Estimating parameters by autosynchronization with dynamics restrictions

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We suggest a general approach to parameter estimation using autosynchronization with some restrictions on system dynamics. This parameter identification method can be extended to estimate parameters from a scalar time series. Furthermore, we propose an average filter method to suppress the influence of noise on parameter estimation. Some limits and extensions of the autosynchronization method are given as well. Several examples are presented to illustrate all methods suggested.

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I. INTRODUCTION

Synchronization [1] as a universal concept in nonlinear sciences has attracted much attention during the past years. The *synthesis* problem has motivated an approach of using synchronization as a parameter estimation method, called *autosynchronization* [2]. Recently, this autosynchronization synthesis issue has regained considerable interest within the nonlinear science research community (cf. Refs. [3–10]). Practical methods [3–10] have previously been suggested to augment the dynamical equations for a pair of synchronously coupled systems with parameter adaptation equations in particular cases, but no general method exists thus far. In this paper, we shall propose a general approach to autosynchronization-based parameter estimation and analyze the convergence of parameter estimation by a general *LaSalle's principle* (see the Appendix) which is less conservative than the typical Lyapunov's direct method [11].

On the other hand, as reported in Ref. [12], the analysis of autosynchronization needs an improvement in theory to ensure parameter estimation. It should be noted that the same drawback in theoretical analysis exhibited also in many recent papers (see, for example, Refs. [8–10]). However, provided that some properties of periodical or chaotic dynamics are applied, we can in theory ensure parameter estimation, as illustrated in Ref. [12] for the Lorenz system. In this paper, we shall extend this idea and show that when the linear dependence of system dynamics is assumed, we can estimate parameters of general systems using autosynchronization.

Furthermore, it is not only of theoretical interest but also of practical value to estimate parameters from only a scalar time series because of information acquisition restriction (or cost). Thus far this autosynchronization synthesis issue is not well understood, however. In this paper we also show that under some restrictions on system dynamics, we can estimate parameters from a scalar time series. Noise usually deteriorates the performance of parameter estimation and leads to fluctuation of parameter estimation around their true values. It is important in practice to suppress the amplitude of fluctuation. However, less attention has not been given to this point. To recover the performance of parameter estimation, here we shall propose an average filter method. Finally, we shall discuss some limits and extensions of the autosynchronization method suggested.

II. GENERAL THEORY

We consider general "real" systems given by

$$\dot{x}_i = c_i(\mathbf{x}) + \sum_{j=1}^{m} p_j f_{ij}(\mathbf{x}), \tag{1}$$

where $i=1,2,\ldots,n$; $\mathbf{x}=(x_1,x_2,\ldots,x_n)^T \in \mathbb{R}^n$ is the state vector; $\mathbf{p}=(p_1,p_2,\ldots,p_m)^T \subset U \in \mathbb{R}^m$ are unknown parameters to be estimated and U is a bounded set. Assume $\mathbf{s}=\mathbf{g}(\mathbf{x})$ is Ω dimensional vector denoting the experimental output of the system.

We attempt to construct a "computational model," given by

$$\dot{y}_i = c_i(\mathbf{y}) + \sum_{j=1}^{m} q_j f_{ij}(\mathbf{y}) + u_i(\mathbf{y},\mathbf{s}),$$

$$\dot{q}_j = N_j(\mathbf{y},\mathbf{s}) \tag{2}$$

to synchronize *identically* with the "real" system (1) from time series $\mathbf{s}$ by finding some appropriate control signals u_i as well as *parameter update* rules N_j. Here $\mathbf{y}=(y_1,y_2,\ldots,y_n)$ is the state vector, $\mathbf{q}=(q_1,\ldots,q_n)$ is an estimate of $\mathbf{p}$, and $u_i\equiv 0$ for all i (e.g., u_i has the form $u_i=-k_i[\mathbf{g}(\mathbf{y})-\mathbf{g}(\mathbf{x})]$ with gain k_i) and $N_j\equiv 0$ for all j when $\mathbf{y}=\mathbf{x}$.

Let $\mathbf{e}=\mathbf{y}-\mathbf{x}$ and $\mathbf{r}=\mathbf{q}-\mathbf{p}$. Then the error equation reads

$$\dot{e}_i = c_i(\mathbf{y}) - c_i(\mathbf{x}) + \sum_{j=1}^{m} p_j[f_{ij}(\mathbf{y}) - f_{ij}(\mathbf{x})] + \sum_{j=1}^{m} r_j f_{ij}(\mathbf{y}) + u_i,$$

$$\dot{r}_j = N_j. \tag{3}$$

When parameter estimation is ensured (i.e., $\mathbf{r}=\mathbf{0}$), the error system (3) becomes

$$\dot{e}_i = c_i(\mathbf{y}) - c_i(\mathbf{x}) + \sum_{j=1}^{m} p_j[f_{ij}(\mathbf{y}) - f_{ij}(\mathbf{x})] + u_i. \quad (4)$$

Therefore, we have to assume that we can design the control signals u_i such that the asymptotic stability of error system (4) is ensured; more precisely, there exists a Lyapunov function $V_o(\mathbf{e})$ decreasing monotonously along the trajectories of system (4), namely, $\dot{V}_o(\mathbf{e})|_{(4)}<0$ when $\mathbf{e}\neq 0$ and $\dot{V}_o(\mathbf{e})|_{(4)}=0$ when $\mathbf{e}=0$. Otherwise, errors of system (4) will destroy parameter estimation.

