Effects of structural periodicity on localization length in one-dimensional periodic-on-average disordered systems

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We investigate the effects of structural periodicity on wave localization in one-dimensional periodic-on-average disordered systems and derive two relations from the properties of the spectral periodicity and symmetry of the underlying periodic systems. These two relations predict equal localization lengths between disordered systems with different randomness. Comparisons with numerically simulated results show good agreement. These relations are used to explain some properties of the frequency dependence of the localization length, such as oscillation, asymmetry, etc.

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I. INTRODUCTION

Anderson localization, which was presented originally [1] for describing the transport behavior of electrons in disordered solid-state systems, has attracted much interest for a long time [2,3]. Due to its tremendous physical meanings, it is still a topic of great interest nowadays and the research has extended to almost all kinds of waves, such as light waves, sound waves, matter waves, and so on [4–7].

A main feature of the localized waves is the exponential decay of their amplitude with the propagating distance. The speed of this decay is denoted by the localization length ξ [2,3]. For a given disordered system, waves at different frequencies generally display different decay speeds, so the frequency dependence of the localization length, $\xi(\omega)$, is one of the most important quantities in the study of wave localization [8–11]. Localization lengths in the long-wavelength and high-frequency limit can be derived analytically [12,13]. In the case of periodic-on-average random systems (PARSs), it is also known [14] that the localization lengths decrease dramatically in the vicinity of the band gaps of the underlying periodic system. Due to the intricate interplay of order and disorder, further understanding of $\xi(\omega)$ mostly relies on results of numerical simulations, where the intrinsic nature may be ignored. Analytical investigations of the localization behavior in disordered systems are always pursued [15,16].

Periodic systems, however, can be analyzed more effectively based on the Bloch theorem, and plenty of features can be derived analytically [17]. In PARSs, the introduction of disorder does not randomize all structural properties, so it is natural to wonder whether some well-known features of periodic systems remain valid to some extent when disorder is introduced. Works following this idea have been surprisingly few, even in the simplest case of one-dimensional (1D) PARSs [18–20].

In this paper, we start from two well-known spectral properties of 1D periodic structure, called symmetry and periodicity, and study their validity in PARSs. Two relations are obtained analytically, which explore the possibility that the localization lengths could be equal when the wave frequencies and structural randomness satisfy some conditions. Comparisons with numerical simulations carried out by using the transfer matrix method show good agreement. These two relations are also used to explain some properties of $\xi(\omega)$ found in numerical results, such as oscillations, asymmetry, etc.

II. MODEL

As mentioned above, the wave localization phenomenon in disordered systems is general for all kinds of waves. Here we deal with the light-wave version and study its localization properties in 1D PARSs. The structures are assumed to be multilayer structures composed of two alternating layers A and B with refractive index n_A and n_B, respectively. The disorder is introduced forcing the layer thickness d_A and d_B to change randomly assuming that d_A and d_B are drawn independently from a uniform distribution, respectively:

$$d_A(i) = \langle d_A \rangle [1 + \delta_A \varsigma_A(i)], \tag{1a}$$

$$d_B(i) = \langle d_B \rangle [1 + \delta_B \varsigma_B(i)]. \tag{1b}$$

where i denotes the ith unit, ς_A and ς_B are supposed to be mutually independent, uniformly distributed stochastic variables in the interval $[-1,1]$, and $0 \le \delta_{A(B)} < 1$ specifies the amount of randomness. So the system is periodic on average with spatial period equal to $\Lambda = \langle d_A \rangle + \langle d_B \rangle$ and disorder parts are controlled by $\Delta d_A = \langle d_A \rangle \delta_A \varsigma_A(i)$ and $\Delta d_B = \langle d_B \rangle \delta_B \varsigma_B(i)$. The schematic sketch of the structure is shown in Fig. 1, and we assume that light waves are incident to the structures normally and the structures are embedded in the air.

