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Controlling chaos for spatiotemporal intermittency

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This paper reports the control of spatiotemporal intermittency in an electroconvective system in a nematic liquid crystal. In the spatiotemporal intermittency, an ordered structure [the defect lattice (DL)] coexists with turbulence. Control of the spatiotemporal intermittency, in which the turbulent state changes to a DL, is achieved by a few percent amplitude modulation of the applied ac voltage. The optimal control frequency is equal to the intrinsic oscillation frequency of the DL. The transition from the turbulent state to a DL occurs not by nucleation of the DL domain, but by penetration of the DL domain into the turbulent one. Control of the spatiotemporal intermittency is achieved through a resonance of the DL oscillation with respect to the modulation frequency that leads to spatial entrainment.

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Controlling chaos has attracted the interest of both engineers and scientists, because chaos is frequently observed in devices and often reduces their efficiencies. It has been shown that application of a small perturbation to a system can suppress the onset of chaos [1]. Such a perturbation can include delayed-feedback signals [2] or periodic external (either additive or parametric) forces [3,4]. The degree of control of the chaos has been quantified by the Lyapunov exponent and attractor dimension, not only in theoretical models [3] but also in experimental systems such as a bistable magnetoelastic system [5], an electronic circuit [6] and a laser [7].

Control of spatiotemporal chaos, i.e., of complex irregular patterns varying in both space and time, has been recently achieved [8]. Such control has applications in various systems such as a plasma [9], a turbulent boundary layer [10] and an electrochemical reaction [11]. However, a clear understanding of the control mechanism is lacking. Spatiotemporal intermittency is an example of this kind of chaos, in which both ordered and disordered states coexist in a dynamic domain structure. It is observed in various systems including a coupled map lattice [12], a partial differential equation system [13], viscous fingering [14], and a convective system [15]. It has been investigated mainly from the viewpoint of statistical physics.

Experimental studies of spatiotemporal intermittency have been mainly performed in quasi-one-dimensional systems so far. Few studies of two-dimensional pattern-forming systems have been undertaken [16], although they are more realistic than one-dimensional systems. An electroconvective system of a planar nematic liquid crystal can exhibit two-dimensional spatiotemporal intermittency consisting of both a defect lattice (DL) pattern (the ordered state) and turbulence (the disordered state). The DL is a structure of periodically aligned defects embedded in a roll pattern [17,18]. With increasing applied voltage (the control parameter), turbulent domains appear locally in the DL (see Fig. 1).

In the present study we report experimental control of two-dimensional spatiotemporal intermittency. We find that turbulent domains in the spatiotemporal intermittency change to lattice state domains when subject to a slowly modulated applied voltage. In other words, the spatiotemporal intermittency can be controlled by parametric modulation.

The experimental setup and sample cell using the nematic liquid crystal *p*-methoxybenzilidene-*p*'-*n*-butylaniline (MBBA) have been described previously [20]. Square electrodes were used with a size of $408d \times 408d$ where the distance d between electrodes is 24.5 μm. The temperature was held at 30.00 ± 0.03 °C. The dielectric constant and the electrical conductivity of the sample measured perpendicularly to the liquid-crystal director were 4.9 and $1.2 \times 10^{-7}\ \Omega^{-1}\ \mathrm{m}^{-1}$, respectively. Hereafter, the initial director (rubbing direction) is defined as the x axis. Images were collected using a charge-coupled device (CCD) camera (Hamamatsu C4880-80) mounted on a microscope. Self-written programs and the ImageJ software package were used for image analysis.