Even when system (4) is asymptotically stable, the asymptotic stability of system (3) in general cannot be ensured and we have to choose functions N_j carefully to eliminate the errors caused by the differences between estimated parameters and their true values. Differentiating the Lyapunov function $V_o(\mathbf{e})$ with respective to the error trajectory (3) results in

$$\dot{V}_o(\mathbf{e})|_{(3)} = \dot{V}_o(\mathbf{e})|_{(4)} + \sum_j r_j \sum_i \left(\frac{\partial V_o}{\partial e_i}\right) f_{ij}(\mathbf{y}), \quad (5)$$

where the second item of the right-hand side comes from the difference between systems (3) and (4).

For error system (3), we choose a Lyapunov function

$$V(\mathbf{e},\mathbf{r}) = V_o(\mathbf{e}) + \sum_j r_j^2/(2\delta_j), \quad (6)$$

where constants δ_j are positive for all j.

Then differentiating $V(\mathbf{e},\mathbf{r})$ with respective to the error trajectory (3) yields

$$\begin{aligned}\dot{V}(\mathbf{e},\mathbf{r})|_{(3)} &= \dot{V}_o(\mathbf{e})|_{(3)} \sum_j r_j N_j/\delta_j \\ &= \dot{V}_o(\mathbf{e})|_{(4)} + \sum_j r_j \left[\frac{N_j}{\delta_j} + \sum_i \left(\frac{\partial V_o}{\partial e_i}\right) f_{ij}(\mathbf{y})\right],\end{aligned} \quad (7)$$

where the last step has used Eq. (5).

It follows that we can design N_j as

$$N_j = -\delta_j \sum_i \left(\frac{\partial V_o}{\partial e_i}\right) f_{ij}(\mathbf{y}) \quad (8)$$

such that $\dot{V}(\mathbf{e},\mathbf{r})|_{(3)} = \dot{V}_o(\mathbf{e})|_{(4)}$, which implies from Theorem 2 (see the Appendix) that $\dot{V}_o(\mathbf{e})|_{(4)}=0$ and thereby $\mathbf{e}=\mathbf{0}$ as $t\to\infty$, where positive constants δ_j determine the updating rate.

Substituting $\mathbf{e}\to\mathbf{0}$ (i.e., $\mathbf{y}\to\mathbf{x}$) into Eq. (3) and noting $u_i\to 0$ when $\mathbf{e}\to\mathbf{0}$, we obtain that as $t\to\infty$, the remaining equation of Eq. (3) actually reads

$$\dot{e}_i = \sum_{j=1}^{m} r_j f_{ij}(\mathbf{x}),$$

which implies that

$$\sum_{j=1}^{m} r_j f_{ij}(\mathbf{x}) \to 0 \text{ (otherwise } \mathbf{e}\to\mathbf{0} \text{ cannot be satisfied)}.$$

It follows $\mathbf{r}\to\mathbf{0}$ provided that the *linear independence* of functions $f_{i,j}(\mathbf{x})$ is assumed.

To summarize the above analysis, the following theorem is thus proved.

Theorem 1. Assume that (i) the control signals u_i are designed such that there exists a Lyapunov function $V_o(\mathbf{e})$ decreasing monotonously along the trajectories of system (4), and (ii) functions N_j in model (2) are designed as Eq. (8). Then $\mathbf{y}(t)\to\mathbf{x}(t)$ and $\sum_{j=1}^m r_j f_{ij}(\mathbf{x})\to 0$ for all i are globally satisfied as $t\to\infty$ for systems (1) and (2) starting from arbitrary initial conditions. Furthermore, $\mathbf{r}\to\mathbf{0}$ if the system dynamics is restricted such that the linear independence of functions $f_{i,j}(\mathbf{x})$ is satisfied for all i.