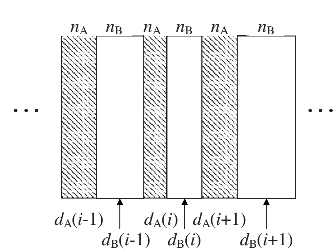

FIG. 1. Schematic sketch of the 1D PARS structure studied in this paper.

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According to the localization theory, all the states in the above 1D PARS are localized and the localization length ξ can be computed in accordance with the following definition [2]:

$$\xi^{-1} = \lim_{L\to\infty} \frac{\ln T}{L}, \tag{2}$$

where L is the length of the structure and T is the transmittance. This equation gives the single value of ξ only in the case of infinite systems $L\to\infty$. However, when L is finite, the calculated result of ξ depends on the choice of realization—i.e., the choice of $\varsigma_A(i)$ and $\varsigma_B(i)$. To obtain a typical value of ξ, generally, one takes a geometrical mean of ξ on more than tens of thousands of samples, which is equal approximately to the value of the infinite system due to the ergodic property of the random quantities ς_A and ς_B. So ξ does not depend on a particular realization [corresponding to particular sequences of $\varsigma_A(i)$ and $\varsigma_B(i)$], but depends on the amount of randomness of the structure, $\delta_{A(B)}$.

The transmittance T can be calculated by means of the transfer matrix method [21]. The state of the system in the kth layer is described by the vector u_k with components representing right- and left-going electromagnetic waves $E_{k,R}$ and $E_{k,L}$, respectively. The evolution of the vector u_k is controlled by the matrix M_k, $u_{k-1}=M_k u_k$, where M_k is determined as follows:

$$M_k = \begin{bmatrix} \cos\varphi_k & j/n_k \sin\varphi_k \\ jn_k \sin\varphi_k & \cos\varphi_k \end{bmatrix}. \tag{3}$$

$\varphi_k=\frac{\omega}{c}n_k d_k$, where d_k and n_k are the layer thickness and refractive index of the kth layer, respectively. ω is the angular frequency and c is the light speed in vacuum.

The transmittance T can be obtained by

$$T = \left| \frac{2}{m_{11}+m_{22}+m_{21}+m_{12}} \right|^2, \tag{4}$$

where m_{ij} $(i,j=1,2)$ are the elements of the matrix $M^{(N)} = \Pi_{k=1}^{N} M_k$,where N is the total layers of the system.

III. PERIODICITY AND SYMMETRY OF PERIODIC STRUCTURE

The spectral properties of 1D periodic structure are described in detail in many works in the literature (Ref. [21], for example). Here, we notice that the transmittance is *periodic*, i.e.,

$$T(\varphi) = T(\varphi + mP), \tag{5}$$

and *symmetric*, i.e.,

$$T(\varphi) = T(mP - \varphi), \tag{6}$$

where m is an integer. In general, φ is the phase shift of one layer: $\varphi=\langle\varphi_A\rangle=\frac{\omega}{c}n_A\langle d_A\rangle$—i.e., the phase shift of layer A (one can choose $\varphi=\langle\varphi_B\rangle$ of course). It should be noted that φ actually denotes the frequency since the parameters c, n_A, and $\langle d_A\rangle$ are fixed. P is the period determined by the ratio of the phase shifts between the A and B layers: $r=\frac{\langle\varphi_A\rangle}{\langle\varphi_B\rangle}=\frac{n_A\langle d_A\rangle}{n_B\langle d_B\rangle}$.

We give a brief demonstration of these two properties as follows. A detailed analysis can be found in many works in the literature (Ref. [21], for example).

