# Intermittency of principal stress directions within Arctic sea ice

Jérôme Weiss
*Laboratoire de Glaciologie et Géophysique de l'Environnement, CNRS, 54 rue Molière, BP 96, 38402 St Martin d'Hères Cedex, France*
(Received 21 February 2008; published 27 May 2008)

The brittle deformation of Arctic sea ice is not only characterized by strong spatial heterogeneity as well as intermittency of stress and strain-rate amplitudes, but also by an intermittency of principal stress directions, with power law statistics of angular fluctuations, long-range correlations in time, and multifractal scaling. This intermittency is much more pronounced than that of wind directions, i.e., is not a direct inheritance of the turbulent forcing.

DOI: 10.1103/PhysRevE.77.056106 PACS number(s): 46.50.+a, 92.10.Rw, 05.45.Df, 92.60.Gn

Arctic sea ice is a very thin plate with an average thickness of few *m* that extends in winter over the entire Arctic basin and beyond, i.e., about $14\times10^6$ km$^2$. The sea ice cover fractures and deforms under the action of several driving forces, whose wind forcing is considered the most important [1]. Most of its deformation is accommodated by a multiscale fracturing and faulting process [2,3] leading to strong spatial heterogeneity and intermittency of strain rates [4,5] and stresses [6]. As an example, intermittency of internal stress amplitudes was revealed from multifractal analysis [6]. However, so far, only fluctuations of the *amplitude* of strain rate or stress invariants have been analyzed, whereas fluctuations of principal *directions* were ignored. In this paper, it is shown that fluctuations of principal stress directions within the sea ice cover are also intermittent, characterized by extreme fluctuations and long-range temporal correlations. This behavior is compared with that of wind direction fluctuations. Van Doorn *et al.* [7] analyzed the statistical properties of wind directions over time scales from hours to years and showed that associated fluctuations resemble Brownian noise when wind speed is high, but are intermittent when wind speed is low. The present analysis indicates that the intermittency of principal stress directions within the ice cover is much more pronounced, i.e., is not a direct and simple inheritance of wind direction intermittency.

We analyze time series of principal stress directions as well as wind directions recorded during the SHEBA field campaign in the Beaufort Sea from October 1997 to July 1998 [8]. Stress sensors were installed on nine sites a few km around the SHEBA camp. The results reported here correspond to the so-called "Baltimore" site [9]. Other sites give similar results. These sensors determine the two-dimensional (2D) stress tensor acting on a material point in the horizontal plane of the ice cover by measuring changes in radial deformation of a cylindrical steel annulus [10]. This allows us to estimate the maximum ($\sigma_1$) and minimum ($\sigma_2$) principal stresses with a resolution of 20 kPa, as well as the direction of $\sigma_1$ with an accuracy of 5° [8]. Each measure corresponds to an instantaneous measure, with a sampling frequency of 1/h. When the sensor is installed, the reference direction is oriented toward magnetic North, but this absolute reference is lost as the ice drifts and rotates, i.e., the fluctuations of direction analyzed here refer to fluctuations of principal stress direction within the material only, not to solid rotation of the ice plate. Note that the solid rotation of the SHEBA camp evolved rather regularly from October 1997 to reach 80° in July 1998 [11]. The second data set is a series of *in situ* wind directions with respect to true North, measured at a height of about 2 m above snow level [11], at about 2 km west of the Baltimore sensor (in Oct. 1997). Values are hourly averaged, and missing data are frequent (40% lacking).

Principal stress or wind directions are by nature circular variables, spanning 180° for principal stresses and 360° for wind. This leads to artificial, large, and sudden fluctuations when the direction changes from –90° to 90° (respectively –180° to 180°) or vice versa for stresses (respectively wind). To reconstruct corrected direction time series, we followed the procedure proposed by van Doorn *et al.* [7]. It is assumed that hourly directional increments were not too large, i.e., $|\Delta\theta|\leq 90°$ for stresses (180° for winds). Any larger increment is considered to result from the crossing of the –90°/90° (respectively –180°/180°) boundary, and is therefore corrected according to $\forall|\Delta\theta|>\theta_{\mathrm{th}}:\Delta\theta\rightarrow\Delta\theta-\frac{\Delta\theta}{|\Delta\theta|}2\theta_{\mathrm{th}}$, where $\theta_{\mathrm{th}}=90°$ for stresses (180° for wind). This assumption is consistent with the fact that large apparent $\Delta\theta$ are generally much larger than $\theta_{\mathrm{th}}$ and occur for $\theta$ close to $\pm\theta_{\mathrm{th}}$, which is particularly true for wind directions [7]. For principal stress directions, owing to the distribution of $|\Delta\theta|$ (see