An ac voltage $V_{ex}(t) = \sqrt{2}V(1+W)\cos(2\pi f_{ac}t)$ was applied to the sample in the z direction using a digital synthesizer (NF1946). For standard electroconvection experiments, W

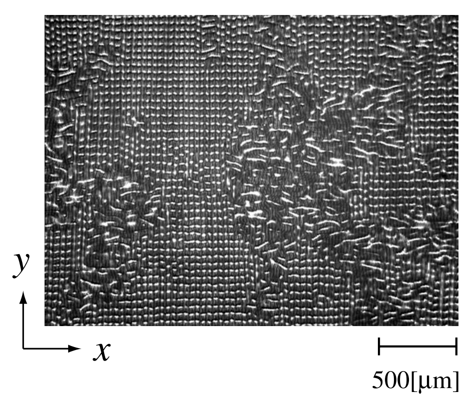

FIG. 1. Spatiotemporal intermittency formed in the DL system for $\varepsilon = 0.80$. The image size is $109d \times 82d$.

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was zero, while for the controlling experiments an amplitude modulation of $W=a_m\cos(2\pi F_m t)$ was established, corresponding to a parametric force. It was found in previous work that a DL pattern appears for applied field frequencies f_{ac} exceeding $0.54f_c$ [18], where f_c is the critical frequency separating the conductive and dielectric regimes. In the present experiment f_{ac} was fixed at $0.65f_c$. A control parameter is defined as $\varepsilon=(V^2-V_c^2)/V_c^2$, where V_c is the threshold voltage for convection.

Based on recent studies of electroconvection in nematics, it has been clarified that the interaction between convective modes and azimuthal rotational modes of the director plays an important role in pattern formation [18,19]. A DL is formed by the interaction of the two modes. The abnormal roll instability [18,19] leads to a domain structure (called the abnormal roll domain) in ϕ that is periodic in the x-direction, where ϕ is the azimuthal angle of the **C** director which is x-y projection of the director. A secondary convective mode $\mathbf{q}_2$ due to the skewed-varicose instability, for which the direction deviates from the x direction, forms a periodic array of shear lines of rolls, and numerous defects are generated along those shear lines [17,18]. Furthermore, the abnormal roll instability induces a "cage effect" [18] in which the gliding motion of the defects is restricted to the interior of the abnormal roll domain. Consequently, the defects align periodically. Thus a DL is formed by the superposition of the convective and azimuthal instability modes.

In the DL the **C** director is rotated away in the direction opposite $\mathbf{q}_2$ due to the anisotropy of the liquid crystal's viscosity. As a result, the convective velocity is reduced because the Carr-Helfrich effect is suppressed. In order to restore the velocity, the rolls must return to their original state with annihilating defects. The nucleation and annihilation of such defects (referred to as a DL oscillation) periodically repeat (see Fig. 3 of Ref. [20]). The oscillation occurs via a Hopf bifurcation by an activator-inhibitor interaction between the **C** director and the convection [21]. A unit cell of the lattice behaves as an individual oscillator. The spontaneous frequencies f_0 of the oscillators are nearly constant for a fixed ε and linearly increase with ε [20]. The oscillation does not continue indefinitely, but is instead intermittent in space and time [20].

When ε is further increased, the lattice collapses locally and becomes turbulent. This state corresponds to spatiotemporal intermittency (Fig. 1). For quantitative size analyses we distinguished two areas: turbulence (dark) and defect lattice (bright) [22]. We measured an area fraction S_T for the turbulent domains, which constitutes an order parameter for spatiotemporal intermittency. From the dependence of S_T on

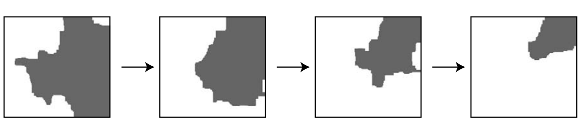

FIG. 2. Change in the spatiotemporal intermittency when subject to amplitude modulation with $a_m=0.04$ and $F_m=f_0$. The black and white areas indicate a turbulent domain and a DL one, respectively. The time interval of adjacent images is 14 min and the observed area is $133d\times125d$.