The periodicity originates in the expansion of the M_k matrix:

$$M_k = \begin{bmatrix} 1 & 1 \\ n_k & -n_k \end{bmatrix} \begin{bmatrix} e^{j\varphi_k} & 0 \\ 0 & e^{-j\varphi_k} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ n_k & -n_k \end{bmatrix}^{-1}. \tag{7}$$

It can be found that the phase shift enters only in the middle matrix (called the phase matrix) in Eq. (7). For the periodic structure with $n_A\langle d_A\rangle=n_B\langle d_B\rangle$, the phase matrices of A and B layers are the same (thus M_k are the same too). If we change the phase shift by an integer multiple of π, the matrices remain invariant or are multiplied by -1, which has no effect on T: $T(\varphi)=T(\varphi+m\pi)$; i.e., the spectra have a period of π. For the periodic structure with $n_A\langle d_A\rangle\neq n_B\langle d_B\rangle$, the ratio $r=\frac{n_A\langle d_A\rangle}{n_B\langle d_B\rangle}$ can be written in the form of a fraction in the lowest term: $r=\nu/\mu$, where ν and μ are relatively prime numbers; then, the period $P=\mu\pi$, which guarantees simultaneous invariance of the phase matrices of the A and B layers under the transformation $\varphi\to\varphi+mP$.

The spectral symmetry is a result of the even symmetry of the transmittance about the angular frequency ω; that is, $T(\omega)=T(-\omega)$ [so $T(\varphi)=T(-\varphi)$ too]. This can be obtained following the equivalence theorems. It is said that an arbitrary layer system may be formally regarded as equivalent to a suitable two-layer system:

$$M_{eff} = \begin{bmatrix} \cos\varphi_1 & \frac{j}{n_1}\sin\varphi_1 \\ jn_1\sin\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} \cos r\varphi_2 & \frac{j}{n_2}\sin r\varphi_2 \\ jn_2\sin r\varphi_2 & \cos r\varphi_2 \end{bmatrix}, \tag{8}$$

where the subscripts 1 and 2 represents two layers, respectively. Substitution of the elements of M_{eff} into the formulation of transmittance gives rise to the even symmetry naturally. It should be mentioned that this even symmetry is true of any 1D system, not only periodic systems. Combined with the spectral periodicity, the spectral symmetry can be obtained easily.

Figure 2 shows the transmittance spectra of two sample structures with $r=1$ (A) and $r=1.5=3/2$ (B). We can find that the spectrum of (A) is a period of $T=\pi$ and that of B is a period of $T=2\pi$. Within each period, the curves are symmetric.

IV. EFFECTS OF STRUCTURAL PROPERTIES IN PARSs

Then we study whether these two properties are still valid to some extent when disorder is introduced. First, let us check the periodicity—i.e., Eq. (5). Following the definition of our disordered structure, Eq. (1a) and (1b), the phase shift of the ith layer $\varphi(i)$ can be separated into two parts:

$$\varphi(i) = \langle\varphi\rangle + \Delta\varphi(i), \tag{9}$$

where $\langle\varphi\rangle$, representing $\langle\varphi_A\rangle$ or $\langle\varphi_B\rangle$, is the periodic part and $\Delta\varphi=\frac{\omega}{c}n\Delta d$, representing $\Delta\varphi_A$ or $\Delta\varphi_B$, is the part caused by disorder. Because the phase shift enters only in the phase matrix M_k in Eq. (7) and the phase matrix is diagonal, it can be separated further into two parts

