Fame emerges as a result of small memory

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A dynamic memory model is proposed in which an agent "learns" a new agent by means of recommendation. The agents can also "remember" and "forget." The memory size is decreased while the population size is kept constant. "Fame" emerged as a few agents become very well known in expense of the majority being completely forgotten. The minimum and the maximum of fame change linearly with the relative memory size. The network properties of the who-knows-who graph, which represents the state of the system, are investigated.

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I. MOTIVATION

One of the observations of complex systems is that they are made out of many interacting agents. Usually, the number of agents is simply too big for an agent to be informed of all the others. Therefore agents act based on limited information. Many real-life examples can be given: A consumer can only have access to a limited number of suppliers. A car can only encounter a small number of other cars in a traffic jam. In the brain, a neuron cannot be connected to all the other 10^{11} neurons [1]. No web page can connect to all the other existing web pages. Similarly, no router can be connected to all other routers on the Internet. Even in many simple models, access to only the local information is a common property. In Bak's sandpile model, a sand particle communicates only with the four sand particles in its neighborhood [2]. Similarly, in Axelrod's two-dimensional culture model, an agent interacts with its four neighbors only [3]. In Conway's game of life a cell checks its eight neighbors in order to decide whether to live or die in the next cycle [4]. Although information exchange is relatively local and the rules of exchange are quite simple, these systems manage to become complex systems.

An individual cannot know the entire population but a small fraction of it. Consider the ratio of the number of people that one knows to the size of the population of the city or the country that she lives in. One expects that this ratio, which will be an important parameter of the model developed in this study, is a very small number [16]. Another observation is that the people that we know constantly changes. We "learn" new people from many sources including people, books, newspapers, radio, television, e-mail, www, and short messaging services (SMS). On the other hand, we do not "remember" all the people that we learn. We have a limited cognitive capacity. A mechanism enables us to "forget" people. Therefore a model should deal with concepts such as population, memory, learning, remembering, forgetting, and interaction of individuals that change their memory content. This paper mainly considers the human population in the development of the model but the findings are applicable to many systems.

Mobile phones are a good example which satisfy many properties of the model presented in this paper [17]. They have a limited memory. When they receive a call, they try to store the caller number. They usually do not store their own phone number, that is, they do not "know" themselves. Another example would be routers in computer networks.

II. RECOMMENDATION MODEL

A reasonable question would be: What happens if an agent is allowed to interact with all the other agents, but remembers only a small fraction of them? In order to investigate this question a simple model is constructed. The dynamics of the system is investigated as the memory size is decreased.

A. Static memory model

Let $\boldsymbol{a_i}$ be an agent. Let $\boldsymbol{A}=\{a_i \mid 1 \leq i \leq n\}$ be a *population* of $\boldsymbol{n}$ agents. Each agent a_i has a memory $\boldsymbol{M_i} \subseteq A$. An agent a_i *knows* agent a_j if $a_j \in M_i$. The *knownness* $\boldsymbol{k_i}$ of agent a_i is the number of agents that know a_i. Then $k_i = |\{a_j \mid a_i \in M_j\}|$. If everybody knows the agent, that is $k_i = n$, then the agent is called *perfectly known*. On the other hand, if nobody knows the agent, that is $k_i = 0$, then the agent is called *completely forgotten*. Knownness depends on n. For example, for $n = 100$, one can be known by at most 100 people but for $n = 1000$, knownness can be as high as 1000. In order to compare populations of different sizes, a metric, independent of n, is needed. The *fame* $\boldsymbol{f_i}$ of agent a_i is defined as the ratio of its knownness to the population size, that is $f_i = k_i / n$. Since $0 \leq k_i \leq n$, it is always the case that $0 \leq f_i \leq 1$. Hence fame is a normalized measure of knownness.

An agent *learns* an agent a_i if it gets a_i in its memory. An agent a_i *remembers* agent a_r, if a_i selects a_r among the agents stored in its memory. An agent *forgets* agent a_f if it removes a_f from its memory.

