Coevolutionary networks with homophily and heterophily

Daichi Kimura* and Yoshinori Hayakawa†
Department of Physics, Tohoku University, Sendai, Japan
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We have investigated a simple coevolutionary network model incorporating three processes—changes of opinions, homophily, and heterophily. In this model, each node holds one of G opinions and changes its opinion, as in the voter model. Homophily is the tendency for connections to form between individuals of the same opinions and heterophily is the opposite effect. If there is no heterophily, this model corresponds to the Holme and Newman model [Phys. Rev. E **74**, 056108 (2006)]. We show that the behavior of this model without heterophily can be understood in terms of a mean field approximation. We also find that this model with heterophily exhibits topologically complicated behaviors such as the small-world property.

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I. INTRODUCTION

In this decade, our knowledge of the structure of interaction in diverse areas, such as social, biological, or technical systems, has developed enormously [1–4]. These complex networks have common features, despite the diversity of context in respective areas, and many models have been proposed to reconstruct these features [5–12]. On the other hand, the dynamics on these complex networks has been investigated energetically and reported as these dynamics are essentially different from that on random or regular networks [13–22]

In real networks, however, the links or connections between individuals are dynamic entities rather than static ones, and often interplay with the dynamics of individuals. For example, in social networks, it seems that each individual attempts to make a new connection to get a new job, information, and so on, while friction breaks existing connections. On these dynamic structures, social interactions such as opinion formation occur. In the biological context, neural networks are typical adaptive networks which optimize their structure under the influence of neuronal activities. Of course, these structural changes affect the dynamics of individuals strongly. Thus, the coevolution of individuals and connections is one of the interesting and important topics. Several authors have investigated models incorporating coevolution and found nontrivial behaviors [23–30].

Recently, Holme and Newman have discussed opinion formation in the real world as a result of combination of two processes—one makes individuals change their opinions due to the influence of their acquaintances, and the other makes connections form between individuals of similar opinions, namely, homophily [25]. They proposed a simple coevolutionary network model incorporating these two processes and found that their model exhibits a nonequilibrium phase transition despite its simplicity. We regard their model as being simple and fundamental enough to understand coevolutionary networks.

However, as well as these two processes, we believe that heterophily is also crucial for opinion formation. In contrast to homophily, heterophily is the tendency for connections to be formed between individuals of different opinions. It is known that optimal heterophily enhances the smooth communication among social networks [31–33]. The concept of weak ties proposed by Granovetter can be considered as an example of heterophily, which has a significant role in making a bridge between communities [34]. If there is no heterophilious communication, the networks are divided into isolated groups, so that heterophily is a fundamental process in social networks.

In this paper, therefore, we propose a coevolutionary network model incorporating these *three* processes, namely, changes of opinions, homophily, and heterophily, and analyze the asymptotic properties of the model.

II. THE MODEL AND ITS PAIRWISE APPROXIMATION

Let us consider a network that consists of N nodes and K links. Each node holds one of G opinions. We assume that the initial topology of the network is a random graph and the mean degree $\bar{k}=2K/N$ is greater than 1. The individuals and connections coevolve as follows.

We choose a node i randomly at each time step. If node i is isolated, we do nothing. Otherwise, we choose a neighbor j of node i randomly, and (1) with probability ϕ reattach the link between node i and j to a randomly chosen node holding same opinion with node i; or (2) with probability ψ reattach the link to a node holding a different opinion from node i; or (3) with probability $1-\phi-\psi$ node i adopts the opinion of node j.

Process 1 represents connection formation between holders of the same opinion, namely, homophily, and process 2 represents heterophily. These two processes make the network topology evolve. Process 3 alters the opinions of nodes due to the influence of their neighbors as in the voter model. If there is no heterophily, $\psi=0$, this model is identical to the Holme and Newman model [25].