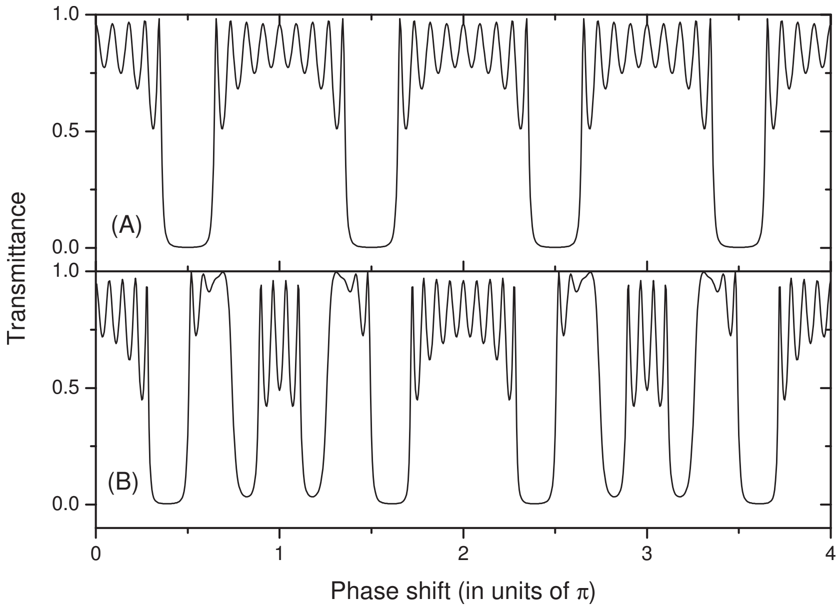

FIG. 2. Transmittance spectra at normal incidence of two periodic systems of ten layers with different ratio r of the layer phase shift of two materials. (a) $r=1$ and (b) $r=1.5$.

$$\begin{bmatrix} e^{j\langle\varphi\rangle} & 0 \\ 0 & e^{-j\langle\varphi\rangle} \end{bmatrix}\begin{bmatrix} e^{j\Delta\varphi} & 0 \\ 0 & e^{-j\Delta\varphi} \end{bmatrix}, \tag{10}$$

which correspond to the periodic and disordered parts, respectively.

According to the periodicity of the periodic structure, the periodic part of the phase matrices remains invariant or is multiplied by -1 under the transformation $\langle\varphi\rangle \to \langle\varphi\rangle' = \langle\varphi\rangle + mP$. However, we cannot draw a conclusion that the total transmittance remains invariant here, since the disorder part of the phase matrices may be affected under this transformation. So in order to keep the invariance of the total transmittance T, there is an additional condition required: $\Delta\varphi = \Delta\varphi'$ under the transformation $\langle\varphi\rangle \to \langle\varphi\rangle'$. According to the definition of a phase shift, the relationship between $\langle\varphi\rangle$ and $\Delta\varphi$ is

$$\Delta\varphi = \frac{\langle\varphi\rangle}{\langle d\rangle}\Delta d. \tag{11}$$

In order to satisfy the condition $\Delta\varphi = \Delta\varphi'$, it gives

$$\frac{\langle\varphi\rangle}{\langle d\rangle}\Delta d = \frac{\langle\varphi\rangle'}{\langle d\rangle}\Delta d'. \tag{12}$$

Substituting the relation $\langle\varphi\rangle' = \langle\varphi\rangle + mP$ into the above equation and after rearrangement, we have

$$\Delta d' = \frac{\langle\varphi\rangle}{\langle\varphi\rangle + mP}\Delta d. \tag{13}$$

It means that if the disorder parts of two structures satisfy the above relation, $\Delta\varphi$ can remain invariant under the transformation $\varphi \to \varphi' = \varphi + mP$, and thus the total transmittances of the two structures are equal:

$$T(\Delta d, \varphi) = T(\Delta d', \varphi') = T\left(\frac{\langle\varphi\rangle}{\langle\varphi\rangle + mP}\Delta d, \varphi + mP\right). \tag{14}$$

It should be noted that the above equation provides a relation of transmittance among systems with different randomness.

The above relation can be extended to the localization length ξ according to its definition, Eq. (2). As we have pointed out above, ξ is calculated by taking a geometrical mean of ξ on more than tens of thousands of sequences and the general behavior of ξ does not depend on the particular sequence due to the ergodic property of the random quantity ς, but depends on the randomness δ. So we have

$$\xi(\delta, \varphi) = \xi\left(\frac{\varphi}{mP + \varphi}\delta, mP + \varphi\right), \tag{15}$$

which means that the localization length of structure with randomness δ at φ is equal to that of structure with randomness $\frac{\varphi}{mP+\varphi}\delta$ at $mP + \varphi$.

