly identical wavelengths and their injection current to threshold current ratios, $I/I_{\mathrm{th}}=p$, were nearly equal, we could establish isochronal synchronization between them.

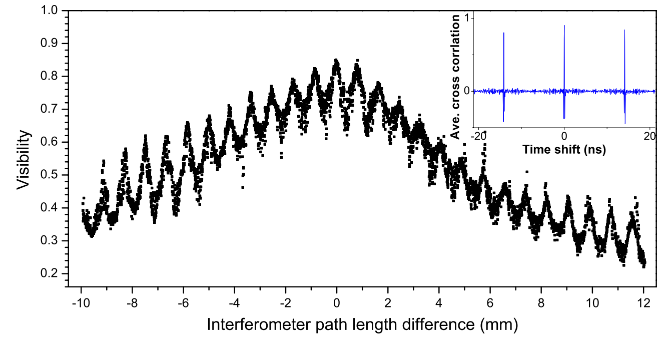

FIG. 2. (Color online) Phase coherence, as measured by fringe visibility of two isochronally synchronized semiconductor lasers with $p=1.2$. The visibility is measured using a Mach-Zehnder interferometer where each laser propagates in one arm of the interferometer and are interfered at an output beam splitter. Inset: The time-shifted intensity cross correlation of the two lasers.

For intensity correlation measurements, we use two fast (50 GHz bandwidth) detectors biased via a 40 GHz bandwidth bias T. The dc current into the bias T is used to measure the average dc power falling on the detector, while the ac currents are measured simultaneously by two channels of a 12 GHz bandwidth, 40 GS/s digital oscilloscope (Tektronix TDS 6124C). We then analyze the data from the digital oscilloscope using time-shifted cross correlation as in Eq. (1). The correlation coefficient is calculated between matching time segments from each detector and then averaged over all segments within the observation time. We arbitrarily choose the size of each segment to be 10 ns, the sampling time to be 25 ps and the total observation time to be 4 μs. The time shift step size (Δt) for the calculation was also 25 ps, so that I_A^i in Eq. (1) is an averaged intensity for a 25 ps wide time window, at time i.

In some cases, when operating close to threshold, breakdowns commonly referred to as low frequency fluctuations (LFF) occur [17–20]. During such breakdowns the two lasers can temporarily desynchronize, as was shown numerically [21] and experimentally [1]. Some of the 10 ns wide segments contain such LFF breakdowns, thus decreasing the value of the average correlation. The affect of LFFs on the value of ρ was small enough, however, that we did not perform a more complex data analysis in which segments containing LFFs would be eliminated. In our measurements the maximum average correlation including LFF containing segments is 0.9 at zero time shift and recovers to 0.82 at $\pm\tau$ as shown in the inset in Fig. 2.

The coherence properties of the light are measured using a modified Mach-Zehnder interferometer, shown in Fig. 1(b). In the interferometer, positioned equidistant from the two lasers, the beam splitter (BS), normally located at the light

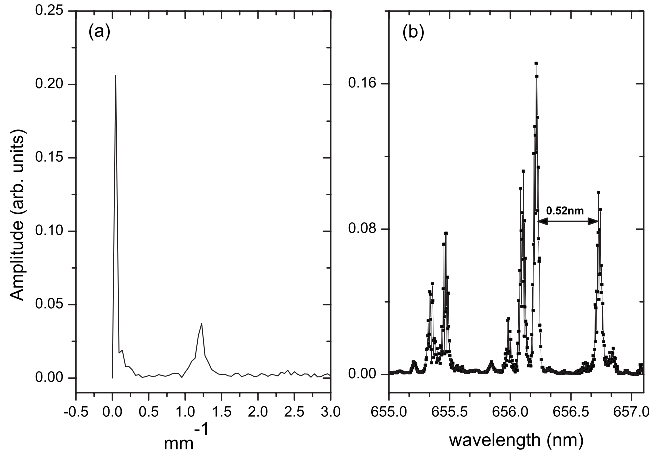

FIG. 3. (a) FFT of visibility vs PLD for the data in Fig. 2. (b) Combined optical spectrum of two coupled lasers in isochronal synchronization.

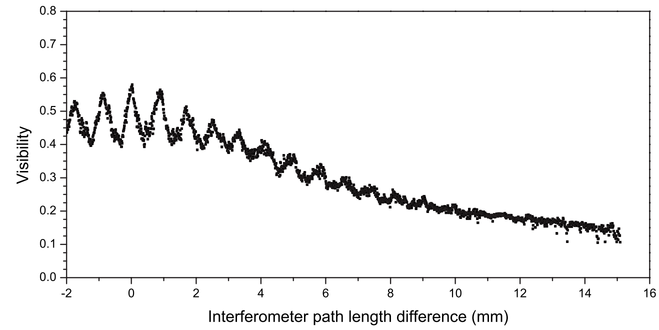

FIG. 4. Visibility measurement of two semiconductor lasers, with $p=1.2$, in isochronal synchronization measured in an asymmetric Mach-Zehnder in which one arm of the interferometer was lengthened by $\tau c=4.24$ m.

input port is removed and each laser beam advances via orthogonal paths inside the interferometer. The coherence length of each individual laser can be measured by blocking the other laser and replacing the input port BS. The path length of one arm of the interferometer, can be varied using a corner cube (CC_1) mounted on a computer controlled stage with a total travel range of 12 mm. A second corner cube (CC_2) is used to adjust the second interferometer arm, so that zero path length difference (PLD) can be achieved within the range of the computer controlled stage motion. The two beams interfere at the exit BS, and if the beams are slightly divergent, create an interference pattern consisting of bright and dark fringes. An audio speaker with an oscillator drive is used to randomly vibrate the exit BS, thus randomly scanning the interference pattern over a small aperture in front of the detector. By collecting data over a sufficiently long time (180 ms in our experiments) the photodetector samples the brightest and darkest fringe of the interference pattern and from these values (which are sampled by an analog-to-digital converter at 50 kHz sampling rate) the visibility can be calculated for any given delay between the interferometer arms. For spectral analysis, an optical spectrum analyzer with maximum resolution of 0.07 nm is used.