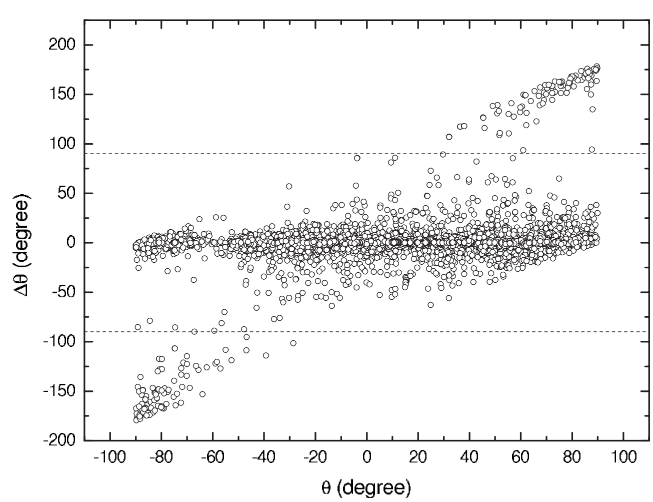


FIG. 1. Scatter plot of $\Delta\theta$ vs $\theta$ for the Baltimore sensor. Most of $|\Delta\theta|>90°$ values occur for $|\theta|$ close to 180°.

 