We now show how to design control signals and parameter update rules in terms of Theorem 1 for the case when all states are measurable. Let $F_i(\mathbf{y},\mathbf{p})=c_i(\mathbf{y})+\sum_{j=1}^m p_j f_{ij}(\mathbf{y})$. We assume that function $F_i(\mathbf{x},\mathbf{p})$ is uniformly Lipschitzian for all i, that is, there exists a constant $\beta>0$ satisfying

$$|F_i(\mathbf{y},\mathbf{p}) - F_i(\mathbf{x},\mathbf{p})| \leq \beta \max_j |y_j - x_j|.$$

One can simply design the control signals as

$$u_i = -k(y_i - x_i),$$

where k is the uniform gain. In this case, the error system (4) actually reads

$$\dot{e}_i = F_i(\mathbf{y},\mathbf{p}) - F_i(\mathbf{x},\mathbf{p}) - ke_i. \quad (9)$$

Choosing as the usual form the Lyapunov function

$$V_o = \sum_i (e_i)^2/2,$$

we obtain

$$\dot{V}_o = \sum_i e_i[F_i(\mathbf{y},\mathbf{p}) - F_i(\mathbf{x},\mathbf{p})] - ke_i^2 \leq (n\beta - k)\sum_i e_i^2.$$

It follows that when $k>n\beta$, the assumption (i) in Theorem 1 is satisfied. Due to $V_o=\sum_i(e_i)^2/2$, we can design $N_j=-\delta_j\sum_i e_i f_{ij}(\mathbf{y})$ for all j. We can conclude from Theorem 1 that $\mathbf{y}\to\mathbf{x}$ and $\sum_{j=1}^m r_j f_{ij}(\mathbf{x})\to 0$ for all i are satisfied. If we assume that *linear independence* of functions $f_{ij}(\mathbf{x})$ is satisfied for all i, then $\mathbf{r}\to\mathbf{0}$ is satisfied.

III. EXAMPLES

To illustrate the suggested parameter estimation method, we first consider the Lorenz system

$$\begin{aligned}\dot{x}_1 &= p_1(x_2 - x_1), \\ \dot{x}_2 &= p_2 x_1 - p_3 x_2 - x_1 x_3 + p_4, \\ \dot{x}_3 &= x_1 x_2 - p_5 x_3,\end{aligned} \quad (10)$$

where parameters p_i are unknown and all states are measurable. As a "computational model" we consider the following equation:

$$\dot{y}_1 = q_1(y_2 - y_1) - k(y_1 - x_1),$$

$$\dot{y}_2 = q_2 y_1 - q_3 y_2 - y_1 y_3 + q_4 - k(y_2 - x_2),$$

$$\dot{y}_3 = y_1 y_2 - q_5 y_3 - k(y_3 - x_3),$$

$$\dot{q}_1 = -\delta_1 (x_2 - x_1)(y_1 - x_1),$$

$$\dot{q}_2 = -\delta_2 x_1 (y_2 - x_2),$$

$$\dot{q}_3 = \delta_3 y_2 (y_2 - x_2),$$

$$\dot{q}_4 = -\delta_4 (y_2 - x_2),$$

$$\dot{q}_5 = \delta_5 y_3 (y_3 - x_3). \tag{11}$$

As analyzed above, when large enough k is chosen, $\mathbf{y} \to \mathbf{x}$ and

$$r_1(x_2 - x_1) \to 0, \tag{12}$$

$$r_2 x_1 - r_3 x_2 + r_4 \to 0, \tag{13}$$

$$-r_5 x_3 \to 0 \tag{14}$$

are satisfied as $t \to \infty$.

It is easy to see from Eq. (12) that when $(x_2 - x_1)$ is not (asymptotically) zero, the linear dependence of $(x_2 - x_1)$ is satisfied and thereby $r_1 \to 0$. Similarly, we can conclude from Eq. (14) that when x_3 is not (asymptotically) zero, $r_5 \to 0$ is ensured. Therefore, if we restrict the dynamics of the Lorenz system such that both $(x_2 - x_1)$ and x_3 are not (asymptotically) zero, we can then identify parameters p_1 and p_3 correctly. For example, parameters p_1 and p_3 can be estimated correctly for chaotic or periodical Lorenz system.

Using $\dot{x}_1 = p_1(x_2 - x_1)$ [cf. Eq. (10)] in Eq. (13) leads to

$$(r_3/p_1)\dot{x}_1 + (r_3 - r_2)x_1 - r_4 \to 0 \tag{15}$$

when $p_1 \neq 0$ (if $p_1 = 0$, the Lorenz system is in stationary state and the linear dependence of functions $f_{ij}(\mathbf{x})$ cannot be satisfied. In this case we cannot identify parameters p_2, p_3, and p_4 correctly.).

We assume that $r_3 = 0$ is not asymptotically satisfied and thus Eq. (15) asymptotically becomes a first-order ordinary differential equation (ODE) whose evolution is exponentially convergent or divergent. If we restrict the dynamics of the Lorenz system such that the Lorenz system is not in stationary state and the evolution of x_1 is neither exponentially convergent nor exponentially divergent, then $r_3 = 0$ is asymptotically achieved. Substituting $r_3 = 0$ into Eq. (15) leads to $r_2 x_1 + r_4 \to 0$. Similarly, when x_1 is linearly dependent, $r_2 = r_4 = 0$ can be ensured as time approaches to infinity. In other words, parameters p_2, p_3, and p_4 can be identified when we restrict the dynamics of the Lorenz system such that the Lorenz system is not stationary and the evolution of x_1 is neither exponentially convergent nor exponentially divergent. For example, for chaotic or periodical Lorenz systems, x_1 is neither exponentially convergent nor exponentially divergent and thereby we can estimate parameters p_2, p_3, and p_4 correctly.