An abstraction which simplifies the model is made. It is assumed that every agent has the same *memory size* $\boldsymbol{m}$, that is $\forall i \, |M_i| = m$. Then the *total memory capacity* of the population is nm. The *memory ratio* $\boldsymbol{\rho}$ is defined to be the ratio of memory size to the population size, $\rho = m/n$. We have $0 \leq \rho \leq 1$, since this work considers m values in the range $0 \leq m \leq n$. Note that ρ corresponds to the ratio of the number of people that one knows to the number of people one possibly knows. As discussed in the "motivation" section, ρ is expected to be a small value which corresponds to small memory sizes.

The *state* of an agent is the content of its memory. Similarly the *state* of the system is the memories of all the agents.

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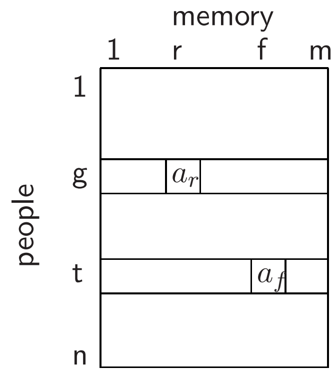

FIG. 1. The state of the system is represented by an $n \times m$ matrix. The row g corresponds to the memory M_g of the agent a_g. During recommendation, the rth item a_r is selected by the giver agent a_g and recommended to a_t. In order to make space for a_r, the taker agent a_t selects the fth item a_f to forget.

The state of the system can be represented by an $n \times m$ matrix as in Fig. 1 where row i corresponds to the memory M_i.

B. Dynamic memory model

The system defined so far is a static one. In order to make it dynamic, interaction between agents is defined by means of recommendation. Agent a_g *recommends* agent a_r to agent a_t as visualized in Fig. 1. The agents $\boldsymbol{a_g}$, $\boldsymbol{a_r}$, and $\boldsymbol{a_t}$ are called the *giver*, the *recommended*, and the *taker*, respectively. The steps of the recommendation process are (i) a_g remembers a_r; (ii) a_g gives a_r to a_t; and (iii) a_t learns a_r if a_t does not already know a_r. "Remembering" and "forgetting" are primitive operations. On the other hand, "learning" is not a primitive operation for $m < n$, since there is no empty space in the memory for the recommended agent. So learning comprises three basic operations: (i) remember some agent a_f; (ii) forget a_f in order to obtain an empty slot; and (iii) put the recommended agent a_r to this slot.

Some remarks about the recommendation operation are needed.

(i) *Selections.* The recommendation operation is carefully defined so that it is open to extensions. There are four selections in every recommendation operation, namely selections of giver-taker (a_g, a_t) and recommended-forgotten (a_r, a_f) as illustrated in Fig. 1. These correspond to selections of g and t from the set of $\{1,2,\ldots,n\}$ and memory positions r and f from $\{1,2,\ldots,m\}$. Different specifications of selections would produce different results. In the simple recommendation model of this paper, all four selections are defined to be randomly chosen from a uniform distribution.

(ii) *Axelrod's culture model.* Another selection criterion could be the case that both the giver and the taker should know the same people in order to interact. Restrict the selection of a_g and a_t in such a way that $|M_g \cap M_t| \geq k$, the case where two agents commonly know at least k agents. For $k = 1$, this leads to a model that is similar to Axelrod's culture model where the culture vector has only one feature and the corresponding set of traits is $\{1,2,\ldots,n\}$.

(iii) *Invariants.* The recommendation operation preserves some global values. Since there are nm memory locations, the summation of the knownnesses of the system is given as $\sum_{i=1}^{n} k_i = nm$. This summation is invariant with respect to recommendation operation, since a recommendation increases the knownness of the recommended by one while decreases that of the forgotten by one.

(iv) *Completely forgotten.* If an agent becomes completely forgotten, then there is no way to be known again.

(v) *Perfectly known.* If an agent becomes perfectly known, it does not mean that it will stay this way unless the system is in one of its "absorbing states."