To investigate the dynamics of the present model, we consider the simplest case in this model, namely, $G=2$, at first. We here apply a pairwise approximation to this model [27,35]. We describe the process of this model as the time evolution of the number of various types of connected pairs and the number of opinion holders,

*daichi@cmpt.phys.tohoku.ac.jp
†hida@cmpt.phys.tohoku.ac.jp

$$\frac{d[AA]}{dt} = (P_{B\to A}[AB] + 2P_{B\to A}[ABA] - P_{A\to B}[AAB]) \times (1-\phi-\psi) + P_{A\to B}[AB]\phi - P_{A\to A}[AA]\psi, \quad (1)$$

$$\frac{d[BB]}{dt} = (P_{A\to B}[AB] + 2P_{A\to B}[BAB] - P_{B\to A}[ABB]) \times (1-\phi-\psi) + P_{B\to A}[AB]\phi - P_{B\to B}[BB]\psi, \quad (2)$$

$$\frac{d[A]}{dt} = \left(\frac{[AB]}{[AB]+2[BB]}[B] - \frac{[AB]}{[AB]+2[AA]}[A]\right)(1-\phi-\psi), \quad (3)$$

where $[X]$ is the number of nodes holding opinion X, $[XY]$ the number of X-Y links, and $[XYZ]$ the number of X-Y-Z triplets (the central Y has X-Y and Y-Z links) with respective states $X,Y,Z \in [A,B]$, and A and B are the opinions each node can hold. $P_{X\to Y}$ denotes the probability that a node X chooses one of its neighbors Y $(X \neq Y)$,

$$P_{X\to Y} = \frac{[X]}{\sum_{W\neq X}[XW] + 2[XX]}, \quad (4)$$

which corresponds to the mean degree of the node holding opinion X. In the right-hand sides of Eqs. (1) and (2), the first term represents the effect of process 3 (voterlike opinion change), the second term the increase of the number of links due to process 1 (homophilious rewiring), and the third term with ψ the decrease due to process 2 (heterophilious rewiring). The factor 2 before term $[ABA]$ implies that in the triplet A-B-A A has two A-B links and therefore the influence of $[ABA]$ should be doubled. We approximate the number of triplets as $[XYX]=[XY]^2/2[Y]$ and $[XXY]=2[XX][XY]/[X]$, where we suppose that triplets are formed from two independent links. The numbers of nodes and links are preserved, i.e., $N=[A]+[B]$ and $K=[AA]+[BB]+[AB]$; thus Eqs. (1)–(3) are closed.

III. PHASE TRANSITION IN THE CASE WITHOUT HETEROPHILY

The behavior of this model significantly depends on ψ. Without heterophily, i.e., $\psi=0$, due to process 1 (homophilious rewiring) and process 3 (voterlike opinion change), this system reaches a state consisting of a set of connected components in which all members hold the same opinion. As Holme and Newman (HN) have already reported in Ref. [25], this model exhibits a nonequilibrium phase transition from a regime in which almost all nodes have the same opinion to one in which the network splits into some groups having different opinions, by changing the parameter ϕ. However, the mechanism of this phase transition has not yet been revealed.

Figure 1 shows the size of the largest connected subnetwork for $G=2$. As the network size N increases, we can observe a steeper slope clearly at the emergence of a large cluster. This might suggest a critical behavior as found in the original HN model where G is kept proportional to N, even in the case that G is fixed. However, to examine the asymptotic behavior, we need to carry out a systematic finite-size scaling for larger system size. Unfortunately, due to the limitations of computational resources, we were not able to prepare such data in the present study. Below we discuss the critical behavior in terms of a mean field approach, which suggests again the existence of a phase transition.

