Master crossover functions for one-component fluids

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(Received 6 August 2007; revised manuscript received 16 October 2007; published 15 February 2008)

By introducing three well-defined dimensionless numbers, we establish the link between the scale dilatation method able to estimate master (i.e., unique) singular behaviors of the one-component fluid subclass and the universal crossover functions recently estimated [Garrabos and Bervillier, Phys. Rev. E **74**, 021113 (2006)] from the bounded results of the massive renormalization scheme applied to the $\Phi_d^4(n)$ model of scalar order parameter $(n=1)$ and three dimensions $(d=3)$, representative of the Ising-like universality class. The master (i.e., rescaled) crossover functions are then able to fit the singular behaviors of any one-component fluid without adjustable parameter, using only one critical energy scale factor, one critical length scale factor, and two dimensionless asymptotic scale factors, which characterize the fluid critical interaction cell at its liquid-gas critical point. An additional adjustable parameter accounts for quantum effects in light fluids at the critical temperature. The effective extension of the thermal field range along the critical isochore where the master crossover functions seems to be valid corresponds to a correlation length greater than three times the effective range of the microscopic short-range molecular interaction.

DOI: 10.1103/PhysRevE.77.021116 PACS number(s): 64.60.A−, 05.10.Cc, 05.70.Jk, 65.20.−w

I. INTRODUCTION

The universal features of three-dimensional $(d=3)$ Ising-like systems with scalar order parameter $(n=1)$, such as magnetic compounds, simple fluids, binary liquids, micellar solutions, polymer mixtures, etc., are now well established by the renormalization group approach [1] of the classical-to-critical crossover behavior [2]. In such a renormalization group approach, the Ising-like universality is linked to the existence of a unique nontrivial (Wilson-Fisher) fixed point [3,4], while one specific trajectory links this well-defined nontrivial fixed point to the trivial (Gaussian) fixed point. Then, it was possible to estimate the complete Wegner-like expansions [5] which interpolate between the critical behavior (controlled by the nontrivial fixed point) and a classical behavior (controlled by the Gaussian fixed point). Such interpolating theoretical expressions were customarily named classical-to-critical crossover functions. The corresponding crossover *within the critical domain* was referred to as the critical crossover [2,6], or the asymptotic crossover [7].

Here we are interested only in the mean crossover functions which are calculated in Ref. [8] (hereafter labeled I) with the objective of avoiding the error-bar propagation due to the theoretical uncertainties in the estimation [9] of universal critical exponents and universal amplitude ratios close to the nontrivial fixed point. As a matter of fact, due to the low convergence of the complete Wegner-like expansions of an infinite number of terms, explicit crossover equations for singular properties are needed to determine accurately the contribution of the confluent corrections. Then, using the minimum and maximum crossover functions (with $d=3$ and $n=1$) recently derived [10] by Bagnuls and Bervillier from a massive renormalization scheme applied to the $\Phi_{d=3}^4(n)$ model, such confluent corrections are made explicit in *ad hoc* mean functions given in I as a simplified form of convenient three-term product leading to the required theoretical precision in a known extension of the Ising-like preasymptotic domain. That means that only the asymptotical two-term Wegner-like expansions of the mean functions are exact within this well-defined Ising-like preasymptotic domain and precisely account for the universal features of the Ising-like universality class where only three (two leading and one confluent) calculated exponents and amplitudes are characteristics of the complete set of mean crossover functions. As already proposed in I, considering for example the homogeneous domain $(t>0)$ along the critical isochore $(h=0)$ where t and h are the two relevant (temperaturelike and magneticlike) scaling fields, we can then select the leading exponent ν and the leading amplitude $(\mathbb{Z}_\xi^+)^{-1}$ related to the dimensionless correlation length $\ell_{\mathrm{th}}(t)$, the leading exponent γ and the leading amplitude $(\mathbb{Z}_\chi^+)^{-1}$ related to the dimensionless susceptibility $\chi_{\mathrm{th}}(t)$, and the lowest confluent exponent Δ and the first confluent amplitude $\mathbb{Z}_\chi^{1,+}$ related to the dimensionless susceptibility $\chi_{\mathrm{th}}(t)$, as independent exponents and independent amplitudes to characterize the universal features calculated within this Ising-like preasymptotic domain. This asymptotic situation is in conformity with the two-scale-factor universality expected for all systems with $d=3$, $n=1$, and short-ranged interaction, and which have an isolated transition point. The asymptotic system-dependent characterization is then accounted for by the two-scale factors introduced through the analytical (linearized) relations between the relevant fields t and h of the model and their corresponding physical variables, provided that the same single length unit is used to make dimensionless the thermodynamic and correlation functions of the physical system [11]. We note that the heat capacity case introduces a negative critical constant acting as a supplementary independent amplitude which can be accounted for by including it in a system-dependent background term.