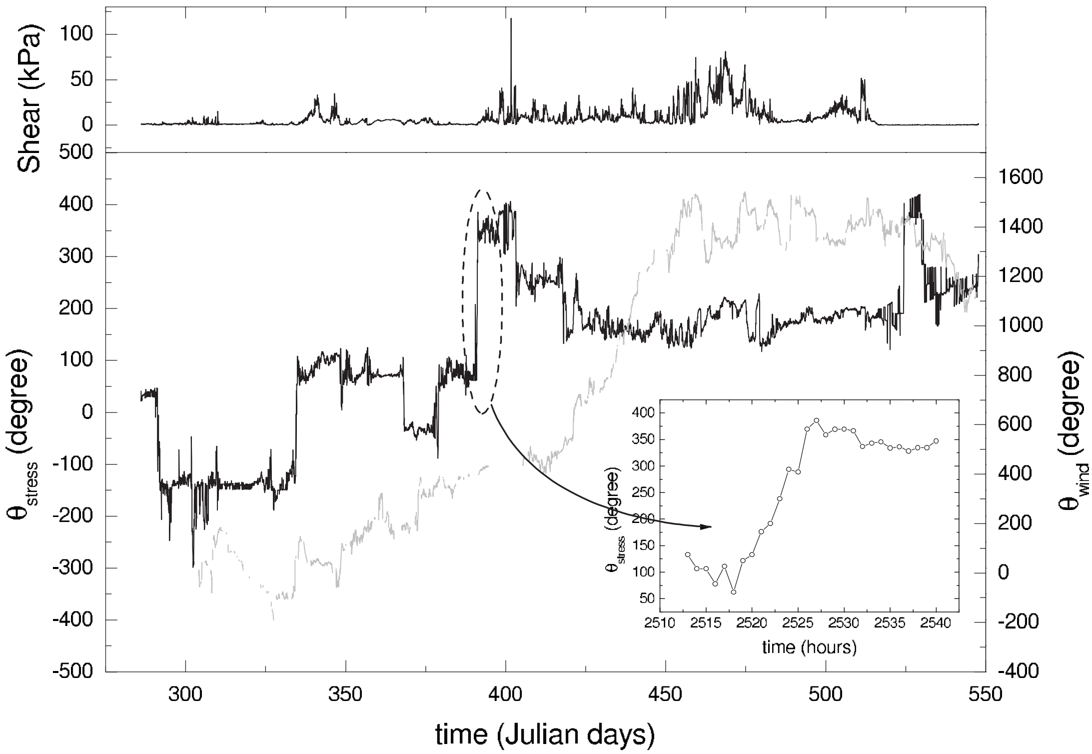


FIG. 2. Top panel: Evolution of the shear stress amplitude at the Baltimore sensor, from October 13, 1997 to June 1, 1998. Bottom panel: Reconstructed principal stress (black line) and 2 m wind (gray line) direction time series over the same period. See text for details about the reconstruction procedure. Inset: Zoom of the principal stress direction time series over a period of about 1 day showing a rotation of about 300° taking place in about 10 h.

below), some ambiguity remains for 16 data points over 6282, when $80^\circ \leq |\Delta\theta| \leq 100^\circ$ (Fig. 1).

The reconstructed principal stress and 2 m wind direction time series are shown in Fig. 2. The principal stress record shows angular fluctuations of various amplitudes, with rotations in one direction as large as 300° taking place over half a day, and suggesting intermittency. The 2 m wind record appears much less intermittent (Fig. 2). This is confirmed by a spectral analysis of the stress record that reveals a $1/f^{\mu}$ power law scaling of the power spectrum $E(f)$ with

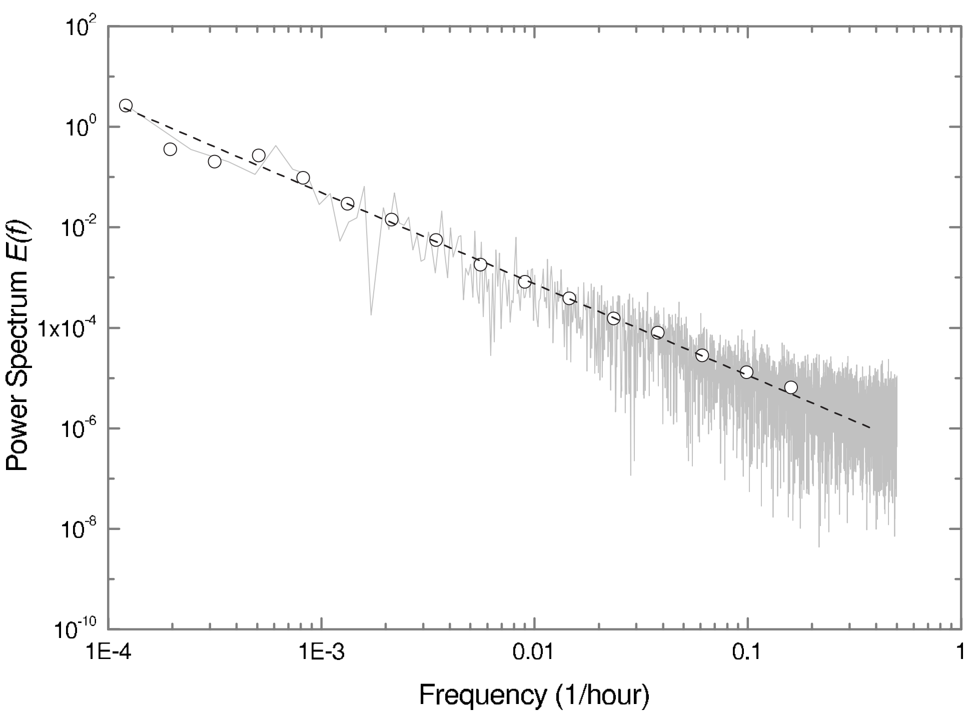


FIG. 3. Power spectrum of the reconstructed principal stress direction record at the Baltimore sensor, shown on Fig. 2, bottom panel. A power law fit (dashed line) is estimated by averaging the spectrum over frequency windows with algebraically increasing width (open circles), yielding $E(f) \sim f^{-\mu}$, with $\mu = 1.76 \pm 0.04$.