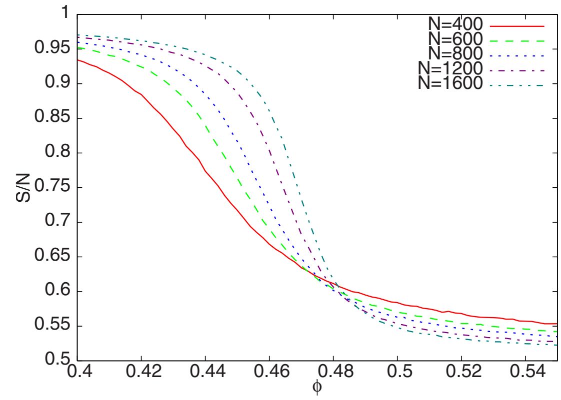

FIG. 1. (Color online) Size of the largest component. N is the number of nodes, and S the size of the largest component. $G=2$, $\bar{k}=4$. $S/N \simeq 1$ for small ϕ implies that almost all nodes have the same opinion, while the network splits into two large groups for large ϕ, i.e., $S/N \simeq 0.5$.

Using Eqs. (1)–(3) for $\psi=0$, we rewrite these equations by substituting the following variables: $u=([AA]+[BB])/K$, $v=([AA]-[BB])/K$, and $w=([A]-[B])/N$,

$$\frac{du}{dt} = \frac{N}{K}\frac{1-u}{1-v^2}[1 - vw + \gamma(1-2u+v^2)], \quad (5)$$

$$\frac{dv}{dt} = \frac{N}{K}(1-2\phi)\frac{1-u}{1-v^2}(v-w), \quad (6)$$

$$\frac{dw}{dt} = 2(1-\phi)\frac{1-u}{1-v^2}(v-w), \quad (7)$$

where $\gamma=2K(1-\phi)/N$ and $0\le u\le 1$, $-1\le v\le 1$, $-1\le w \le 1$. The variable u stands for the fraction of no "conflicting" links, namely, neighboring nodes holding the same opinion, and w stands for "magnetization," when opinion-A holders are regarded as +1 spins and B as −1. These equations have sets of fixed points on

$$u = \frac{1}{2}\left(1-\frac{1}{\gamma}\right)v^2 + \frac{1}{2}\left(1+\frac{1}{\gamma}\right), \quad v = w, \quad (8)$$

$$u = 1, \quad (9)$$

respectively. The set of fixed points (8) corresponds to the state where $P_{A\to B}=P_{B\to A}$, namely, the transition probabilities of both opinions are equal. Equation (9) corresponds to $[AB]=0$, where the network reaches an "absorbing" state. The eigenvalues at the set of fixed points (8) are

$$\lambda = \begin{cases} 0, \\ -\dfrac{N}{K}(\gamma-1) \lessgtr 0, \\ \left(1-\dfrac{1}{\gamma}\right)\left(\dfrac{N}{2K}(1-2\phi)-(1-\phi)\right) \lessgtr 0 \quad (\gamma \gtrless 1), \end{cases} \tag{10}$$

and, for (9),

$$\lambda = \begin{cases} 0, \\ 0, \\ -\dfrac{N}{K}\dfrac{1}{1-v^2}[1-vw+\gamma(v^2-1)] \gtrless 0 \quad (\gamma \gtrless 1). \end{cases} \tag{11}$$

Figure 2 shows a schematic view of sets of fixed points and flows.

For $\gamma > 1$, Eq. (8) represents a one-dimensional attractor which has one neutral mode in the tangential direction of the curve. Considering the fluctuation due to random sampling of nodes, we can regard the dynamics on the attractor as a random walk under the constraint given by (8) with an absorbing boundary condition at $u=1$. On the other set of fixed points, one can find that $v, w \to \pm 1$ when $u \to 1$; thus, all nodes have same opinion whenever $\gamma > 1$.