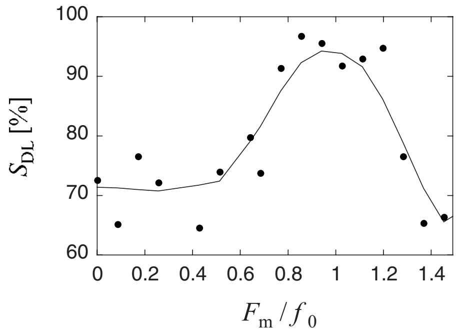

FIG. 3. Resonance curve in S_{DL} of the defect lattice area, measured after applying the amplitude modulation $a_m=0.02$. The observed area was $106d\times79d$.

ε, it became clear that the turbulent domains grow continuously with ε above a threshold value ε_0^* [22].

When the parametric modulation W with its frequency $F_m\sim1$ Hz much smaller than f_{ac} was applied to the system, the sizes of the turbulent domains decrease by the formation of the lattices shown in Fig. 2. It demonstrates control of spatiotemporal intermittency. We measured the area fraction S_{DL} $(=1-S_T)$ of the DL domains after applying an amplitude modulation with a changing value of F_m (Fig. 3). After S_{DL} was set to 70% by selection of suitable ε, W with $a_m=0.02$ was applied to the system. Each data point was acquired in 10 min, after application of the amplitude modulation. At a frequency F_m of f_0, resonant behavior can be observed as shown in Fig. 3. This result suggests that control of spatiotemporal intermittency is achieved by entraining the lattice oscillations at the slowly varying frequency of the amplitude modulation. Accordingly, $F_m=f_0$ is the optimal frequency for controlling spatiotemporal intermittency.

The temporal change in S_T for various values of a_m is plotted in Fig. 4. It typically takes several hours to achieve complete control (i.e., $S_T=0$). This control time is much longer than the characteristic times of the system (i.e., the director relaxation time and the value of f_0^{-1}). For $a_m=0.02$, complete control was not possible for 5 hours. As a_m increases, the control time shortens.

The bifurcation values ε^* in the spatiotemporal intermittency when subject to amplitude modulation were measured for various values of a_m. In each measurement the applied

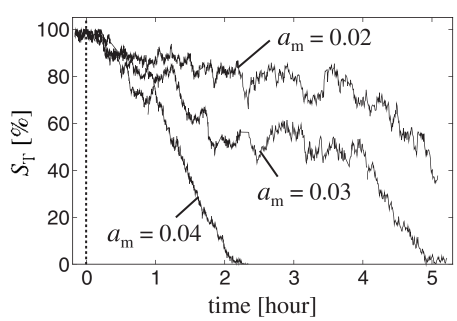

FIG. 4. Change in the value of S_T used to control spatiotemporal intermittency for $a_m=0.02$, 0.03, and 0.04. An amplitude modulation was applied to the system at $t=0$. The initial values of S_T were about 97% at $\varepsilon=0.86$.

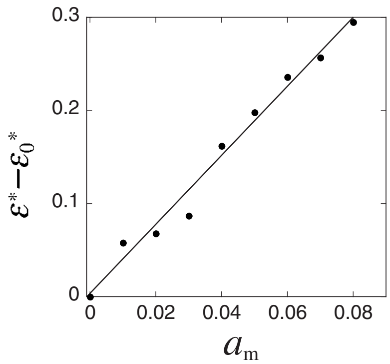

FIG. 5. Dependence of the shift $\varepsilon_0^* - \varepsilon^*$ in the bifurcation points on a_m.

voltage V was increased at the rate of 0.5 mV/s. The dependence of the shift $\varepsilon^* - \varepsilon_0^*$ on a_m is shown in Fig. 5, where ε_0^* is the bifurcation point for $W=0$. A smooth linear relation is observed with no jump in a_m.

The response of a single lattice oscillation to the amplitude modulation was investigated in order to understand the control mechanism. Since a DL exhibits spatiotemporally intermittent oscillations, the oscillator cells do not synchronize with each other. When the amplitude modulation is applied, both the stationary and oscillating cells begin to be entrained to its frequency and phase, as shown in Fig. 6. Thus the inducement (a) and phase shift (b) of the oscillators occur throughout the entrainment. The entrainment induces coherent oscillations of the lattices in the whole DL area and spatially uniform DLs form as a result.