(vi) *Recommending items.* Note that although the memory model is presented as agents recommending agents, it can be extended to a model for a general case of agents recommending any type of items, such as books or songs, to other agents. Concepts such as "completely forgotten" would be difficult to explain for a human population since a person would know herself even if the rest forgets her. On the other hand, it is not hard to talk about a song, a book, a cultural tradition, or even a language that is completely forgotten. In science, there are many examples of concepts discovered, forgotten, and rediscovered. A few changes would be needed to extend the model. Let $\boldsymbol{B}$ be the set of items. Then, an agent $a_i \in A$ would have items $b_j \in B$ in her memory, that is $M_i \subseteq B$. The memory ratio, that is the ratio of the actual number of memorized to the number of possibly memorized, would be $\rho = m/|B|$. The recommendation operation would be defined as an agent a_g recommends item b_r to agent a_t. In the rest of the paper, we assume that agents recommend agents to agents, that is $B = A$.

C. Simple recommendation model

Many models can be built on these concepts. One of the simplest models, called the *simple recommendation model*, is obtained by defining all the selection mechanisms as random selections. There are four random selections for each recommendation. The giver, g, and the taker, t, are selected randomly from the set of $\{1,2,\ldots,n\}$. The giver a_g selects the recommended agent a_r from its memory by selecting r from $\{1,2,\ldots,m\}$ randomly. This is the remembering process. If the taker a_t already knows a_r, then it does nothing. Otherwise it has to learn it. Learning calls for selecting a memory slot. This selection of f is also done randomly from $\{1,2,\ldots,m\}$.

This definition implies a number of properties. (i) The selection rules do not prefer one agent to another. That is, the process is symmetric with respect to agents. (ii) Any agent can get a recommendation from any other agent. Note that this may be an oversimplification, since in real-life examples an agent can get in touch with only a limited number of agents. On the other hand, increase in communication (e.g., via e-mail) may enable one to communicate with almost anybody.

D. Termination conditions

When to terminate a simulation is a difficult issue. Defined this way, the memories of the agents are kept changing as long as the recommendations continue. There are some special cases in which continuing recommendations cannot change the state of the system. In these cases, the simulation can be terminated.

Absorbing states

A state where every agent has exactly the same memory content, that is $\forall i,j M_i = M_j$, is called an *absorbing state*. In an absorbing state case, nobody can recommend anything new since everybody knows exactly the same m agents and the remaining $n-m$ agents are completely forgotten. So there is no point continuing the simulation. Therefore an absorbing state is a termination point. Since the system asymptotically converges into one of these absorbing states, absorbing states are theoretical termination points. Note that $m=0$ and $n=m$ cases are special cases of absorbing states.

Simulations show that there are two regimes in the system [5]. In the beginning, the system tends to forget. This forgetting mechanism works so powerful that many agents become completely forgotten at the very early stages of the simulation. As simulations proceed, the number of known people becomes much less than the population size. Then the system reverses this behavior. This time it tries not to forget. This is an expected behavior since the system converges to an absorbing state asymptotically. In this paper the second regime is investigated. The *average recommendation per agent* $\boldsymbol{\nu}$ is defined to be the ratio of total number of recommendations to the population size n. Throughout this study $\nu = 10^6$ is used.

III. RANDOM INITIAL MEMORY

The initial configuration of the agent memory is important for the model. The memories of the agents are initially filled with randomly selected indexes from $\{1, 2, \ldots, n\}$. Repetitions are not allowed. The model is simulated for different values of n and m. Population sizes of $n=10^2$, 10^3, and 10^4 and memory ratios of $\rho=0.50$, 0.30, 0.20, 0.10, 0.05, and 0.01 are used.

Effects of memory size

For the same population size n, effects of changing memory size m in the interval $0 < m < n$ is investigated. Since the memories of the agents are initially randomly filled, the initial fame of an agent is around the average value of $\langle f \rangle = m/n$. As m is decreased, some agents become more known than others, at the expense of others becoming less known. Further decrease of m increases the degeneration further.