$\mu=1.76\pm0.04$, i.e., significantly below the Brownian noise exponent $\mu=2$ (Fig. 3). This value can be compared with $\mu=1.42$ previously obtained for the corresponding shear stress amplitude record [6]. A similar analysis performed on the 2 m wind record revealed an exponent $\mu=2.02\pm0.05$, i.e., wind direction fluctuations cannot be distinguished from Brownian uncorrelated noise on this basis, in agreement with previous work [7]. Therefore the intermittency of the principal stress direction is not a direct inheritance of wind forcing.

The non-Brownian character of principal stress direction fluctuations is also revealed by the distributions of $\Delta\theta$ that are clearly non-Gaussian [Fig. 4(a)]. These distributions are symmetric and centered around zero ($\langle\Delta\theta\rangle=0.03°$), meaning that there is no preferential long term rotation trend for principal stress directions within sea ice. Interestingly, these distributions are well described by a power law tail, $P(|\Delta\theta|)\sim|\Delta\theta|^{-\beta}$ with $\beta=1.6\pm0.1$ [Fig. 4(b)]. Note that these statistics might have been marginally perturbed by our reconstruction procedure that imposes $|\Delta\theta|\leq 90°$. However, an analysis of the distributions of raw angular fluctuations, before reconstruction, shows the robustness of this power law scaling: Compared to the distributions plotted in Fig. 4(b), the only difference is a bump above 150°, corresponding to the crossing of the $-90°/90°$ boundary. Wind direction fluctuations deviate from Gaussianity to a lesser extent, with distributions correctly described by exponential tails [Fig. 4(b)], as for velocity increments in the inertial range of turbulence [12].

The shear stress amplitude record, $\tau=\frac{(\sigma_1-\sigma_2)}{2}$, is also shown on Fig. 2. This stress invariant was selected, as Coulombic shear faulting is the main fracturing mechanism within sea ice [2,13], and consequently the deviatoric part of the strain tensor dominates [14]. No obvious correlation between large stress amplitudes and large rotation increments is visible. A more detailed correlation analysis using a comparison with randomly reshuffled data sets instead reveals a slight anticorrelation: large $|\Delta\theta|$ are preferentially associated with small shear stress amplitudes as well as small divergence. A similar analysis revealed that principal stress direction fluctuations are uncorrelated with 2 m wind direction fluctuations. These results suggest that large fluctuations of principal stress directions are more likely during "quiet" periods of low fracturing and deformation activity. They stress again that this intermittency is not the direct result of the fluctuations of direction of the local wind.

Principal stress direction fluctuations within Arctic sea ice deviate from Brownian random noise as they exhibit (i) long-range time correlations (Fig. 3) and (ii) power law statistics [Fig. 4(b)]. To fully characterize this intermittency, it seems therefore pertinent to perform a multifractal analysis of the signal that allows us to measure how the distributions of direction increments vary with the time scale $\Delta t$ considered. The moments of direction increment are defined as $\langle|\Delta\theta|^q\rangle_{\Delta t}=\langle|\theta(t+\Delta t)-\theta(t)|^q\rangle$. A scaling behavior $\langle|\Delta\theta|^q\rangle\sim\Delta t^{\zeta(q)}$ is observed over the entire scale range, i.e., more than three orders of magnitude [Fig. 5(a)]. The moment function shows curvature, i.e., multifractality in the time domain, the fingerprint of intermittency [Fig. 5(b)]. Theoretical arguments lead to $\zeta(2)=\mu-1$ [12], in excellent agreement with our results, $\zeta(2)=0.75$ and $\mu=1.76$. $\zeta(q)$ is well fitted by the