Similarly, based on the transmittance symmetry of 1D periodic structure—i.e., Eq. (6)—we can obtain another relation about the localization length. It should be mentioned that the even symmetry of transmittance is true for any 1D system, not only the periodic case. According to Eq. (2), the localization length has the property of even symmetry too:

$$\xi(\varphi) = \xi(-\varphi). \tag{16}$$

Combining Eq. (16) with (15), we obtain another relation easily:

$$\xi(\delta, \varphi) = \xi\left(\frac{\varphi}{mP - \varphi}\delta, mP - \varphi\right), \tag{17}$$

which provides another relation of the localization length among realizations with different randomness. By combining Eq. (15) with (17), we have a more general relation

$$\xi\left(\frac{\varphi}{mP + \varphi}\delta, mP + \varphi\right) = \xi\left(\frac{\varphi}{nP - \varphi}\delta, nP - \varphi\right), \tag{18}$$

where m and n are integers.

We have done some numerical simulations to check whether these relations are correct. We first consider the case of $n_A\langle d_A\rangle = n_B\langle d_B\rangle$, where the spectrum of the underlying periodic system has been shown in Fig. 2(A). It should be

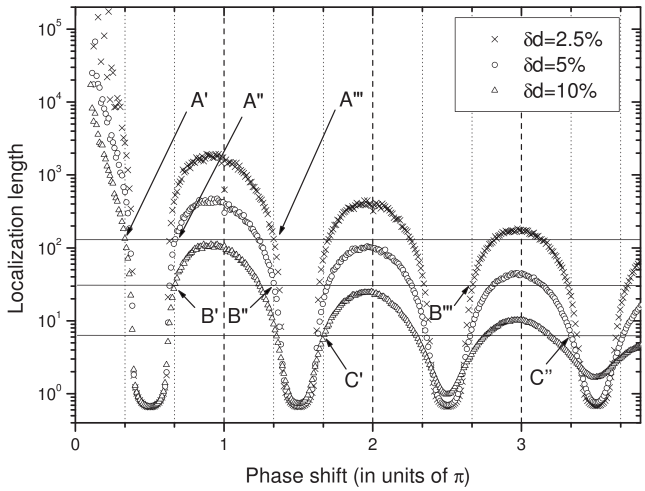

FIG. 3. Localization length as a function of phase shift for three 1D PARSs with different randomness: $\delta_A=\delta_B=\delta d=2.5\%,5\%,10\%$. The structural parameters of the underlying periodic system are the same as those of Fig. 2(A).

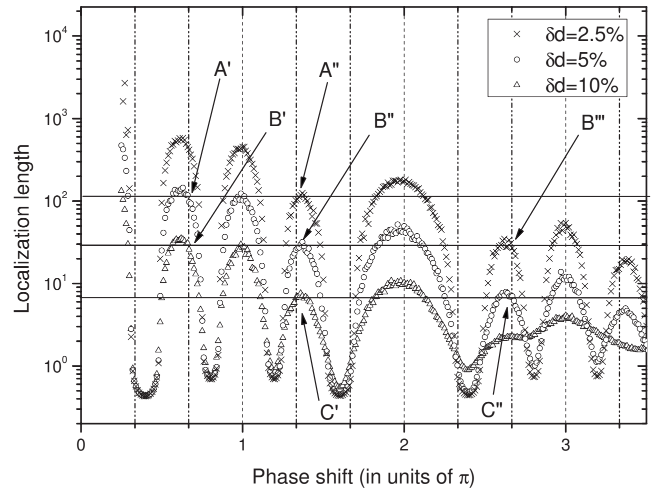

FIG. 4. Localization length as a function of phase shift for three 1D PARSs with different randomness: $\delta_A=\delta_B=\delta d=2.5\%,5\%,10\%$. The structural parameters of the underlying periodic system are the same as those of Fig. 2(B).

noted that the period of the spectrum is π. The refractive indices of the two materials are assumed to be $n_A=1.4$ and $n_B=3$, respectively. For convenience, the disorder is introduced into the layer thicknesses of the two materials simultaneously with the same randomness—i.e., $\delta_A=\delta_B=\delta d$. The localization lengths of three disordered structures with randomness $\delta d=10\%$, 5%, and 2.5% are calculated by means of the transfer matrix method and shown in Fig. 3, represented by different marks. Each point plotted is averaged over 500 realizations, and each realization contains layers whose number is about 5–6 times the localization length. The localization lengths in the figures are in units of $\Lambda=\langle d_A\rangle+\langle d_B\rangle$.