To summarize the above analysis, when the dynamics of Lorenz system is restricted such that functions $f_{ij}(\mathbf{x})$ in the left-hand sides of Eqs. (12)–(14) are linearly dependent, all parameters p_i can be estimated using the "computational model" (11) with large enough k. Figures 1(b)–1(f) show the parameter estimation for chaotic Lorenz system, where $k = 10$ and $\delta_i = 1$ for all i (the convergence of parameter identification is improved when larger δ_i are chosen).

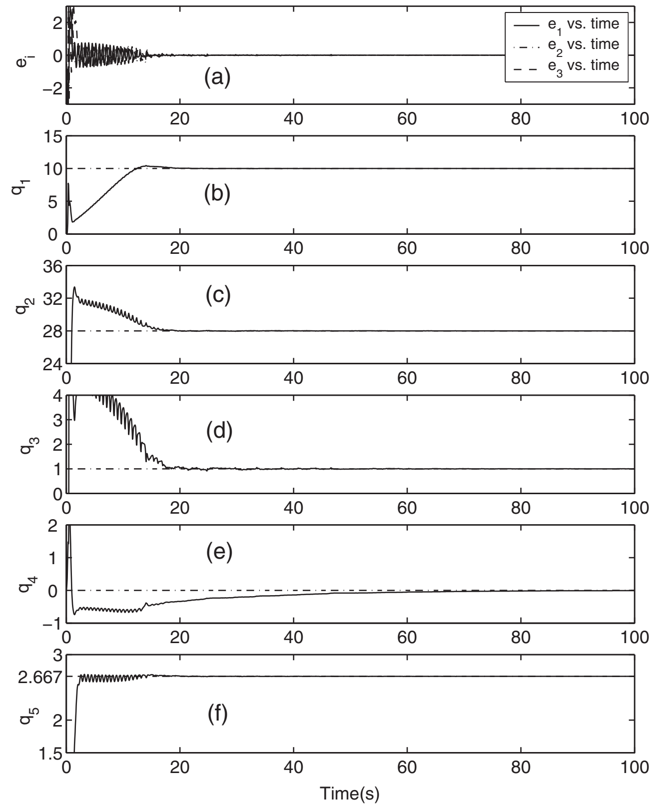

FIG. 1. Parameter estimation of chaotic Lorenz system with $p_1 = 10$, $p_2 = 28$, $p_3 = 1$, $p_4 = 0$, and $p_5 = 2.667$. (a) State synchronization errors e_i versus time. (b)–(f) q_i versus time.

We now treat parameter estimation when only partial states are measurable and we consider the following Rösseler system as an illustrating example:

$$\dot{x}_1 = -x_2 - x_3,$$

$$\dot{x}_2 = x_1 + p_1 x_2,$$

$$\dot{x}_3 = p_2 - p_3 x_3 + x_1 x_3, \tag{16}$$

where p_i are unknown parameters and $\mathbf{s} = (x_2, x_3)^T$ is assumed. As a computational model we consider the following equation:

$$\dot{y}_1 = -y_2 - y_3 + u_1,$$

$$\dot{y}_2 = y_1 + q_1 y_2 + u_2,$$

$$\dot{y}_3 = q_2 - q_3 y_3 + y_1 y_3 + u_3,$$

$$\dot{q}_1 = -\delta_1 y_2 (y_2 - x_2),$$

$$\dot{q}_2 = -\delta_3 (y_3 - x_3),$$

$$\dot{q}_3 = \delta_3 y_3 (y_3 - x_3), \tag{17}$$

where $u_1=(1-y_3)(y_3-x_3)$, $u_2=-k_1(y_2-x_2)$, and $u_3=-k_2(y_3-x_3)$.

Then the error system reads

$$\dot{e}_1 = -e_2 - y_3 e_3,$$

$$\dot{e}_2 = e_1 + r_1 y_2 - (k_1 - p_1) e_2,$$

$$\dot{e}_3 = y_3 e_1 - (k_2 - x_1 + p_3) e_3 + r_2 - r_3 y_3,$$

$$\dot{r}_1 = -\delta_1 y_2 e_2,$$

$$\dot{r}_2 = -\delta_2 e_3,$$

$$\dot{r}_3 = \delta_3 y_3 e_3, \tag{18}$$

where the derivative of the third equation used the fact $y_1 y_3 - x_1 x_3 = y_3 e_1 + x_1 e_3$.