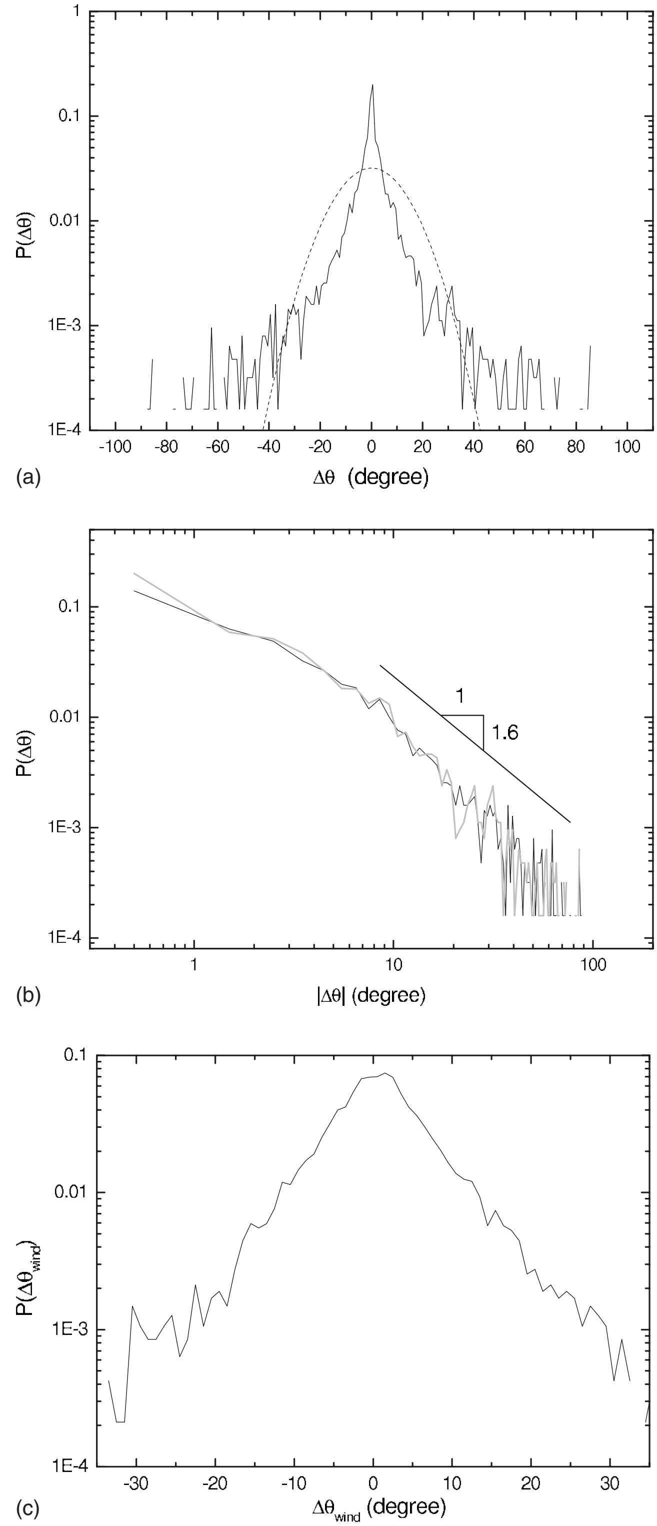


FIG. 4. Probability density function (pdf) of principal stress direction increments $\Delta\theta$ (Baltimore sensor). (a) The pdf (solid line) deviates significantly from the corresponding Gaussian distribution (dashed line). (b) The pdf of the absolute values of the direction increments $|\Delta\theta|$ in a log-log plot, revealing power law statistics, $P(|\Delta\theta|)\sim|\Delta\theta|^{-\beta}$, with $\beta=1.6\pm0.1$ (black line: pdf of $|\Delta\theta|$ for $\Delta\theta<0$; light gray line: pdf of $|\Delta\theta|$ for $\Delta\theta>0$). (c) the pdf of the 2 m wind direction fluctuations $\Delta\theta_{\rm wind}$ is shown on a semilog plot for comparison. This pdf is characterized by an exponential tail.