According to Eq. (15),

$$\xi(10\%,\pi/3)=\xi\left(\frac{\pi/3}{\pi+\pi/3}10\%,\pi+\pi/3\right)=\xi(2.5\%,4\pi/3). \tag{19}$$

That is, the localization length at the point of $\delta d=10\%$, $\varphi=\pi/3$ [we use the pair $(10\%,\pi/3)$ for convenience in the following] is equal to that at point $(2.5\%,4\pi/3)$. At the same time, according to Eq. (17), they are equal to that at point $(\%5,2\pi/3)$ too:

$$\xi(10\%,\pi/3)=\xi\left(\frac{\pi/3}{\pi-\pi/3}10\%,\pi-\pi/3\right)=\xi(5\%,2\pi/3). \tag{20}$$

The above three points have been denoted in Fig. 3 by A' $(10\%,\pi/3)$, A'' $(\%5,2\pi/3)$, and A''' $(2.5\%,4\pi/3)$, respectively. It can be found that the numerical results of the localization lengths at these three points do equal each other, showing good agreement with the above analysis. We can also find agreement between the analytical and numerical results from points B' $(10\%,2\pi/3)$, B'' $(\%5,4\pi/3)$, and B''' $(2.5\%,8\pi/3)$ and from points C' $(10\%,5\pi/3)$ and C'' $(5\%,10\pi/3)$.

We also calculated the case of $n_A\langle d_A\rangle=1.5n_B\langle d_B\rangle$, where the spectrum of the underlying periodic system has been shown in Fig. 2(B). All other parameters are the same as those of Fig. 3. In this case, the period of the spectrum is 2π. According to Eq. (15),

$$\xi(10\%,2\pi/3)=\xi\left(\frac{2\pi/3}{2\pi+2\pi/3}10\%,2\pi+2\pi/3\right)=\xi(2.5\%,8\pi/3),$$

and according to Eq. (17),

$$\xi(10\%,2\pi/3)=\xi\left(\frac{2\pi/3}{2\pi-2\pi/3}10\%,2\pi-2\pi/3\right)=\xi(5\%,4\pi/3).$$

These three points have been denoted by B' $(10\%,2\pi/3)$, B'' $(\%5,4\pi/3)$, and B''' $(2.5\%,8\pi/3)$, respectively, in Fig. 4. It can be found that the localization lengths at these three points do equal each other, showing agreement again between the numerical and analytical results. At the same time, agreement can be found between points A' $(5\%,2\pi/3)$ and A'' $(\%2.5,4\pi/3)$ and between points C' $(10\%,4\pi/3)$ and C'' $(5\%,8\pi/3)$.

V. DISCUSSION

The above relations can be used to investigate some properties of $\xi(\omega)$ in PARSs. From Figs. 3 and 4, it can be found that the curves of $\xi(\omega)$ oscillate. For example, in Fig. 3, the curves approach a maximum near the points $\varphi=\pi,2\pi,3\pi,\ldots$, while they approach a minimum near the points $\varphi=\pi/2,3\pi/2,5\pi/2,\ldots$. This oscillation is caused by the band structures of the underlying periodic system. Generally, states from passbands have relatively large localization lengths, while states from stop bands have small localization lengths, especially for the case of small degrees of randomness. So the curves of $\xi(\omega)$ in general oscillate and the minimum values correspond to the center of the original band gaps. However, this oscillation decreases with an increase of the band number, which can be seen from Figs. 3 and 4. For example, in Fig. 3, the amplitude of the oscillation during the zone $[0,\pi]$ is larger than those during $[\pi,2\pi]$ and $[2\pi,3\pi]$. This is consistent with the well-known property that the localization length will tend to be a constant in the limit of high frequency [13], which implies that the oscillation will disappear in the high-frequency region.