Choosing a Lyapunov function

$$V = \sum_i e_i^2/2 + \sum_i r_i^2/(2\delta_i),$$

we obtain

$$\dot{V} = -(k_1 - p_1)e_2^2 - (k_2 - x_1 + p_3)e_3^2.$$

This indicates from Theorem 2 (see the Appendix) that when large enough k_i are chosen, we obtain

$$e_2 \to 0, \quad e_3 \to 0, \tag{19}$$

which implies

$$\dot{r}_i \to 0, \forall\, i.$$

Substituting Eq. (19) into Eq. (18) results in that as $t \to \infty$,

$$e_1 + r_1 x_2 \to 0,$$

$$y_3 e_1 + r_2 - r_3 x_3 \to 0. \tag{20}$$

It follows from the first equation of Eq. (20) that $x_2 \to -e_1/r_1$ when $r_1 \neq 0$. To estimate parameter p_1 correctly, we have to restrict the dynamics of the Rössler system such that $x_2 \to -e_1/r_1$ cannot be satisfied. For example, for chaotic or periodical state x_2, $x_2 \to -e_1/r_1$ is not ensured due to $\dot{e}_1 \to 0$ and $\dot{r}_i \to 0$ for all i. In this case, $r_1 \to 0$ and $e_1 \to 0$ as $t \to \infty$.

Substituting $e_1 \to 0$ to the second equation of Eq. (20) yields

$$r_2 - r_3 x_3 \to 0.$$

If we restrict the dynamics of the Rössler system such that x_3 is linear dependent, then we similarly obtain $r_2 \to 0$ and $r_3 \to 0$, as illustrated in the above Lorenz example.

Figure 2 summarizes our results. It is easy to see from Fig. 2(a) that all state synchronization errors approach to zero asymptotically. Figure 2(b) shows that all parameters p_i can be estimated correctly.

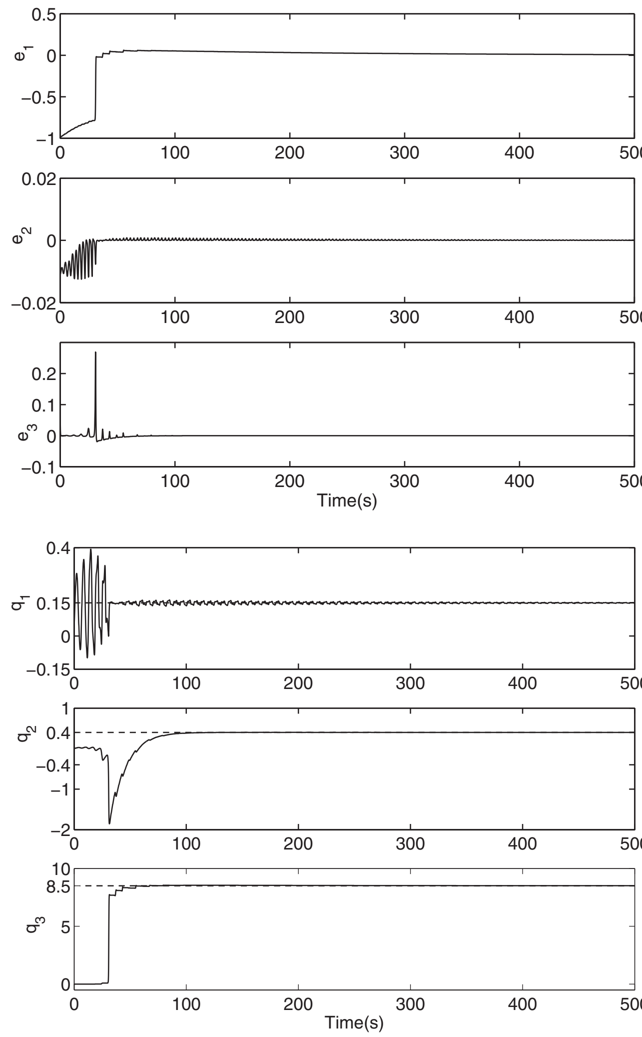

FIG. 2. Parameter estimation of chaotic Rössler system with $p_1=0.15$, $p_2=0.4$, and $p_3=8.5$. (a) State synchronization errors e_i versus time. (b) q_i versus time.

It should be noted that under proper restrictions on system dynamics, we can in theory ensure parameter estimation even when the convergence of only partial state errors can be concluded from the general LaSalle's principle (see Theorem 2 in the Appendix). Furthermore, it is possible to ensure parameter estimation when only partial states of systems are measurable, as illustrated in Fig. 2(b). Therefore, it is nontrivial to apply proper restrictions on system dynamics for convergence analysis of parameter estimation.