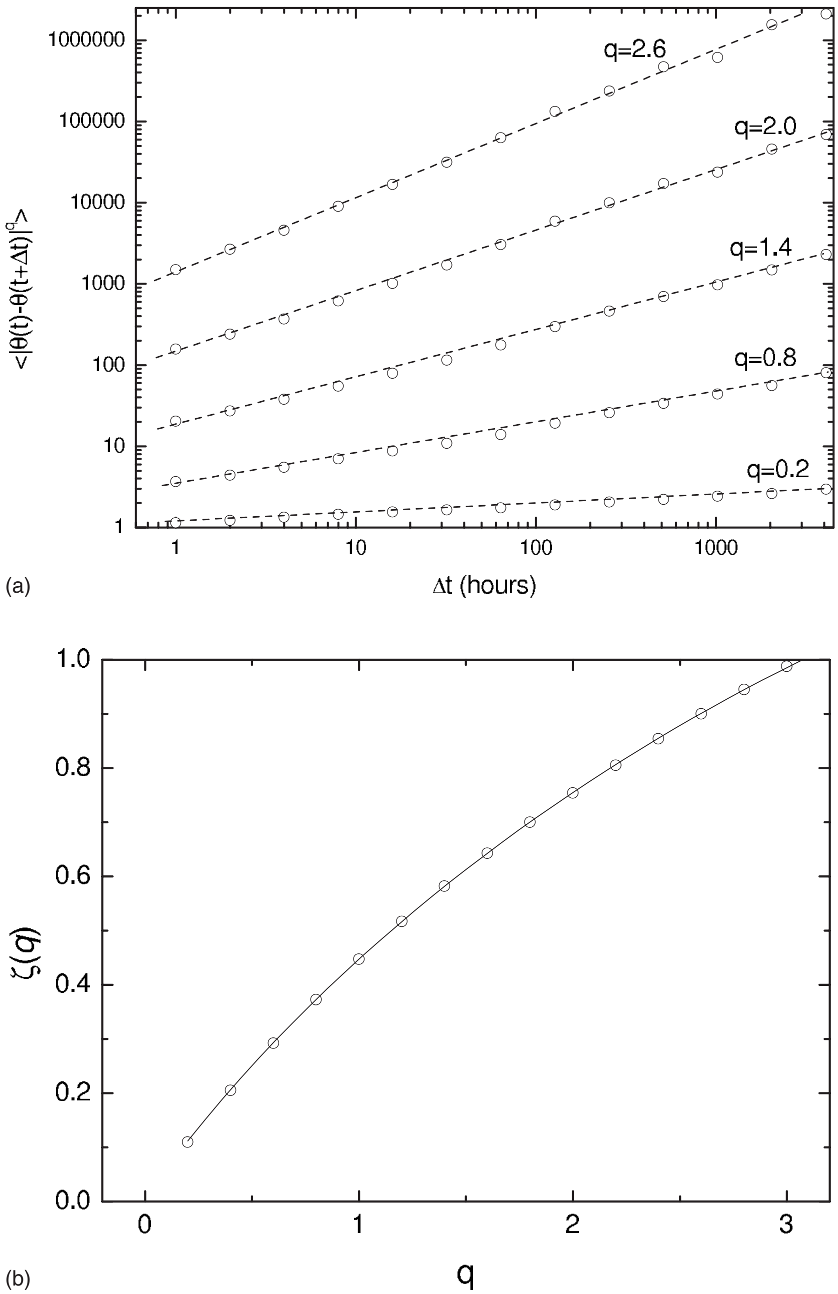


FIG. 5. Multifractal analysis of principal stress direction fluctuations (Baltimore sensor). (a) Moments $\langle|\Delta\theta|^q\rangle_{\Delta t}=\langle|\theta(t+\Delta t)-\theta(t)|^q\rangle$ as a function of the time scale $\Delta t$ for different value of $q$ over the range 1–4096 h, and the corresponding least-square fits (dashed lines) giving the values of $\zeta(q)$, $\langle|\Delta\theta|^q\rangle\sim\Delta t^{\zeta(q)}$. (b) Moment function $\zeta(q)$ showing curvature, i.e., multifractality in the time domain. The solid line represents the universal multifractal model $\zeta(q)=qH-\frac{C}{\alpha-1}(q^\alpha-q)$ with $H=\zeta(1)=0.447$, $\alpha=1.34\pm0.02$, and $C=0.09\pm0.001$.

so-called universal multifractal model, $\zeta(q)=qH-\frac{C}{\alpha-1}(q^\alpha-q)$, where $\alpha$ is the degree of multifractality ($\alpha=0$ for a monofractal) and $H=\zeta(1)$ is the Hurst's exponent [15]. The multifractality revealed here ($\alpha=1.34\pm0.02$) is less marked than the multifractality observed for shear stress amplitudes [6] ($\alpha=1.69\pm0.04$), but very similar to that observed for turbulent wind velocities ($\alpha=1.3\pm0.1$) [15]. $|\Delta\theta|/\Delta t$ can be interpreted as a rate of direction change measured at time scale $\Delta t$. The moments of this rate scale as $\langle(\frac{|\Delta\theta|}{\Delta t})^q\rangle\sim\Delta t^{-[q-\zeta(q)]}$, i.e., the higher moments grow faster toward small time scales than for monofractal scaling, meaning that direction changes are increasingly localized in time toward small scales.