For $n=100$, ρ is changed and the change in fame f is observed. Figure 2 is an example of various simulations which produce similar results. In this visualization, agent number 1 is the most famous one and agent number 100 the least famous. Note that the area under the curve is equal to the total memory capacity $n \times m$. As m decreases, agents on the right become completely forgotten, as a result the agents on the left become increasingly famous. Around $\rho=0.5$, the knownness of some agents becomes very low. Completely forgotten agents starts to appear around $\rho=0.35$. From that point on, decrease in m increases the number of completely forgotten agents. Since the total memory capacity is fixed, a few agents become very well-known as a result of this process. Hence fame emerges as an effect of small memory size.

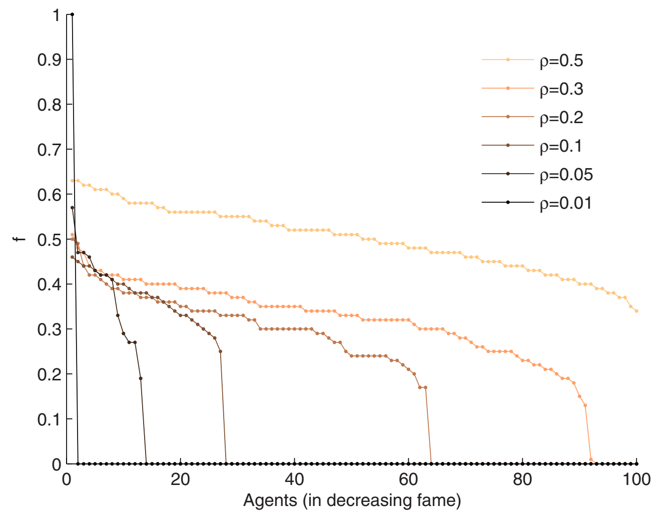

FIG. 2. (Color online) Even distribution of fame degrades as ρ decreases. The model is simulated for various m values where $n=100$ and $\nu=10^6$. At the end of the simulation, memory dumps of agents provide who-knows-who information. Fame of each agent is calculated and for better visualization the agents are sorted in decreasing order of fame.

Eventually m decreases to the extreme case of $m=1$ where an agent can remember only one agent. In this $\rho=0.01$ case, the dynamics of the system goes to an extreme. All the agents become completely forgotten, except only one. That lucky agent is known by all other agents. This is the expected absorbing state since the number of known agents is $m=1$. In order to check this finding, simulations with larger values of n are done for $m=1$. It is observed that as the population size gets larger; reaching an absorbing state becomes harder.

IV. REGULAR INITIAL MEMORY

One may suspect that these findings are due to small fluctuations of the random initial memory. Although random initialization does not prefer any agent systematically, it has some statistical variation. As a result of that, some agents could be slightly more known then others. This initial unbalance may affect the dynamics. In order to check this possibility, a perfectly symmetrical memory initialization scheme is used. In the regular initial memory scheme, each agent a_i is allowed to know its m-neighbor, that is $M_i = \{a_k | k \equiv i + j \pmod{n} \text{ for } 1 \leq j \leq m\}$, similar to the case of [6]. In this way, it is guaranteed that the knownness of every agent is exactly m.

For regular initial memory, an $n=100-1000$ range with increments of 100 is simulated. For each n, a $\rho=0.10-0.90$ range with increments of 0.05 is studied. Additionally, a $\rho=0.01-0.05$ range with increments of 0.01 is simulated in order to see the behavior at very small values of ρ. Every n and ρ combination is simulated 20 times. Interestingly, both random and regular initial memory strategies produce similar results.

