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All-optical noninvasive chaos control of a semiconductor laser

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We demonstrate experimentally control of a chaotic system on time scales much shorter than in any previous study. Combining a multisection laser with an external Fabry-Perot etalon, the chaotic output transforms into a regular intensity self-pulsation with a frequency in the 10-GHz range. The control is noninvasive as the feedback from the etalon is minimum when the target state is reached. The optical phase is identified as a crucial control parameter. Numerical simulations agree well with the experimental data and uncover global control properties.

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The control of chaos in dynamical systems has received much attention since its introduction in the seminal paper of Ott, Grebogi, and Yorke [1]. The possibility of directing the motion of a chaotic system by small perturbations is not only interesting from a fundamental point of view, but also of immense practical importance. A large variety of control methods has been developed in the meantime for different purposes [2,3]. Among them, time-delayed feedback control (TDFC) plays an outstanding role [3–11]. It transforms chaotic motion into regular oscillations of a period T that equals the delay time τ by simply applying at any time t a control force proportional to the difference $s(t)-s(t-\tau)$, s being a state variable of relevance. An essential feature of TDFC is its noninvasive character: The control force disappears when the target state is reached, while deviations automatically create feedback driving the system back to periodic motion.

TDFC is relatively easy to implement in the electrical domain and has been exploited in many different areas of science and engineering [3]. It is suited for fast systems [5], and optoelectronic TDFC of lasers on time scales down to 10 ns has been achieved [3,8].

In high-speed data communication, semiconductor lasers operate in the 10-GHz range and beyond. Such ultrafast time scales demand inevitably all-optical feedback. In the optical domain, the delay time is only limited by the speed of light, however a new degree of freedom—the wave phase—comes into play. It alters the scenario even qualitatively making all-optical TDFC conceptually different from the standard approach. In this Rapid Communication, utilizing feedback from a Fabry-Perot (FP) cavity, we demonstrate stabilization of intensity pulsations of a chaotic multisection semiconductor laser. Operating in the ps domain, this is the fastest chaos ever controlled in a practical device. We also present a theoretical analysis of the conditions that have to be specifically matched in all-optical TDFC.

The experimental control setup is schematized in Fig. 1(a). The device is an $\lambda=1.55$ μm integrated tandem laser (ITL) as used in ultrafast optical communication [12]. It is composed of three sections. Two distributed feedback (DFB) lasers are connected via a passive waveguide section. Each section is individually biased by a direct current. Variation of these currents as well as temperature give rise to a four-dimensional parameter space with a rich variety of nonlinear regimes. A glass etalon of reflectivity $R=0.76$ placed in front of the ITL serves as FP [Fig. 1(a)]. The device output is collimated and coupled into the FP. The reflected light is reinjected in the laser and provides the control force. The delay time τ is defined by the round-trip time in the FP cavity. A neutral gray filter is used to adjust the feedback strength K. The distance between laser and FP can be tuned with subwavelength precision by a piezoactor. A large-area photodiode measures the power transmitted through the FP, while the signal emitted from the opposite laser facet is launched into a 40-GHz spectrum analyzer (ESA) after optoelectronic conversion in order to record power spectra.

In the present study, the ITL is set to an operation regime where it undergoes a bifurcation sequence as shown in Fig. 1(b) when one laser current is tuned. First, continuous-wave (cw) emission turns into an intensity self-pulsation mode at a Hopf bifurcation displayed by the emergence of a prominent peak in the power spectrum at a frequency of $f=16$ GHz and its higher harmonics. Here, relaxation oscillations in the carrier-photon dynamics become undamped. At larger current, appearance of a feature at $f/2$ (and its mirror with respect to f) uncovers a period-doubling (PD) bifurcation. This is followed by generation of a period-4 pulsation $(f/4)$ beyond which the device runs quickly into a chaotic regime characterized by a broad power spectrum. Such a PD scenario is present in a wide parameter range and observable in the emission from both DFB sections.