IV. PARAMETER ESTIMATION IN THE PRESENCE OF NOISE

The influence of noise on synchronization is an important aspect especially for applications. Numerically "high-quality" synchronization can be achieved by the approach proposed. Then noise is tolerated at a level that does not take the system out of the basin of attraction of the synchronization manifold. However the estimated values for unknown parameters will fluctuate around their true values when the experimental outputs are disturbed by noise. As an illustrat-

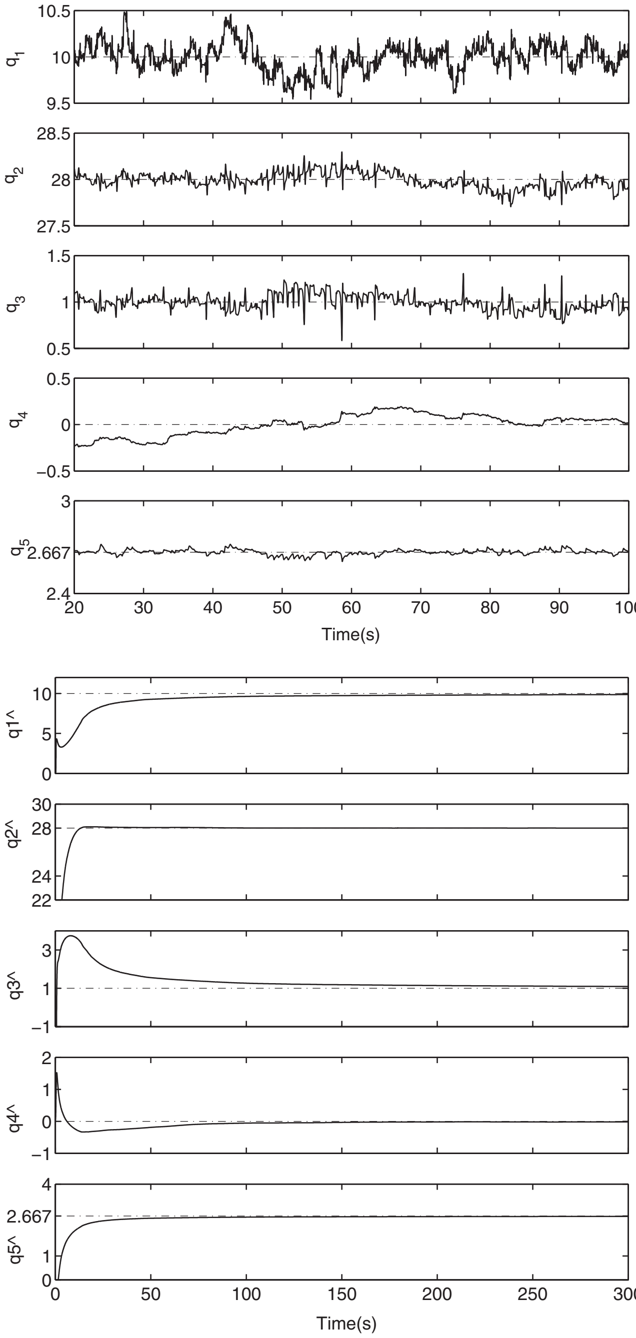

FIG. 3. Parameter estimation in the presence of measure noise. (a) Estimated parameters fluctuate around their true values $p_1=10$, $p_2=28$, $p_3=1$, $p_4=0$, and $p_5=2.667$ in the presence of noise. (b) Parameter estimation after average filtering.

ing example we revisit the Lorenz system (10) and its computational model (11) but assume the output x_1 is disturbed by uniformly distributed random noise with amplitude ranging from -1 to 1. Figure 3(a) shows that the estimated parameters q_i fluctuate around their true values. Furthermore the amplitude of fluctuation increases if that of noise increases. Hence the parameter estimation performance of developed autosynchronization methods [3–10] deteriorates dramatically in the presence of noise with large amplitude.

To suppress the estimation fluctuation caused by the noise, here we suggest an average filter,

$$\hat{q}_i(kT) = (1/k)\sum_{i=1}^{k} q_i(iT), \tag{21}$$

where T denotes the sampling time. It is clear to see from Fig. 3(b) that unknown parameters p_i can be estimated with high accuracy even in the presence of large random noise.

V. PARAMETER ESTIMATION FROM A SCALAR TIME SERIES

Theorem 1 ensures the global attraction of $\mathbf{y}=\mathbf{x}$, provided that each partial derivative $\partial V_o/\partial e_i$ is known. For the usual form $V_o \equiv \Sigma_i (e_i)^2/2$, the least requirement is that x_i be known if the equation for $\dot{x}_i$ contains parameters that one seeks to estimate. If only partial states of the system (1) are measurable, then only the parameters that involved in the evolution equations of these measurable states can be estimated globally (staring from arbitrary initial estimates). However it is possible to estimate model parameters from a scale time series if some restrictions on system dynamics are added such that system (1) can be transformed into the well-known Brunovsky's canonical form [13]. As an illustrating example, we consider a more general system, given by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{p}), \quad y = \phi(\mathbf{x}), \tag{22}$$

where $\mathbf{x} \in \mathbb{R}^N$ is the state vector, $\mathbf{f}=(f_1,\ldots,f_N)^T$ is a known vector-field function, and $y \in \mathbb{R}$ denotes a scalar experimental output time series. We assume that $\mathbf{f}$ and ϕ are sufficiently smooth such that the output is Nth order continuously differentiable.