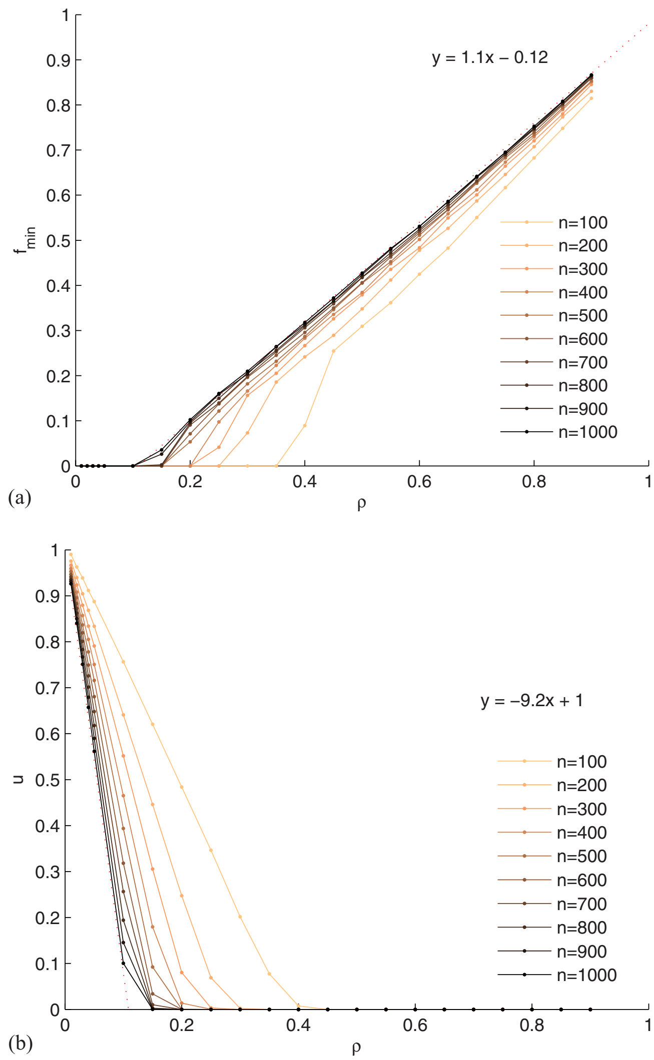

FIG. 3. (Color online) Averages of 20 simulations with various n values where $\nu=10^6$ and m changes as ρ does. (a) Change of the minimum fame, and (b) change of the percentage of the completely forgotten agents as ρ changes.

A. Minimum fame

The minimum fame $\boldsymbol{f_{min}}$ in the population for a range of parametric settings is investigated. As ρ decreases, the minimum value of fame decreases as in Fig. 3(a). This decrease turns out to be linear. As n increases the linear region becomes more visible and n values 800–1000 produce almost the same line. For $n=1000$, the line is given as $f_{min}\approx 1.1\rho-0.12$. The minimum value of fame is $f=0$ when the first agent becomes completely forgotten. Occurrence of the first $f=0$ case depends on n and it has quite a dynamic range. The first $f=0$ case occurs when ρ is around 0.35 for $n=100$. As n increases, the first $f=0$ case moves to smaller values of ρ. For $n=1000$, it happens at $\rho=0.1$.

B. Percentage of forgotten agents

As ρ is decreased beyond the point where at least one agent is forgotten, the minimum fame does not provide any further information. For those values of ρ, the number of