In order to achieve noninvasive control, the Fourier components of the self-pulsation spaced by $2\pi/T$ have to match exactly the minima of the FP reflectivity spectrum. This requires that the ratio τ/T must be integer, which is the standard condition of TDFC [4]. However, in addition and specific of all-optical TDFC, the carrier frequency ω_0 [15] of the laser emission has to obey $\exp(i\omega_0\tau)=1$. The experimental challenge is to meet both conditions simultaneously. An essential advantage of the ITL in this regard is that the carrier frequency as well as the period of the self-pulsations can be rather independently tuned through the device temperature and/or the currents on the laser sections. This enables us to satisfy both requirements at given τ.

The experimental control scenario is summarized in Fig. 2. In a first step, the device is adjusted just beyond the PD bifurcation and τ matches the 26 GHz frequency of the fun-

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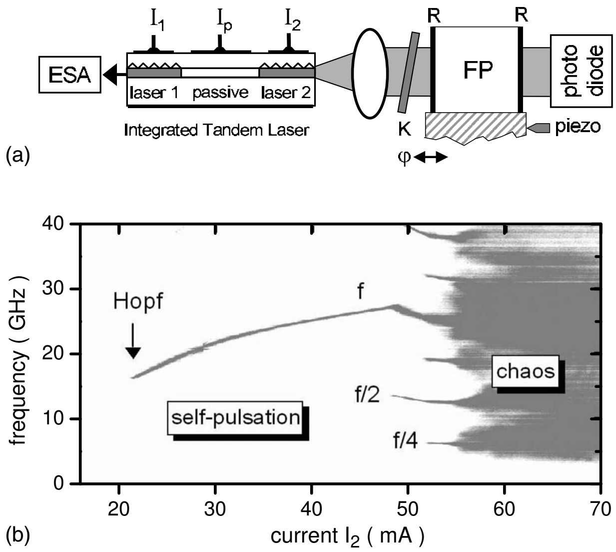

FIG. 1. (a) Sketch of the experimental setup; for explanations, see text. (b) Period doubling route to chaos of the solitary ITL: Power spectra represented in the frequency-current plane. Gray: signal strength more than 4 dB above noise level. I_1=50 mA, I_p =0 mA.

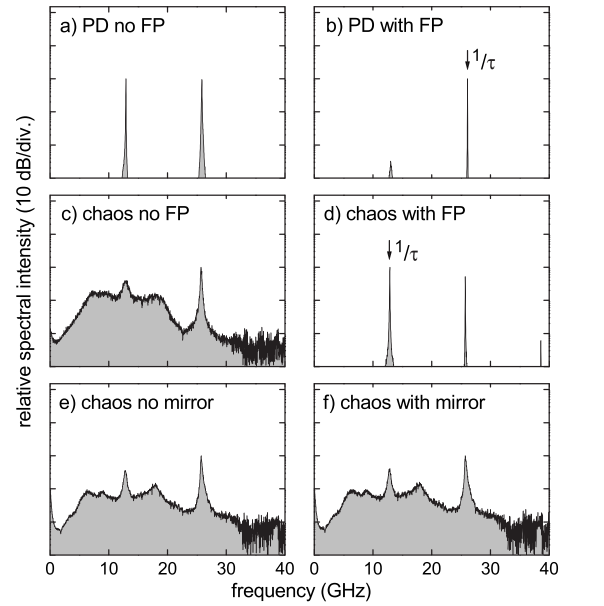

FIG. 2. All-optical TDFC (experiment). (a,b) Stabilization of the period-1 pulsation beyond the PD bifurcation with a 4-mm etalon. $I_{1,p,2}$=(47.8,67.8,74.0) mA. (c,d) Stabilization of the period-2 pulsation in the chaotic regime. An 8-mm etalon is used here. $I_{1,p,2}$ =(57.0,65.1,74.4) mA. (e,f) No change of operation mode when the FP is replaced by a simple mirror with the minimum reflectivity of the cavity. The height of the strongest feature in all spectra is arbitrarily scaled. Note the log scale.

damental period-1 pulsation (a). Without FP, the power spectrum is dominated by two peaks at f and $f/2$. Feedback from the cavity suppresses the period-2 peak by almost three orders of magnitude and restores thus cleanly the fundamental self-pulsation (b). Next, the solitary ITL is driven in the chaotic regime (c). Weak features at f and $f/2$ still superimposed to the otherwise broad power spectrum clearly indicate that the self-pulsations are indeed embedded as unstable elements in the chaotic attractor. Adding now an FP with τ matching the period-2 pulsation, the broad chaos contribution declines, whereas the discrete features increase strongly in height and become much sharper (d), demonstrating that a nearly perfect periodic regime is recovered.