Let

$$\mathbf{H}(\mathbf{x}) = (y,\dot{y},\ldots,y^{(N-1)})^T = (\phi(\mathbf{x}),L_{\mathbf{f}}\phi(\mathbf{x}),\ldots,L_{\mathbf{f}}^{N-1}\phi(\mathbf{x}))^T, \tag{23}$$

where $y^{(i)}$ is the ith order differential of y, and L denotes the Lie derivative operator defined by

$$L_{\mathbf{f}}^{j}\phi(\mathbf{x}) = \sum_{i=1}^{N} \frac{\partial(L_{\mathbf{f}}^{j-1}\phi)}{\partial x_i} f_i(\mathbf{x}).$$

We can conclude from the well-known implicit function theorem that if $\partial\mathbf{H}/\partial\mathbf{x}$ is nonsingular and continuous everywhere on a certain open set, the system (22) can be transformed into the well-known Brunovsky's canonical form [13]

$$\dot{z}_i = z_{i+1}, \quad 1 \leqslant i \leqslant n-1,$$

$$\dot{z}_n = g(\mathbf{c},z_1,\ldots,z_n), \tag{24}$$

where $z_1=y$ and $\mathbf{c}$ is a function of $\mathbf{p}$.

If we assume further that g is a linear function of the parameters $\mathbf{c}$ and is linear independent, then, in terms of Theorem 1, we can identify parameter vector $\mathbf{c}$ of system (24) and therefore can identify parameter vector $\mathbf{p}$ of system (22) from the scale time series y because $z_{i+1}=y^{(i)}$ can be observed from y using differential estimators. To illustrate this method, we analyze the Rössler system

$$\dot{x}_1 = -x_2 - x_3,$$

$$\dot{x}_2 = x_1 + ax_2,$$

$$\dot{x}_3 = b + x_3(x_1 - c),$$

$$y = x_2, \tag{25}$$

where parameters a, b, and c are unknown. Let $z_1=y$, $z_2=\dot{y}$, and $z_3=\ddot{y}$. Then system (25) can be transformed into

$$\dot{z}_1 = z_2,$$

$$\dot{z}_2 = z_3,$$

$$\dot{z}_3 = -z_2 + z_1 z_2 + z_2 z_3 + p_1 + p_2(z_1 + z_3) + p_3 z_2 + p_4 z_1 z_2 + p_5(z_1^2 + z_2^2 + z_1 z_3 - z_3), \tag{26}$$

where $p_1=-b$, $p_2=-c$, $p_3=ac$, $p_4=a^2$, and $p_5=-a$.

As a computational model we consider the following equation:

$$\dot{v}_1 = v_2 + u_1,$$

$$\dot{v}_2 = v_3 + u_2,$$

$$\begin{aligned}\dot{v}_3 &= -v_2 + v_1 v_2 + v_2 v_3 + q_1 + q_2(v_1 + v_3) \\ &\quad + q_3 v_2 + q_4 v_1 v_2 + q_5(v_1^2 + v_2^2 + v_1 v_3 - v_3) + u_3 \\ &\triangleq -v_2 + v_1 v_2 + v_2 v_3 + u_3 + \sum_{i=1}^{5} q_i \tau_i(\mathbf{v}),\end{aligned}$$

$$\dot{q}_i = N_i, \quad i = 1, \ldots, 5, \tag{27}$$

with

$$u_i = -k(v_i - \hat{y}^{(i-1)}), \tag{28}$$

$$N_i = -\delta_i \tau_i(\mathbf{v})(v_3 - \hat{y}^{(2)}), \tag{29}$$

where $\hat{y}^{(i-1)}$ denotes the estimation of $y^{(i-1)}$ by using differential estimation techniques.

It is easy to see that for chaotic or periodical Rösslers, all p_i of system (26) can be estimated correctly when large enough k is chosen. It follows that the parameters $a(=-p_5)$, $b=(-p_1)$, and $c(=-p_2)$ can then be estimated, cf. Fig. 4.