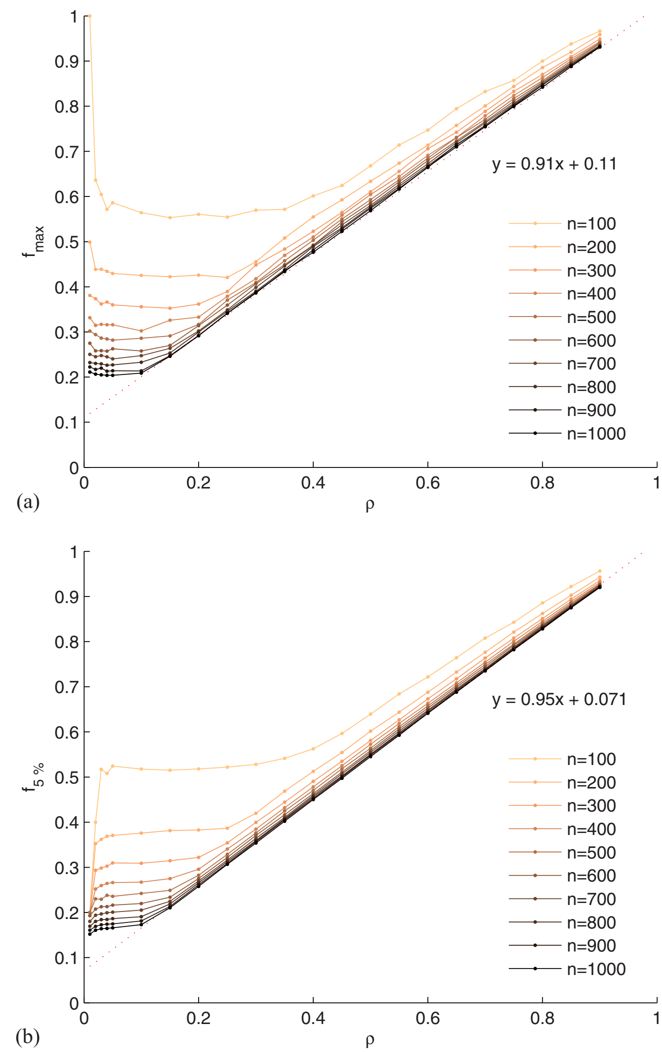

FIG. 4. (Color online) Averages of 20 simulations with various n values where $\nu=10^6$ and m changes as ρ does. (a) Change of the maximum fame, and (b) change of the cumulative fame of the top 5% as ρ changes.

completely forgotten agents can be investigated.

The percentage $\boldsymbol{u}$ of the population that is completely forgotten is used and Fig. 3(b) is obtained. Note that the graphs in Figs. 3(a) and 3(b) complement each other for any particular value of n. As expected, for any values of ρ, one graph has nonzero values whenever the other graph has zeros. Here again, as n increases, the curves converge to a line which is given as $u\approx -9.2\rho+1$ calculated for $n=1000$.

C. Maximum fame

The maximum value of fame $\boldsymbol{f_{max}}$ has an interesting behavior as ρ changes. For $n=100$, f_{max} slowly decreases as ρ decreases from 0.9 to 0.1. It reaches a minimum value around $\rho=0.1$. Interestingly, further decrease of ρ causes f_{max} to increase. This pattern can be seen by tracing Fig. 4(a) from right to left, where the emergence of fame can be observed as the relative memory size (indicated by ρ) is decreased. This unexpected behavior can be explained: When

$\rho=1$, every agent is known by everybody else so the fame is 1. As ρ decreases, the memory size of the agents decreases. Since no one dominates the memories yet, people are almost evenly distributed in the memories. So the reduction of the maximum fame is due to the decrease of the memory size; but as ρ keeps decreasing, after a certain point some people become completely forgotten and some others become the dominating ones. As ρ approaches to the limit of 0, more people become completely forgotten and fewer people dominate the memories. Those that dominate take all the references. So the rapid increase of maximum fame in the vicinity of $\rho=0$ can be explained due to this positive feedback.

Another observation is that the decrease of maximum fame is also linear. As in the case of minimum fame, as n gets larger, the linear pattern becomes more apparent. For $n=1000$, it is given as $f_{max}\approx 0.91\rho+0.11$.

D. Cumulative fame

Maximum fame is a measure of the dominance of one agent. Dominance of a group of famous agents is investigated by means of the cumulative fame of the top $\boldsymbol{p}$ percent of agents ordered according to their fames. The top $p\%$ of the population is selected. The *cumulative fame* $\boldsymbol{f_{p\%}}$ is obtained by adding their fames. The maximum possible value for the cumulative fame is $np/100$ when all top $p\%$ are completely known, that is, each has fame of $f=1$. This value is used for normalization.