An important point is whether the control is indeed noninvasive. In a practical setup, the reflectivity at the FP minima remains finite. From the power transmitted through the FP and the separately measured losses along the optical pathway, we find that the feedback is less than a 10^{-3} portion of the laser emission. To check on the role of this residual signal, we have replaced the FP by a simple mirror providing the same feedback level. In marked contrast to the FP setup, no change of the power spectrum is observed (e,f). Furthermore, locating the ITL out of the chaotic regime, no modification of the self-pulsations is found. These findings provide conclusive evidence that the periodic motion present under control is not related to a shift of the operation point, but associated with an unstable orbit existing in the chaos stabilized by noninvasive feedback from the FP.

From a mathematical point of view, this orbit is quasiperiodic with a pulsation frequency and an optical frequency. It represents a modulated wave [14]. As a consequence, optical phase shifts appear as extra control parameters which have to be treated carefully both in theory and experiment. This has been considered so far only for the special case of steady states (cw operation) [9,10,13]. In general, the optical field emitted by the laser is

$$\mathcal{E}(t) = \mathrm{Re}\{E(t)e^{-i\omega_0 t}\}, \tag{1}$$

where, in the case of a periodic intensity pulsation, it holds $E(t+T)=E(t)$ for the slow amplitude. The feedback field entering the laser when the carrier frequency is on resonance with a reflection minimum of the FP reads as

$$E_{\mathrm{b}}(t) = Ke^{i\varphi}\sum_{n=0}^{\infty} R^n[E(t_n) - E(t_{n+1})], \tag{2}$$

with $t_n=t-\tau_l-n\tau$ [13]. τ_l is the latency time needed for one round trip between laser and FP. It is easily seen from the above expression that a too large latency overrules the feedback from the FP weakening the ability of control. In the experiment, geometrical constraints set a lower limit of $\tau_l \approx 80$ ps which is almost equal to τ and thus not yet critical. K comprises all losses between laser and FP including the factor $\sqrt{R}$ for a single reflection. The phase shift $\varphi=\omega_0\tau_l$ associated with the latency round-trip is not present in conventional TDFC. It makes the total control gain $K\exp(i\varphi)$ a complex quantity. Variation of the latency phase over a period of 2π involves thus a sign change of the control force implying a finite φ-range where all-optical TDFC is possible. The experimental data follow entirely this expectation. Figure 3 assembles a series of measurements in the chaotic regime where the position of the FP is moved in small steps of about 47 nm. The sub-ps alteration of τ_l itself is unimportant, but φ changes over one period when the total translation equals the wavelength. Periodicity, both in the FP transmission as well as in the power spectra, is clearly recognizable.