From Fig. 2 it is apparent that when isochronal intensity correlation is established between the lasers (as indicated in the inset) the optical phases of the two lasers are also synchronized. In this measurement, the interferometer step size was 0.004 mm and the maximum optical PLD between the interferometer arms was 22 mm. The visibility at zero PLD is 0.85 and decreases as the PLD increases to yield a coherence length of the synchronized lasers of 14.5 mm, measured at fullwidth at half-maximum (FWHM). We noted experimentally that there is a strong correspondence between the measured intensity correlation, $\rho(\Delta t=0)$, and visibility, $V(\Delta l=0)$, values. Though the two measured values are slightly different (0.9 versus 0.85) this is probably due to experimental parameters not fully controlled. These include the difference in the spatial mode structure of the lasers and thus their interfering overlap can have a small background intensity which lowers the value of V. We also note that □ is measured using windows of 10 ns whereas the visibility is measured over a time window of 180 ms.

The oscillation of the visibility, with a ~0.8 mm PLD, in Fig. 2, is indicative of the presence of laser modes separated by a mode spacing, $\Delta\lambda$, which is expected to give an oscillation in the visibility with a PLD given by

$$\Delta L = \frac{\lambda_0^2}{\Delta\lambda}, \tag{3}$$

where λ_0 is the average laser wavelength [22]. To confirm this we calculated the fast Fourier transform (FFT) of the data shown in Fig. 2 (without the bias term) and obtained that the oscillation period is 1.22 mm^{-1} as shown in Fig. 3(a). (The peak at 0.04 mm^{-1} corresponds to the overall width of the visibility curve.) From Eq. (3), we predict that the laser spectrum should have modes separated by $\Delta\lambda \sim 0.525$ nm, which is in good agreement with the measured wavelength difference (0.52 nm) of the mode clusters, as shown in Fig. 3(b). The relative amplitudes of the clusters of modes shown in Fig. 3(b) varies over time and snapshots of the laser spectrum show varying intensity distributions between the modes. The wavelength of each mode and the difference between them, however, remains constant. The mode spacing of ~0.12 nm in Fig. 3(b), would imply an oscillation in the visibility with a period of 3.4 mm. However, we did not detect such periodicity in the visibility measurement. This could be because the lasers instantaneously operate on a single mode within each of the mode clusters while the spectrum of Fig. 3(b) shows the integrated spectrum of each wavelength over a long time (~2 ms).

From measurements of the visibility for varying p values, we have observed that the coherence length of the laser decreases with increasing p. At $p=1.35$, the coherence length of the coupled chaotic lasers is 13 mm, while at $p=1.2$ the coherence length is 14.5 mm (Fig. 2). Remarkably the maximum visibility at zero PLD, $V(\Delta L=0)\sim 0.85$, is independent of p. Similarly the maximum intensity correlation, $\rho(\Delta t=0)\sim 0.9$, is also nearly independent of p.

For isochronal synchronization, the intensity correlation revives after a propagation loop time, τ (as can be seen in the inset in Fig. 2), and thus we would expect the phase coherence of the lasers to also reappear when measured with a very asymmetric Mach-Zehnder interferometer, in which the PLD is near $\tau c = 4.237$ m. Since the intensity correlation decreases as $n\tau$ increases, however, we also expect that the visibility will decrease as the interferometer asymmetry is increased. In Fig. 4 we show the visibility measured for an interferometer in which the path of laser B inside the interferometer was extended by 4.24 m. The maximum visibility measured in this configuration is 0.58, corresponding to a maximum intensity cross correlation at τ time shift of 0.82 (Fig. 2 inset). The oscillations in the visibility measurement are similar to the oscillations measured in the symmetric Mach-Zehnder and are consistent with wavelength differences between the mode clusters of the lasers.

In the case of achronal synchronization, the intensity correlation is zero at $\Delta t = 0$ and is maximum at $\pm\tau$. Thus we expect that the visibility in a symmetric Mach-Zehnder should be zero, as is indeed the case. At a time shift of $\pm\tau$ the intensity correlation is maximum and has a value of 0.79 and as expected the asymmetric Mach-Zehnder shows a visibility of 0.6. For a single laser with delayed self-feedback, at zero PLD, the visibility approaches 1 [23,24]. For such a laser at a PLD=τc, the visibility drops to 0.66, while the intensity correlation reduces to $\rho = 0.85$.

In conclusion, we have shown that intensity correlations between two chaotic lasers also necessitate phase correlations. The degree of phase correlation is found to be closely related (possibly equal to) the degree of intensity correlation. The maximum correlations are observed at zero time shifts for two lasers isochronally synchronized and are shifted by the light propagation delay times for achronally synchronized lasers. These findings have important applications to cryptography and high bandwidth communication protocols, where in addition to the pulse amplitude and time slot, also the phase of the carrier can be used to code information.

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