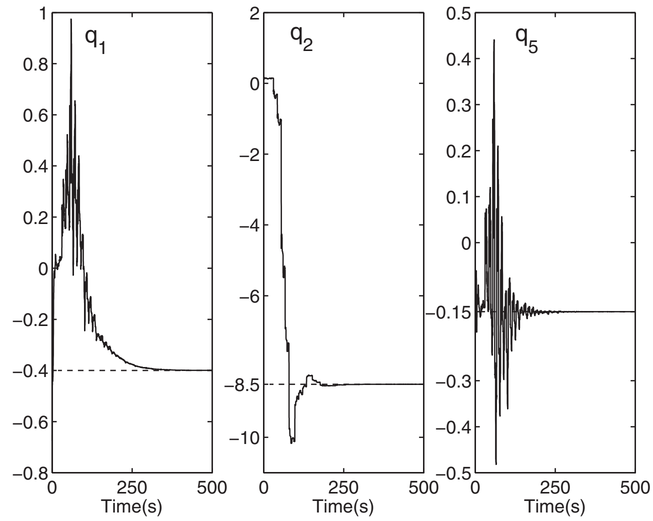

FIG. 4. Parameter estimation from a scalar time series. The computational model can identify parameters $a=0.15$, $b=0.4$, and $c=8.5$ of Rössler system due to $a=-p_5$, $b=-p_1$, and $c=-p_2$.

VI. DISCUSSIONS AND EXTENSIONS

The main assumption of the autosynchronization method, that one can know the system structure, in practice is really a restriction. We have validated that the parameter estimation performance will deteriorate dramatically if we cannot know the system structure accurately. However, this drawback can be relaxed by using our recently developed system structure identification method [14]. Detailed analysis will be reported in a later paper.

Another restriction is associated with the assumption that one can design the control signals u_i such that system (4) is globally and/or locally asymptotically stable, which in practice cannot always be satisfied using the general LaSalle's principle. However, this restriction can be relaxed a little by the use of the conditional Lyapunov exponents [15] (or transverse Lyapunov exponents [16]) as the least conservative criterion. Local synchronization is ensured when all conditional Lyapunov exponents are less than zero.

Furthermore, when $\mathbf{f}$ in system (22) is a nonlinear function of the parameters $\mathbf{p}$, it becomes very difficult to ensure parameter estimation globally. However, it is still possible to design proper parameter update rules using local linearization around the true values for ensuring local parameter estimation.

VII. CONCLUSIONS

We systematically investigated autosynchronization as an approach to parameter estimation and gave the rules to design control signals and parameter update rules for general systems. We showed that autosynchronization as an effective method can be applied to parameter identification if proper restrictions on system dynamics are added. We also argue that under some restrictions on the system dynamics, we can estimate parameters from a scalar time series. Furthermore, we suggested an average filter method to suppress the influence of noise on parameter estimation. Some limits and extensions of the autosynchronization method suggested are discussed as well. It should be stressed that many interesting problems can be transformed into an autosynchronization synthesis issue. Our recent work [17], for example, has shown that network connectivity can be identified using autosynchronization methods. Therefore, our results are nontrivial and provide a guidance for parameter identification which is crucial for various engineering applications.

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APPENDIX: GENERAL LASALLE'S PRINCIPLE

Lyapunov's direct method (cf. Ref. [11]) is widely used for synchronization synthesis in the literature and gives analytical criteria for local or global synchronization by constructing a Lyapunov function decreasing monotonically along synchronization error system trajectories (that is, the Lyapunov function has strictly negative definite differential with respective to the synchronization error system trajectories). However, for the autosynchronization synthesis problem, the time differential of the Lypapunov function does not include parameter estimation errors and thereby is only negative semidefinite rather than being negative definite [12]. In this case, the classical Lyapunov's direct method cannot be applied and instead we can use a general *LaSalle's principle* [18] admitting that Lyapunov functions have only negative semidefinite different for general time-varying systems. This general LaSalle's principle [18] can be described as follows:

Theorem 2. Consider general nonlinear time-varying systems, given by

$$\dot{\mathbf{x}} = \mathcal{F}(\mathbf{x},t), \tag{A1}$$

where $\mathcal{F}$ is locally Lipschitz continuous in $\mathbf{x}$, uniformly in t, in a ball of radius r defined by $B_r = :\{\mathbf{x} : \|\mathbf{x}\| \leq r\}$. Assume that for $\mathbf{x} \in B_r$, there exists a function $v(\mathbf{x},t)$ such that (i) for functions α_1 and α_2 of class $\mathcal{K}$,

$$\alpha_1(\|\mathbf{x}\|) \leq v(\mathbf{x},t) \leq \alpha_2(\|\mathbf{x}\|),$$

and (ii) for some nonnegative function $w(\mathbf{x})$,

$$\dot{v}(\mathbf{x},t) = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial \mathbf{x}}\mathcal{F}(\mathbf{x},t) \leq -w(\mathbf{x}) \leq 0.$$

Then for all $\|\mathbf{x}(t_0)\| \leq \alpha_2^{-1}(\alpha_1(r))$, the trajectories $\mathbf{x}(\ )$ are bounded and

$$\lim_{t\to\infty} w(x(t)) = 0.$$

It should be noted that the classical LaSalle's invariance principle [19] is restricted in applications because it holds only for time-invariant and periodical systems and cannot directly be applied for chaos synchronization synthesis. However, the general LaSalle's principle (Theorem 2) can be applied to general time-varying systems (including chaotic systems).

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