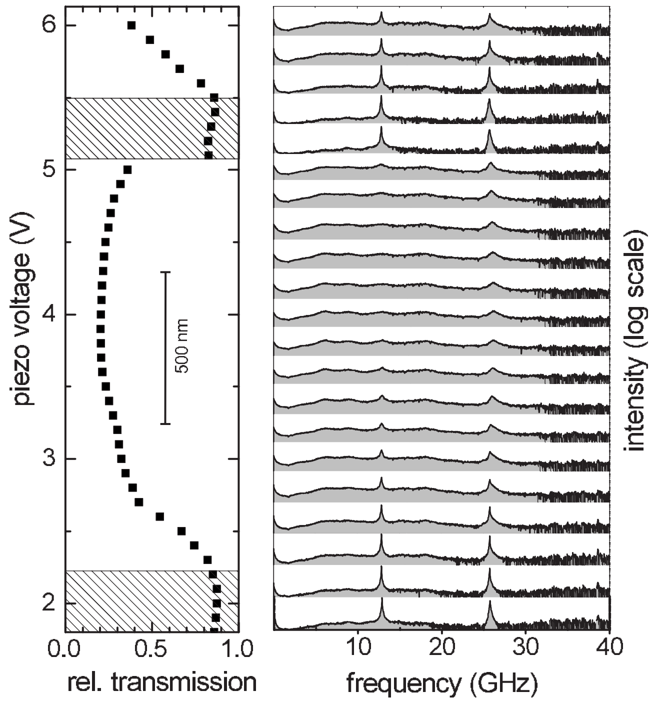

FIG. 3. Effect of latency phase on control. Left: Power transmitted through the FP (horizontal axis) versus piezovoltage (vertical scale). The power is given relative to the laser output. Shaded areas mark the regions of control. Right: Power spectra corresponding to selected piezovoltages in the left part.

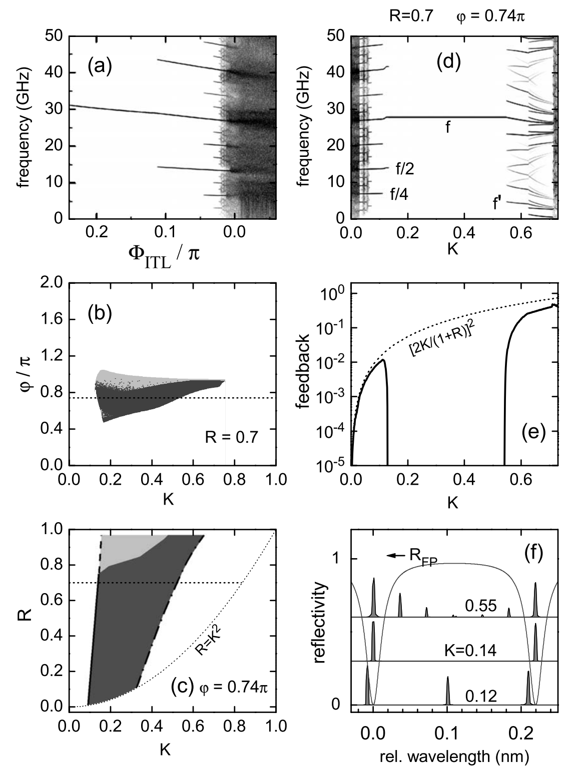

FIG. 4. All-optical TDFC (theory). (a) PD route to chaos of the solitary ITL. The internal phase shift Φ_{ITL} induced by the current in the ITL is used as control parameter. (b,c) Domain of control in the (φ,K) and (K,R) plane at given R and φ, respectively. $\Phi_{\mathrm{ITL}}=0$. Domain boundaries: super- and subcritical PD bifurcations (solid and dashed, respectively), supercritical torus (Hopf) bifurcation (dash-dotted). Light gray area: bistable region (hysteresis), dotted line: $K=\sqrt{R}$. (d) Evolution of the power spectra when moving along the thin dashed horizontal lines in (b) and (c). (e) The same for the feedback measured in units of the ITL output. Dashed curve: Maximum feedback available at given K and R. (f) Optical emission spectrum compared to the FP reflectivity spectrum at selected feedback strength. The emission peaks are artificially broadened for better visibility. Parameters as in Ref. [10] except $\tau_l=127$ fs, $\tau=35\,973$ fs; laser 1: $I=100.4$ mA, $\delta=19\,341\ \mathrm{m}^{-1}$; passive: $L=500\ \mu$m, $\gamma=25\ \mathrm{cm}^{-1}$; laser 2: $I=80$ mA, $\delta=16\,321\ \mathrm{m}^{-1}$.

The range of control is identified by a transmission close to 1 and sharp peaks in the power spectrum. In the region in between, the power spectra are even less structured than in the uncontrolled device indicating an increased irregularity of the emission. Such behavior might be interesting for applications demanding an extremely flat spectral response.