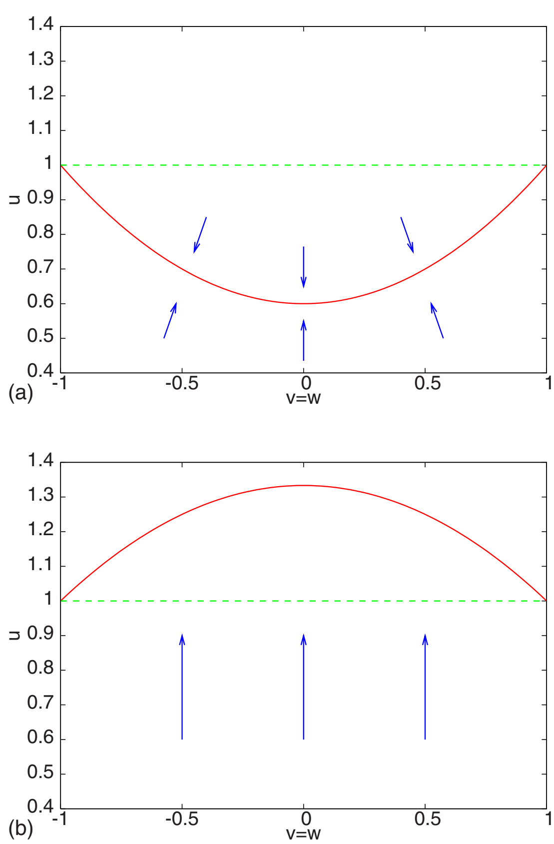

FIG. 2. (Color online) Schematic view of sets of fixed points of Eqs. (5)–(7) for $\gamma > 1$ (upper) and $\gamma < 1$ (lower). The solid line represents Eq. (8) and dashed line Eq. (9). The arrow indicates the flow.

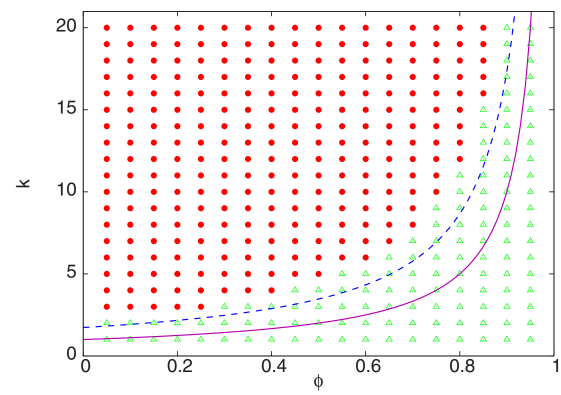

FIG. 3. (Color online) Phase diagram of final state for ϕ and $\bar{k}$ from numerical simulations. $N=1000$, $G=2$, and the number of holders of each opinion is $N/2$ at the initial state. Filled circles indicate that the size of the largest component at the end of the simulation is greater than $0.9N$, and open triangles less than $0.9N$. Solid line shows $\gamma=1$ $[\bar{k}=1/(1-\phi)]$ and dashed line $\bar{k}=\sqrt{3}/(1-\phi)$.

To the contrary, for $\gamma < 1$, the set of fixed points (8) disappears within the meaningful range of u, i.e., $0 \leq u \leq 1$. The numerical solution with the initial condition $u=0.5$, $v=\delta v$, $w=\delta w$ $(\delta v, \delta w \ll 1)$, where both opinion holders occupy about half of all nodes initially, indicates that u converges to 1 while v and w do not change their values significantly. As a result, the network finally splits into two components; one consisting of opinion-A holders and the other consisting of opinion-B holders, for $\gamma < 1$.

Therefore, the location of the sets of fixed points in the phase space determines the asymptotic behavior of this system. The behavior of this system abruptly changes from the voterlike stochastic process on the attracting set with a neutral mode to a simple relaxation process to the attractor (9).

Figure 3 shows the phase diagram of the final state for ϕ and the mean degree $\bar{k}$. The solid line in Fig. 3 represents the condition $\gamma=1$ $(\bar{k}=\frac{1}{1-\phi})$ where the disappearance of the parabolic attractor (8) occurs. A calculation incorporating higher-order structures rather than the pairwise approximation improves the accuracy for the phase boundary. Considering the time evolution of the number of triplets $[XYZ]$, one can estimate the boundary of phase as $\bar{k}=\frac{\sqrt{3}}{1-\phi}$ (dashed line in Fig. 3; see Appendix A for the derivation). As can be seen, the phase boundary obtained from numerical simulations agrees with the theory.