The multiscale fracturing process associated to the deformation and dynamics of Arctic sea ice is not only characterized by the spatial heterogeneity and intermittency of strain-rate and stress amplitudes, but also by an intermittency of principal stress directions with power law statistics and long-range correlations in time. This is consistent with the persistence of active fractures with nearly constant orientations over long time scales (>1 month) [16], as well as with the sudden shifts of orientation that can be seen on the animations of satellite-derived strain-rate fields [17]. These results suggest that this intermittence is not an inheritance of the turbulent wind forcing. Instead, it essentially results from the fracturing process itself: within an elastic plate whose inelastic deformation is accommodated by multiscale fracturing [2,3], the local stresses result from the superposition and interaction of many long-range elastic stress fields associated to fractures of various sizes whose activation is itself characterized by complex space and time scaling laws. If a fractured plate is loaded by a forcing whose principal directions change through time in a relatively random, Brownian-like manner, it will not respond simply and immediately to these random fluctuations of the forcing field because of the presence of pre-existing, persistent fractures that act as preferential paths to accommodate the deformation and control the elastic stress field. Sudden shifts of principal stress directions are more likely to happen after quiet periods of low stress and fracturing activity, when the refreezing of the fractures has partly effaced the underlying structure.

I thank J. Richter-Menge and B. Elder who provided the stress records of the SHEBA experiment.

[1] M. Steele, J. Zhang, D. Rothrock, and H. Stern, J. Geophys. Res. **102**, 21061 (1997).
[2] J. Weiss, E. M. Schulson, and H. L. Stern, Earth Planet. Sci. Lett. **255**, 1 (2007).
[3] J. Weiss, D. Marsan, and P. Rampal, in *Scaling in Solid Mechanics*, edited by F. Borodich (Springer, New York, in press).
[4] D. Marsan, H. Stern, R. Lindsay, and J. Weiss, Phys. Rev. Lett. **93**, 178501 (2004).
[5] P. Rampal, J. Weiss, D. Marsan, R. Lindsay, and H. Stern, J. Geophys. Res. **113**, C03002 (2008).
[6] J. Weiss and D. Marsan, C. R. Phys. **5**, 683 (2004).
[7] E. van Doorn, B. Dhruva, and R. Sreenivasan, Phys. Fluids **12**, 1529 (2000).
[8] J. Richter-Menge *et al.*, J. Geophys. Res. **107**, 8040 (2002).
[9] http://www.crrel.usace.army.mil/sid/SeaIceDynamics/SHEBA.htm
[10] G. F. N. Cox and J. B. Johnson (CRREL, Hanover, NH, 1983).
[11] P. O. G. Persson, C. W. Fairrall, E. L. Andreas, P. S. Guest, and D. K. Perovich, J. Geophys. Res. **107**(C10), 8045 (2002).
[12] U. Frisch, *Turbulence, The Legacy of A.N. Kolmogorov* (Cambridge University Press, Cambridge, England, 1995).
[13] E. M. Schulson, J. Geophys. Res. **109**, C07106 (2004).
[14] H. L. Stern and R. E. Moritz, J. Geophys. Res. **107**, 8028 (2002).
[15] F. Schmitt, D. Lavallée, D. Schertzer, and S. Lovejoy, Phys. Rev. Lett. **68**, 305 (1992).
[16] M. Coon, R. Kwok, G. Levy, M. Pruis, H. Schreyer, and D. Sulsky, J. Geophys. Res. **112**, C11S90 (2007).
[17] R. Kwok, in *IUTAM Scaling Laws in Ice Mechanics and Ice Dynamics*, edited by J. Dempsey (Kluwer Academic Publishers, Fairbanks, AK, 2001), p. 315.