Among the $d=3$, $n=1$ Ising-like physical systems, the one-component fluids are of special interest. Indeed, a phenomenological approach proposed by Garrabos [12–14] displays an equivalent description of the preasymptotic domain, using two-term Wegner-like expansions to reproduce the master crossover behaviors observed for any one-component fluid as a function of the master thermal field $\mathcal{T}^* \to 0$ along the master critical isochore ($\mathcal{H}^*=0$). In such a master description, the asymptotic renormalized fields $\mathcal{T}^* \to 0$ and $\mathcal{H}^* \to 0$ are obtained by using appropriate scale dilatation of the two relevant fluid variables, namely, the asymptotic reduced temperature distance to the critical temperature $\Delta\tau^* \to 0$ and the asymptotic reduced ordering field distance to the critical ordering field $\Delta h^* \to 0$ (in the pure fluid case, the ordering field is proportional to a chemical potential difference; see below). As already illustrated in Ref. [14], when selecting now the updated mean values $\nu=0.630\,387\,5$, $\gamma=1.239\,693\,5$, and $\Delta=0.501\,89$ [9,10] of the universal exponents used for the mean crossover functions (see I), all the master crossover behaviors are asymptotically characterized by the leading amplitude $\mathcal{Z}_\xi^+$ related to the master correlation length $\ell_{qf}^*(\mathcal{T}^*)$, the leading amplitude $\mathcal{Z}_\chi^+$, and the confluent amplitude $\mathcal{Z}_\chi^{1,+}$ related to the master susceptibility $\chi_{qf}^*(\mathcal{T}^*)$, in the homogeneous domain ($\mathcal{T}^*>0$), along the critical isochore ($\mathcal{H}^*=0$), in conformity with the above universal features calculated from the massive renormalization scheme. A noticeable interest from an experimental point of view comes from the nature and the number of characteristic parameters of each pure fluid since this asymptotic singular behavior is thus entirely described using an energy unit, a length unit, and two dimensionless numbers as scale factors. In addition, all these four parameters are unequivocally defined from a critical set $Q_{c,a_{\bar{p}}}^{\min}=\{T_c, p_c, v_{\bar{p},c}, \gamma_c'\}$ made of four critical coordinates (see below) of the liquid-gas critical point (we ignore the quantum effects to simplify the presentation at this introductive level). Moreover, the two dimensionless numbers act in an analogous manner to the two-scale factors introduced in the renormalization scheme. Then, the practical question is how to modify the mean crossover functions to account exactly for both the quantity and the nature of these characteristic parameters of the one-component fluids. Formulated in terms of the universal features calculated within the Ising-like preasymptotic domain, the problem is to establish an unambiguous link between the three-amplitude set $\mathbb{S}_A^{\{MR\}}=\{(\mathbb{Z}_\xi^+)^{-1}, (\mathbb{Z}_\chi^+)^{-1}, \mathbb{Z}_\chi^{1,+}\}$ which characterizes the mean crossover functions calculated in the massive renormalization scheme, and the three-amplitude set $\mathcal{S}_A^{\{1f\}}=\{\mathcal{Z}_\xi^+, \mathcal{Z}_\chi^+, \mathcal{Z}_\chi^{1,+}\}$ which characterizes the two-term master crossover functions valid for all one-component fluids. This paper addresses this basic problem. Moreover, a correlated problem is to develop a general method to identify, in the theoretical scheme, the quantity and the nature of the free parameters needed by the system-dependent description of the asymptotic singular behavior of any system belonging to the complete universality class. This system-dependent parameter set, denoted $Q_{c,\mathrm{th}}$, must then be mandatorily related to $Q_{c,a_{\bar{p}}}^{\min}$ for the one-component fluid case.

The introduction of the system-dependent parameter set $Q_{c,\mathrm{th}}$ for the practical use of the theoretical functions estimated from the massive renormalization scheme was discussed in a detailed manner in Refs. [15]. More precisely, it was shown in Ref. [16] and in I that the Ising-like universal features [9], accurately estimated in the Ising-like preasymptotic domain close to the nontrivial fixed point, require each system to be characterized by introducing at least four parameters to describe the singular behavior along the thermodynamic line equivalent to $h=0$.