One can generalize the same argument for the case $G > 2$. The pairwise approximation predicts that the phase boundary is given by $\bar{k}=\frac{1}{1-\phi}$ for $G>2$ as well as $G=2$ (see Appendix B for the derivation). We expect that the behavior of the system with $G>2$ is basically identical to the case $G=2$. In fact, it seems that the value of G does not affect significantly the critical value ϕ_c in numerical simulations [25]. The equations obtained from the pairwise approximation have also trivial fixed points in the subspace with

$[XY]=0$ for every $X \neq Y$, where X and Y take one of the G opinions. These fixed points correspond to the state with no conflicting pairs, i.e., the final state of this model. Although the stability of these fixed points has not been examined for general G in the present study, at least for $G=2$ all eigenvalues become zero at the transition point $\gamma=1$. Thus, we may conclude that this neutrality causes a power law size distribution near the critical point as claimed in Ref. [25].

IV. DYNAMICAL EQUILIBRIUM IN THE CASE WITH HETEROPHILY

For $\psi > 0$, the network is not divided into isolated groups since the connections between different opinion groups arise from process 2 (heterophilious rewiring). In the case $G=2$, Eqs. (1)–(3) have a stable fixed point at $u=u_0$, $v=w=0$, where

$$u_0 \equiv \frac{3\bar{k}(1-\phi-\psi)+1-\frac{1}{2}\psi-\sqrt{\left[\bar{k}(1-\phi-\psi)+\frac{1}{2}\psi-1\right]^2+4\bar{k}\psi(1-\phi-\psi)^2}}{4\bar{k}(1-\phi-\psi)}; \quad (12)$$

thus the coexistence of two states is stable. Indeed, we find by numerical simulation that the network does not converge to one opinion. Figure 4 shows the time-averaged value of u for different ψ. As can be seen, Eq. (12) captures the trend well. Therefore, for even very small ψ, the heterophilious rewiring drastically changes the asymptotic behavior of the present model from that of Holme and Newman model.

Furthermore, the network exhibits the small-world property within a certain regime of parameters ϕ, ψ in the case $G \propto N$. Figure 5 shows the time development of L/L_0 and C/C_0, where L is the average path length and C the average clustering coefficient of this model with $G=0.05N$, $\phi=0.7$, $\psi=0.15$, and L_0 and C_0 of the random graph, respectively. We can find $L/L_0 \simeq 1$ and $C/C_0 \gg 1$ for each N so that the network shows the small-world property. We assumed the scaling form $N^b x/x_0 = f(N^a t)$ where $x=L, C$, and roughly estimated $a \simeq -1.5$, $b \simeq -0.015$ for L, $a \simeq -1.5$, $b \simeq -0.9$ for C. This implies that the growth of L with increasing N is very slow while C becomes significantly larger than that of the random graph. Thus, we expect this model to exhibit the small-world property even in the limit of $N \to \infty$. Figure 6 shows the ternary plots of $L/L_0, C/C_0, S/N$ and the regime that exhibits a small-world property. As Fig. 6 shows, a quite large regime of parameters ϕ, ψ shows such a small-world property. In this regime, process 1 (homophilious rewiring) is stronger than process 3 (voterlike opinion change) so that the network will split into some isolated communities whose nodes hold the same opinion if there is no heterophily

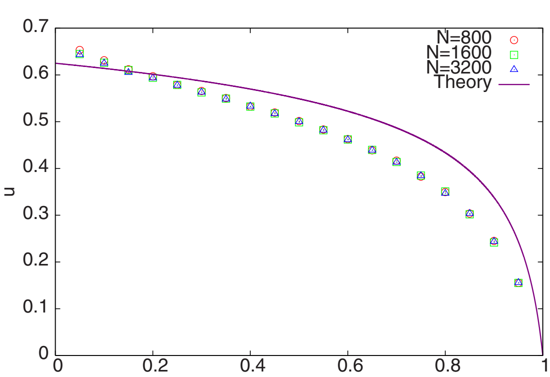

FIG. 4. (Color online) Time-averaged value of u for different ψ and N. Solid line shows Eq. (12). $\phi=0, \bar{k}=4$.