This property of decreasing oscillation can be explained by the above relation (15), which can be changed easily to

$$\xi(\delta,mP+\varphi)=\xi\left(\frac{mP+\varphi}{\varphi}\delta,\varphi\right). \tag{21}$$

Since mP is always positive, $\frac{mP+\varphi}{\varphi}>1$, then $\frac{mP+\varphi}{\varphi}\delta>\delta$. If we confine φ to the first period—i.e., $0<\varphi<P$—then the above equation means that for a given sequence with randomness δ, the value of $\xi(\omega)$ in high-order zones (i.e., $mP+\varphi$ with $m>1$) can be related to that in the first period of another sequence with relatively larger randomness. A larger randomness should disorder the structural properties more seriously and thus reduce the oscillation amplitude of $\xi(\omega)$. With an increase of m, $\frac{mP+\varphi}{\varphi}\delta$ will tend to be large enough to destroy the structural property totally, leading to disappearance of the oscillation.

There is another property of $\xi(\omega)$ which can be explained by the relations we derived. It has been demonstrated above [i.e., Eq. (6)] that the spectra of periodic systems are symmetric within each period (i.e., the intervals of $[0,P]$, $[P,2P]$, $[2P,3P]$, …). However, $\xi(\omega)$ are asymmetric in each zone. This point can be found easily from Fig. 3, where the asymmetry is obvious for states from passbands and band edges. Let us take the curves in Fig. 3 during $[\pi,2\pi]$, for example. The curves during the left half of this zone—i.e., $[\pi,\frac{3}{2}\pi]$—are clearly different with those during the right half—i.e., $[\frac{3}{2}\pi,2\pi]$. Furthermore, this asymmetry decreases with an increase of the band number. This can be seen from the difference between two maximum values within each zone. In $[\pi,2\pi]$, $\xi(\varphi=\pi)-\xi(\varphi=2\pi)\approx 1500-400=1100$, whereas in $[2\pi,3\pi]$, $\xi(\varphi=2\pi)-\xi(\varphi=3\pi)\approx 400-150=250$.

This property of asymmetry can also be understood by the relations derived above. For convenience, we define φ_l to denote the phase shifts located at the left half of the first period—i.e., $0\le\varphi_l<P/2$. Then $mP+\varphi_l$ always locates in the left half of each period and $(m+1)P-\varphi_l$ in the right half. From Eq. (18) we have $\xi(\frac{\varphi_l}{mP+\varphi_l}\delta,mP+\varphi_l)=\xi(\frac{\varphi_l}{nP-\varphi_l}\delta,nP-\varphi_l)$. We choose $n=m+1$, then $\xi(\frac{\varphi_l}{mP+\varphi_l}\delta,mP+\varphi_l)=\xi(\frac{\varphi_l}{(m+1)P-\varphi_l}\delta,(m+1)P-\varphi_l)$, which actually provides a relation between $\xi(\omega)$ within two halves of each period. We can define the ratio of two periods of randomness as

$$r=\frac{\varphi_l}{mP+\varphi_l}\delta:\frac{\varphi_l}{(m+1)P-\varphi_l}\delta=\frac{(m+1)P-\varphi_l}{mP+\varphi_l}. \tag{22}$$

Since $0\le\varphi_l<P/2$, we have $1<r\le 1+\frac{1}{m}$. $1<r$ means that the states within the right half always suffer more disorder than those within the left half, leading to the asymmetry of the curve of $\xi(\omega)$ within each period. In addition, with an increase of k, r decreases and tends to be equal to 1, meaning that the curve of ξ tends to be more symmetrical within regions of large frequency, which is also consistent with the numerical results shown in Fig. 3.

VI. CONCLUSION

In summary, we have presented analytically two relations of the frequency dependence of the localization length in 1D PARSs based on the spectral periodicity and symmetry of the underlying periodic systems. Numerical simulations are performed and show good agreement with the analytical results. These relations are used to explain some properties of the frequency dependence of the localization length. Since the relations originate from the properties of the underlying periodic structure, they are expected to be generic properties of 1D PARSs and thus probably facilitate a further understanding of the complicated behavior of wave localization. Based on these relations, further quantitative research work is being carried out and more nontrivial results are anticipated.

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