Two of them are dimensional parameters, namely, the critical temperature T_c, which acts as the energy unit (introducing the universal Boltzmann constant k_B) to express the Hamiltonian in dimensionless form, and the unknown inverse coupling constant $(g_0)^{-1}$ of the fourth-order term of the dimensionless Hamiltonian. As a matter of fact, any Hamiltonian representative of a physical system at near criticality, such as a one-component fluid near its liquid-vapor critical point, is driven to the nontrivial fixed point under the action of the renormalization transformations [4]. Due to the fact that renormalizable field theories are short-distance insensitive, universality emerges in a regime $\xi\Lambda_0 \gg 1$ in which the correlation length ξ is much larger than the physical microscopic scale which plays the role of the inverse wave number cutoff $(\Lambda_0)^{-1}$ in the renormalization scheme. This universality is non-mean-field-like in nature (at least for the three-dimensional systems which are of present interest), because the actual molecular interaction range at the microscopic scale of the physical system cannot be completely eliminated. Implicitly, $(\Lambda_0)^{-1}$ remains the single natural length unit in the theoretical scheme. Therefore, $(g_0)^{-1}$ takes a convenient length dimension at $d=3$ to act as the adjustable length unit, and to express the physical correlation length in dimensionless form.

The other two parameters are dimensionless coefficients, namely, the scale factors ϑ and ψ, which provide the asymptotic analytical (linear) proportionality between the two dimensionless physical fields $\Delta\tau^* \to 0$ and $\Delta h^* \to 0$ of the Ising-like fluid and the two renormalized relevant fields $t \to 0$ and $h \to 0$ of the $\Phi_{d=3}^4(n=1)$ model, respectively [3,4] (using customary field notations). In such an asymptotical situation, the universal features close to the liquid-gas critical point are estimated in conformity with the two-scale-factor universality, using only theoretical values of two asymptotic critical exponents and one confluent exponent as independent exponents. We can select, for example, the above set $\{\nu, \gamma, \Delta\}$, already used in I. Introducing all these four adjustable parameters in the mean crossover functions $\ell_{\mathrm{th}}(t)$ of the correlation length and $\chi_{\mathrm{th}}(t)$ of the susceptibility, it is then "theoretically" possible to determine a four-parameter set, i.e., $Q_{c,\mathrm{th}}=\{T_c,(g_0)^{-1}$ or $(\Lambda_0)^{-1},\vartheta,\psi\}$, by fitting the asymptotic experimental behaviors of the correlation length $\xi_{\mathrm{expt}}(\Delta\tau^*>0)$ and the isothermal compressibility $\kappa_{T,\mathrm{expt}}(\Delta\tau^*>0)$ of the selected one-component fluid along its critical isochore in the homogeneous domain (the thermodynamic line equivalent to $h=0$ in the pure fluid case).

However, the practical introduction of these four parameters in the fitting equations needs careful attention, except for the critical temperature T_c, which is generally the "measured" value for the selected one-component fluid, leading to the natural energy unit $(\beta_c)^{-1}=k_BT_c$ by introducing the universal Boltzmann constant k_B. Conversely, the finite scale $(\Lambda_0)^{-1}$, or the coupling constant $(g_0)^{-1}$, is generally unknown for a real microscopic interaction at short-range distance in pure fluids. Then, the unambiguous choice of a single microscopic length unit to make dimensionless the thermodynamic variables remains open. Simultaneously, the macroscopic size L of the fluid sample where the singular behaviors are "measured" should be larger than ξ, i.e., $L\gg\xi\to\infty$, and special attention to account for the extensive nature of the thermodynamic properties of the physical system is also needed. Moreover, thanks to the general point of view of the thermodynamics for three-dimensional systems, the dimensionless forms of any physical density variable X/V (where X is the total extensive variable and $V\propto L^d$ is the total volume of the system) can be obtained without reference to the unknown wavelength numbers g_0 or Λ_0 defined at the critical point. Indeed, introducing the total amount of matter N (or the total mass $M=Nm_{\bar{p}}$, where $m_{\bar{p}}$ is the mass of the particle, while the subscript $\bar{p}$ refers to a particle property) of a system filling a total volume V, the dimensionless order parameter conjugated to the dimensionless ordering field can always be defined if the amount of matter in a *reference* volume is known (here the reference volume can be chosen, for example, as the volume of a mole, a particle, a cell lattice, a mass unit of matter, etc.). Therefore, any reference length a_0, defined such that n_0 is the amount of matter in the volume $(a_0)^d$, can be used as the explicit length unit for the thermodynamic and correlation functions. As a result, when T_c and a_0 are known, the nondimensional singular description of each three-dimensional simple fluid is closed, and its particular isochoric behavior is characterized by a constant amount of matter filling the volume $(a_0)^d$. Thus, the use of the massive renormalization results generates a third adjustable dimensionless scale factor—namely u_f^*, such as $u_f^*=g_0a_0$ (see I)—which relates the dimensionless correlation length $\xi(\Delta\tau^*)/a_0$ of the physical system to the corresponding dimensionless theoretical function $\ell_{\mathrm{th}}(t)$ derived from the massive renormalization scheme. As a correlated result, when a_0 takes its physical sense as representing the effective range of the microscopic molecular interaction between the n_0 particles, i.e., $a_0\propto(\Lambda_0)^{-1}$, while n_0 is proportional to the coordination number, the Ising-like singular nature of the physical system can be characterized by a set $S_{\mathrm{SF}}^{\{MR\}}=\{u_f^*,\vartheta,\psi\}$ of three dimensionless scale factors, which complements the dimensional set $\{T_c,a_0,n_0\}$ needed to express thermodynamic functions in their appropriate dimensionless forms for a comparison with theoretical functions. However, in such a three-dimensionless-parameter characterization of the physical system, it is then essential to recall that the theoretical estimations of the universal features are valid only within the Ising-like preasymptotic domain. In this well-defined asymptotic situation, each dimensionless theoretical function which represents a complete Wegner-like expansion of an infinite number of terms can be approximated by its restricted two-term form, to provide the universal features related to only three independent critical exponents (i.e., our selected set $\{\nu,\gamma,\Delta\}$).