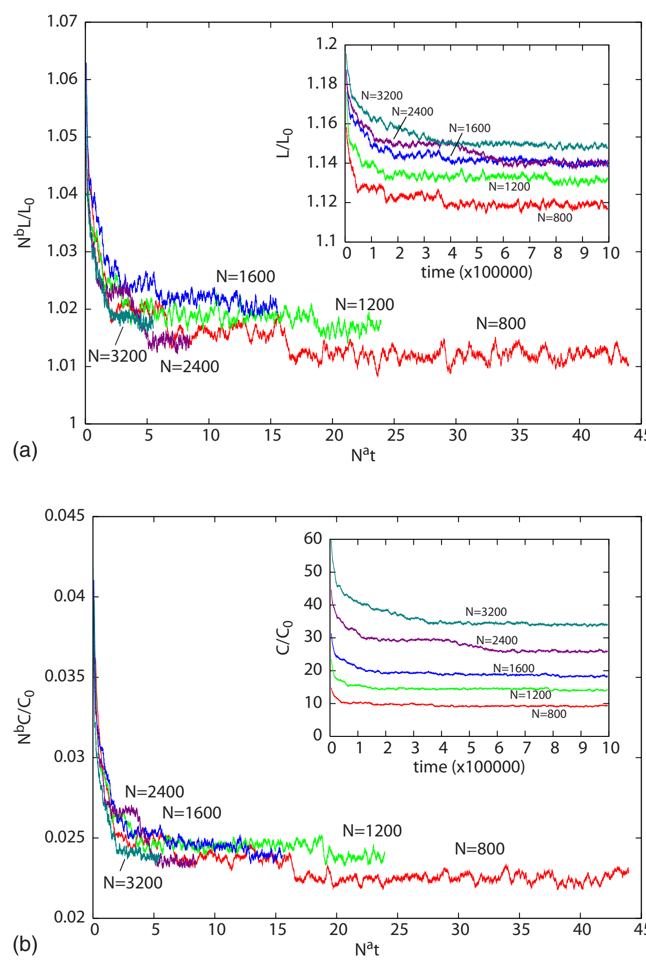

FIG. 5. (Color online) Time evolution of normalized path length L/L_0 and clustering coefficient C/C_0 for several network sizes N, assuming the scaling form $N^b x/x_0 = f(N^a t)$ $(x=L,C)$. $a=-1.5$, $b=-0.015$ for L, and $a=-1.5$, $b=-0.9$ for C. $G/N=0.05$, $\bar{k}=4$, $\phi=0.7$, $\psi=0.15$. Inset is the same plot with $a=b=0$ for both L and C.

($\psi=0$). This effect increases C. On the other hand, some amount of heterophily in the model seems to make bridges among such communities, which decreases L and increases S. Thus, we expect that the network retains the small-world property dynamically for these reasons.