In this study, $p=5$ is used and Fig. 4(b) is obtained. As n increases, the curves converge to a line which is given as $f_{5\%}\approx 0.95\rho+0.071$ for $n=1000$. Figures 4(a) and 4(b) are quite similar as expected. On the other hand, the $f_{5\%}$ line decreases slightly faster than the f_{max} line as ρ decreases. As ρ decreases to $\rho=0.1$, the curves become saturated. They stay this way for awhile and then as ρ approaches 0, they start to decrease again. This behavior near $\rho=0$ can be explained by the memory size. As m decreases, at some point there is no space to keep 5% of the population. Whenever that happens, the cumulative fame of the top 5% starts to decrease towards 0. This final decrease is much sharper. This behavior can be seen more clearly for small values of n in the figure. For example, for $n=100$, top 5%, means five agents. If m becomes less than 5, that is $\rho<0.05$, a sharp decrease is expected as in the figure.

V. NETWORK ISSUES

A who-knows-who graph is another representation of the state of the system. The directed graph $G(A,E)$ where A is the set of agents and $E=\{(a_i,a_j)|a_j\in M_i\}$ is called the *who-knows-who graph*. The graph is a directed graph, since the corresponding relation "to know" is not symmetric.

In this directed graph, out-degree is not interesting since all vertices have the same out-degree of m, independent of recommendations. On the other hand, in-degree of a vertex changes by the recommendations and has a dynamic range starting from 0 and it can be as large as n. For both random and regular initial memory cases, the initial in-degree distribution is uniform since every agent has the same knownness of m. As a result of recommendations, in-degrees of a few agents increase while the majority decreases to 0. So as a result of recommendations, uniform in-degree distribution degrades to the one with two peaks around 0 and n. At an absorbing state, there would be exactly two nonzero points in the in-degree distribution, namely 0 and n. There is a nucleus of m vertices in which a vertex is connected to other $m-1$ vertices and itself. The remaining $n-m$ vertices are connected to this m-vertex nucleus. The m vertices of the nucleus have an in-degree of n and $n-m$ vertices have 0.

The undirected graph underlined by the directed who-knows-who graph is topologically investigated. In the random case, the initial network is a random graph. In the regular case, the initial graph is regular. As the recommendation dynamics takes place and fame emerges, both initial graphs transform into one common graph structure. The few famous vertices, which are in the process of forming the nucleus, become hubs. The rest of the vertices are connected to these hubs. Giving this picture, the graph is more towards star-connected rather than power-law degree distribution. Therefore the average distance is very low. The clustering coefficient is also low. The recommendation is a transformation that uses local information only. There are some network growth models that also use local information only but they produce power-law degree distribution [7,8].

VI. ABSORBING STATES

The main focus of this work is the behavior of the system as it approaches but never reaches an absorbing state. In this section a brief investigation of the system at the absorbing state is done and the rest is left as future work. The system simply takes too much time to reach an absorbing state for n values considered so far. On the other hand, if one reduces n, absorbing states become obtainable within reasonable durations. Simulations are done for small values of n and m such as $n\in\{20,30,40,50,60\}$ and $m\in\{1,2\}$. As a measure of time, the number of simulation cycles required to reach an absorbing state is measured.

It is clear that the system reaches an absorbing state asymptotically. Therefore, near the absorbing state, forgetting the next person becomes increasingly harder. Let t_i be the time the ith person is forgotten. Then, define the *time needed to forget the next person* after the ith person as $\Delta t(i)=t_{i+1}-t_i$ where $i\in\{1,\ldots,n-m\}$. As expected, $\Delta t(i)$ rapidly increases as i approaches $n-m$.

In some systems, system size makes a big effect on the behavior. When the parameters are scaled with the systems sizes, then some regularities become visible [9]. In this system n turns out to be an important parameter. Figure 5 provides the behavior of $\Delta t(i)$ as an average of 40 simulation runs. In the x axis the number of persons forgotten is scaled by n which is the percentage of forgotten, that is u. In the y axis $\Delta t(i)$ is scaled as $\Delta t(i)/m$ for various values of n.