The feedback parameters ϕ, K, and R form a three-dimensional manifold. In order to figure out the domains where control is possible, we have numerically analyzed the ITL-FP configuration by solving the corresponding partial-differential traveling-wave equations [13]. The solitary ITL is excellently described by such analysis as seen from the calculated power spectra in Fig. 4(a) reproducing almost perfectly the experimental PD route to chaos [cf. Fig. 1(b)]. In the numerics, the ITL is first set in the chaotic regime and then the FP is added. Successful control is identified by the occurrence of a regular intensity pulsation with a period τ and disappearance of the feedback signal subsequent to a short switching transient. In this way, the domain of control in the K-φ plane at given R shown in Fig. 4(b) is obtained. Here, the period-1 pulsation is locally stable. The chaotic initial state belongs to its basin of attraction only in the dark regions of the island. Light gray with isolated dots indicates an area where the ability of control depends on the initial state because of hysteretic behavior. As in the experiment, control is limited to a small range of the latency phase. For all suitable φ, there is a lower and upper border for the feedback strength. In order to uncover the role of R, control domains in the K-R plane at proper φ are ascertained [Fig. 4(c)]. To detect hysteresis, K is varied in small steps forward and backward between zero and $K=\sqrt{R}$ using the final state of the previous step as initial value. The effect of the sole FP with no extra attenuation in the latency path is described by the curve $R=K^2$. When moving along the curve, control is first attained and later lost at critical cavity finesses. The existence of a lower threshold is intuitively clear as the control force has to be strong enough to drive the system back to the target state. Beyond the upper border, spectral emission components off-resonant to the FP become undamped by the increased feedback at the wings of the reflection minimum [cf. Fig. 4(f)]. Here, a reduction of K, corresponding to adjusting the filter in the experiment, recovers control.

The right panels of Fig. 4 illustrate the control scenario along a line of given φ and R close to the experimental values (dashed horizontals in b and c). Computed power spectra confirm that the pulsation frequency f is locked to $1/\tau$ in the control domain for K (d). The reinjected feedback power is more than five orders below the laser emission (e) and the optical spectrum is a comb of lines fitting exactly to the FP resonances (f). When approaching the domain boundary from below, chaos disappears in a reverse PD sequence. At the upper boundary, a second frequency f' emerges in a Hopf bifurcation. The torus creates satellite lines in the optical spectrum increasing quickly the feedback signal so that the torus finally breaks up into chaotic behavior.

At the above choice of parameters, no hysteresis is found. A more extended analysis covering the range of Fig. 4(b) provides that the domain boundary at high K is always constituted by a torus (Hopf) bifurcation with a soft transition to quasiperiodic pulsations. The boundary at low K is more complex. Beyond a certain FP reflectivity, the PD bifurcation becomes subcritical and bistability occurs. Here, the stabilized pulsation coexists with feedback-modified complex dynamics and control can only be achieved when smoothly decreasing K from above. The nature of the upper border of the hysteretic region could not be identified from the numerical data. Studies on conventional TDFC for a general class of unstable periodic orbits have yielded very similar control domains in the range of small K and R [6,7]. However, in contrast, bistability is absent on the low-K side, but occurs at the opposite border where the Hopf bifurcation turns from super to subcritical at sufficiently large R. Whether this discrepancy is caused by a specific constellation of internal parameters in the optical setup or a fundamental difference between the TDFC of periodic orbits and modulated waves is an open question deserving further investigation.

The above analysis is made for the stabilization of the period-1 pulsation in order to uncover the systematic trends in three-dimensional control parameter space. Exemplary calculations for longer latency times, with spontaneous-emission noise, and control of the period-2 pulsation provide results consistent with the general trends.

In conclusion, we have implemented and demonstrated all-optical and noninvasive TDFC on a practical laser device operating in the 10-GHz frequency range. Both the experimental and theoretical results emphasize the role of the optical phase in chaos control. Latency switching between a regular and irregular mode of operation by refractive index modulation or a programmable piezo-actor can be useful, e.g., in cryptography concepts. Another aspect lies in the potential of overcoming limitations inherent to standard TDFC [11]. Using shorter FPs, the scheme can be quite easily extended to even higher frequencies. Therefore, the present proof-of-principle experiment has revealed various specific features of ultrafast chaos control in the optical domain, with possible applications in future high-speed optical communication systems.

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