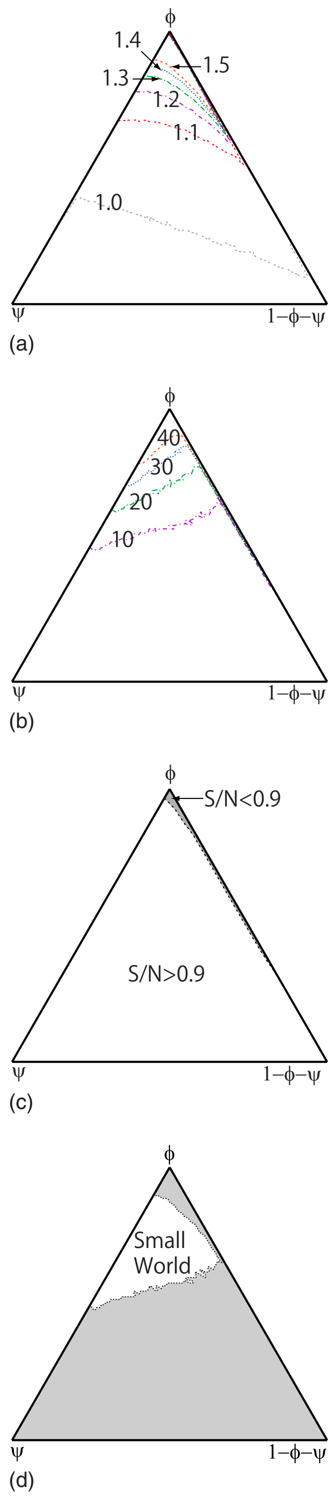

FIG. 6. (Color online) Ternary plots of (a) L/L_0, (b) C/C_0, (c) S/N, and (d) small-world regime where $L/L_0<1.5$, $C/C_0>10$, $S/N>0.9$. $N=1600$, $\bar{k}=4$, $G=0.05N$, $t=5\times10^5$. Dashed lines with labels are contours.

V. CONCLUSION

We have investigated a simple coevolutionary network model incorporating homophily and heterophily. If there is no heterophily ($\psi=0$), corresponding to the Holme and Newman model, this model exhibits a nonequilibrium phase transition. We have derived the phase diagram and clarified that the asymptotic behavior of this model abruptly changes from a voterlike stochastic process to a simple relaxation process in terms of the pairwise approximation. We have found that the behavior of this model with heterophily ($\psi>0$) is qualitatively different from the case without heterophily ($\psi=0$). For $\psi>0$, the network does not converge into one opinion. Furthermore, we have also found that this model exhibits a small-world property within a certain parameter regime in the case $G\propto N$.

The pairwise approximation cannot predict the appearance of the small-world property for $\psi>0$, because this approach deals only with the population of nodes and links, not topological properties. The derivation of L and C is not straightforward for the present model. Although we may be able to estimate the number of loops that consist of three nodes to give an estimate of C, these equations are too complicated for analytical studies. Further study of the analysis of topological changes for $\psi>0$ will be the subject of future work.

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APPENDIX A: CALCULATION INCORPORATING HIGHER-ORDER STRUCTURES

The time evolution equations of the number of triplets can be described as

$$\frac{d[AAA]}{dt} = (P_{B\to A}[AAB] + 2P_{B\to A}[ABA] - P_{A\to B}[AAAB_{\mathrm{L}}] + P_{B\to A}[AABA_{\mathrm{L}}] - P_{A\to B}[A,AAB_{\mathrm{T}}] + 3P_{B\to A}[B,AAA_{\mathrm{T}}])(1-\phi) + \left(P_{A\to B}[AAB] + P_{A\to B}\frac{2[AA][AB]}{[A]}\right)\phi, \tag{A1}$$

$$\frac{d[ABA]}{dt} = \{-2(P_{B\to A}+P_{A\to B})[ABA] - P_{A\to B}[ABAB_{\mathrm{L}}] + 2P_{B\to A}[ABBA_{\mathrm{L}}] + P_{A\to B}[A,AAB_{\mathrm{T}}] - 3P_{B\to A}[B,AAA_{\mathrm{T}}]\}(1-\phi) - 2(P_{B\to A}+P_{A\to B})\times[ABA]\phi, \tag{A2}$$