Interestingly, there are two families of curves. The upper family belongs to $m=2$. It starts with a slight decrease which corresponds to the initial trend of forgetting. Then the regime changes and forgetting becomes harder and harder as u approaches 1. The $m=1$ family does not have this pattern. Un-

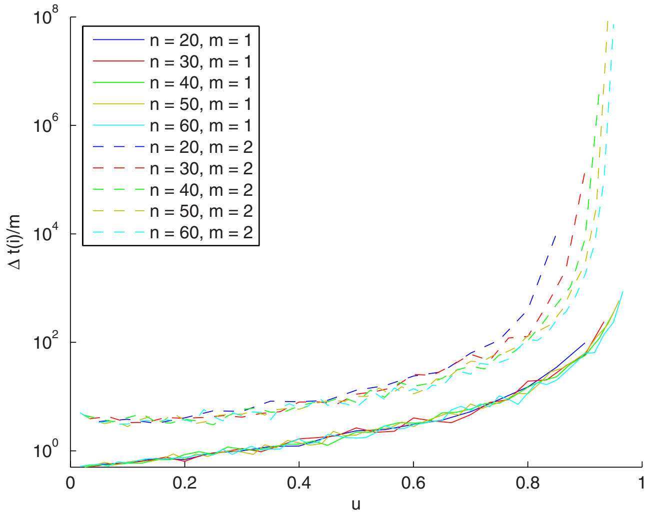

FIG. 5. (Color online) Time to forget next person scales with n for small n and m values. Average of 40 simulations.

fortunately, these n and m values are too small to observe the patterns that are focused in this work.

VII. RELATED WORK ON FAME

Recently some studies on fame have been done. The difficulty starts with the definition of fame. An innovative metric for fame is defined as the number of hits returned from a search of a person's name on Google [10,11]. In this study, the fames and the achievements of WWI fighter pilots are examined. "Fame" F is defined as the number of Google hits. "Achievement" is the number of opponent aircrafts destroyed. It is found that fame grows exponentially with achievement. The distribution of fame is given as $P(F) \propto F^{-\gamma}$ where $\gamma \approx 2$. A similar study on scientists gives another distribution, $P(F) \propto e^{-\eta F}$ where $\eta = 0.001\,02$ [12,13].

Scientific papers can be "famous" by getting cited. A study on scientific papers published in *Physical Review D* in 1975–1994 has been done [14]. There are 24 000 papers, 350 000 citations, that is 15 citations per paper on the average. Yet, 44 papers are cited 500 times or more. It is found that copying from the list of references used in other papers has an impact. A paper that is already cited has more chances to get cited again.

Early results of the simple recommendation model such as the fast and slow forget regimes, asymptotic approach to absorbing states, and degeneration of the distribution of knownness to fame as ρ decreases were presented in [5].

VIII. CONCLUSIONS

"Too many to remember" is quite the common case in many complex systems. A dynamic memory model is defined where agents interact by exchanging recommendations. A random-selection based model is described as the simplest instantiation of the general model. Although the model does not prefer any agent, some agents become increasingly famous as the memory gets smaller. This observation can be interpreted as the *emergence of fame*.

The model can be used in some practical applications. Suppose some agents are preferred in the recommendations. Then, their fame is expected to increase and last longer. This can be used to model the social dynamics of advertisement. Essential questions in advertisement such as how frequently to advertise or how widely to advertise can be better estimated. Voting or election results are studied in opinion dynamics [15]. Emergence of fame can be considered as a formation of an opinion through interactions of agents.

The general model will serve as a basis for building sophisticated models as different selection criteria are adopted and the agent interaction scheme is restricted with new assumptions. For example, it is possible to define selections so that only agents with a common friend can interact. This leads to a version of Axelrod's culture model [3]. The model can be modified so that the giver always recommends itself rather than some agent from its memory. Then it becomes very close to the small-world model presented in [6]. Another possibility is to place the agents on the vertices of an interaction graph, possibly with small-world or scale-free properties in order to introduce real-world flavor.

ACKNOWLEDGMENTS

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[16] For example, one may know 2×10^3 people while there are 15×10^6 in Istanbul that makes the ratio of 1.3×10^{-4}.
[17] The example of mobile phone was proposed by one of the anonymous referees to whom the author would like to thank.