Generally speaking, low-dimensional chaos bifurcates from the stable limit cycles. An unstable limit cycle may remain, however, as a leading frequency in the chaos, and has been used as a controlling frequency in the resonance control for the low-dimensional chaos. Such a frequency plays an important role also in controlling spatiotemporal chaos in coupled chaos systems [4]. In that case, the subsystems partly retain the property of low-dimensional chaos.

On the other hand, due to melting of each lattice in the turbulent domain, there is no clear subsystem in the present spatiotemporal intermittency. A leading frequency f_0 is no longer observed in the temporal spectrum measured in a turbulent domain, and the turbulent state cannot be entrained to the amplitude modulation with f_0.

Indeed Fig. 2 shows that the nucleation of a DL does not occur inside the turbulent domain. Instead the coherent lattice oscillation of the DL changes into a turbulent state at the boundary of the turbulent domain, and the DL domain invades the turbulent one. Namely, the temporal entrainment induces spatial entrainment, and a more ordered state is realized in the spatiotemporal intermittency. This spatial entrainment may be partly achieved through the mediation of elastic interaction of the director.

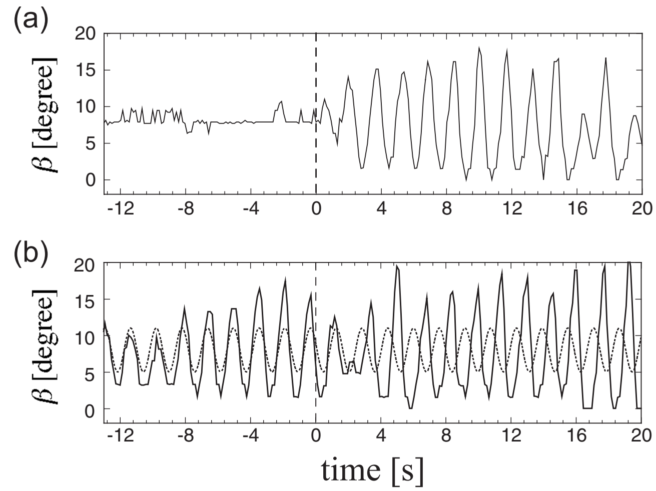

FIG. 6. (a) Induced oscillation and (b) phase shift of the oscillation of a DL by amplitude modulation. The oscillation amplitude is measured by the deflection angle β of a roll with respect to the y direction (see Fig. 3 in Ref. [20]). The broken line in panel (b) indicates the original phase before application of the amplitude modulation. The modulation with $F_m = f_0$ was applied at $t=0$ with $\varepsilon=0.60$.

The above argument suggests that control is not possible for the turbulent state in the absence of a DL. Actually controlling spatiotemporal intermittency would thus not succeed at $S_T = 100\%$. One needs at least a small, instantaneous patch of DL to control the spatiotemporally intermittent chaos. It can thus be said that control of spatiotemporal intermittency is different from conventional control of chaos.

In this Rapid Communication, we have reported the experimental realization of the control of spatiotemporal intermittency appearing in the defect lattice in an electroconvective system in a nematic liquid crystal. The controlling spatiotemporal intermittency, for which the turbulent state changes to a defect lattice, can be achieved by a slow, weak modulation of the applied ac field. As the amplitude of the modulation increases, the time to achieve control shortens. The bifurcation point at which a turbulent domain first appears is shifted by the application of the modulation and a linear dependence of the shift is observed as a function of the amplitude of the modulation.

These result from entrainment that may be specific to the electroconvective system of nematic liquid crystals. The structure changes from one of turbulence to a defect lattice only at the boundary without any nucleation of lattice states inside the turbulent domain. This dynamic originates from the interaction between the convection and the azimuthal rotation of the **C** director.

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