$$\frac{d[AAB]}{dt} = \{-(P_{B\to A}+P_{A\to B})[AAB]+P_{B\to A}[ABB] + 2P_{B\to A}[BAB]+P_{A\to B}[AAAB_{\mathrm{L}}]+P_{B\to A}[ABAB_{\mathrm{L}}] - 2P_{A\to B}[BAAB_{\mathrm{L}}]-P_{B\to A}[AABA_{\mathrm{L}}] + 2P_{B\to A}[B,AAB_{\mathrm{T}}]-2P_{A\to B}[A,ABB_{\mathrm{T}}]\}(1-\phi) + \left(2P_{A\to B}[BAB]+P_{A\to B}\frac{[AB]^2}{[A]}-(P_{B\to A}+P_{A\to B}) \times[AAB]\right)\phi, \tag{A3}$$

where $[WXYZ_{\mathrm{L}}]$ is the number of in-line quadruplets whose nodes are in the order W-X-Y-Z, and $[W,XYZ_{\mathrm{T}}]$ the number of three-pronged quadruplets centered at W. Equations for $[BBB],[BAB]$, and $[ABB]$ are similarly derived as above. We assume here that the network is a random graph so that we take account of only trees, because a random graph includes few loops. We approximate the number of quadruplets using the numbers of triplets, links, and nodes; thus these equations are closed. These equations have a fixed point at

$$u=\frac{1}{2}\frac{\frac{4K^2(1-\phi)^2}{N^2}+\frac{4K(1-\phi)}{N}+1}{\frac{4K^2(1-\phi)^2}{N^2}+\frac{2K(1-\phi)}{N}-1}. \tag{A4}$$

This fixed point disappears for $\bar{k}\le\frac{\sqrt{3}}{1-\phi}$ within $0\le u\le 1$.

APPENDIX B: BOUNDARY OF PHASE FOR GENERAL G

For $G>2$ and $\psi=0$, using the same approach as in the main text, we can also derive time evolution equations for the numbers of various types of connected pairs and the number of opinion holders as

$$\frac{d[XX]}{dt}=\sum_{Z\neq X}[(P_{Z\to X}[XZ]+2P_{Z\to X}[XZX] - P_{X\to Z}[XXZ])(1-\phi)+P_{X\to Z}[XZ]\phi], \tag{B1}$$

$$\frac{d[XY]}{dt}=(-P_{X\to Y}([XY]+2[YXY]-[XYY])-P_{Y\to X}([XY]+2[XYX]-[XXY]) + \sum_{Z\neq X}\{(P_{Z\to X}+P_{Z\to Y})[XZY]-P_{X\to Z}[YXZ]-P_{Y\to Z}[XYZ]\})(1-\phi)-(P_{X\to Y}+P_{Y\to X})[XY]\phi, \tag{B2}$$

$$\frac{d[X]}{dt}=\sum_{Z\neq X}\left(\frac{[XZ]}{\sum_{W\neq Z}[ZW]+2[ZZ]}[Z] - \frac{[XZ]}{\sum_{W\neq X}[XW]+2[XX]}[X]\right)(1-\phi), \tag{B3}$$

where X,Y,Z, and W take one of the G opinions and $X\neq Y$. If we approximate triplets as product of independent pairs such as $[XYZ]=[XY][YZ]/[Y]$, we obtain a closed set of equations. Although Eqs. (B1)–(B3) are too complex to use in deriving fixed points generally, one can still find that these equations have a common fixed point at $([XX],[XY],[X])=(\mu^*,\nu^*,N/G)$, which means that each opinion is equilibrated with one another, where

$$\mu^*=\frac{2K(1-\phi)+(G-1)N}{2G^2(1-\phi)}, \tag{B4}$$

$$\nu^*=\frac{2K(1-\phi)-N}{G^2(1-\phi)}. \tag{B5}$$

Here we used $N=\Sigma_X[X]$ and $K=\Sigma_X[XX]+\Sigma_X\Sigma_{X\neq Y}[XY]$. This fixed point disappears when $\bar{k}<\frac{1}{1-\phi}$, corresponding to $\nu^*<0$ and $\mu^*>K/G$. Equations (B1) also have fixed points in the subspace with $[XY]=0$ for every $X\neq Y$.

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