Indeed, in the 1970s, it was clearly shown by experimentalists that the singular properties of pure fluids close to their liquid-gas critical point were satisfied by power laws with universal features comparable to the ones estimated for the uniaxial three-dimensional Ising system used as a predictive model (for a review, see for example [17]). It was then revealed that two independent leading amplitudes, attached to the universal values of two independent critical exponents, are the only two fluid-dependent parameters needed to characterize the asymptotic singular behavior of each one-component fluid. Therefore, selecting the dimensional correlation length $\xi(\Delta\tau^*)$ and the dimensionless isothermal compressibility $\kappa_T^*(\Delta\tau^*)=p_c\kappa_{T,\mathrm{expt}}(\Delta\tau^*)$ in the homogeneous domain $(\Delta\tau^*>0)$ along the critical isochore, each Ising-like critical fluid can then be characterized by the related leading amplitudes ξ_0^+ and Γ^+ (introducing the critical pressure p_c and using standard notations for critical fluids [11]). This asymptotic situation was in conformity with the two-scale-factor universality expected for all systems, with short-ranged interaction, and which have an isolated transition point.

Correspondingly, it was demonstrated [18] that the two-scale-factor universality related to the Ising-like nature of the critical phenomena in pure fluids is "observed" in a very limited range of temperature and densities around their liquid-gas critical points. Obviously, since the asymptotic critical domain associated with this limit is so narrow that it is difficult to achieve experimentally, it was fundamental to account for the possible nonuniversal character of the system through the confluent singularities in the corrections to scaling [5] (ignoring here the background contributions, which are significant only in the case of the specific heat [16]). That leads to express the singular properties as truncated forms of the Wegner-like series. That precisely corresponds to the Ising-like limit of the asymptotic crossover mentioned just above and investigated in detail in the renormalization theory, where the resummation of the Wegner-like expansions should yield complete crossover functions from asymptotic (Ising-like) critical behavior to the classical (mean-field) critical behavior. Different theoretical approaches have been adopted by many investigators to obtain solutions resumming the complete Wegner series (see, for example, a review in Ref. [7] for their application to the fluid case). The practical results essentially depend on the approximations used in the renormalization scheme and the way the cutoff effects are taken into account. However, as in I, we do not consider here the resulting differences, for example, from the implicit forms of the crossover functions determined by the minimal subtraction renormalization scheme [19], except to show in Appendix A that the well-defined Ising-like limit can be accounted for in a similar manner. Indeed, despite these technical differences in treating the asymptotic crossover, the Ising-like universal feature was related to the lowest confluent exponent Δ where only one (fluid-dependent) confluent amplitude is needed to characterize the first-order term of the confluent correction to scaling [5]. Therefore, in a fluid characterization where the leading amplitudes ξ_0^+ and Γ^+ are selected as independent leading amplitudes in conformity with the asymptotic two-scale-factor universality, it was necessary to add, for example, the confluent amplitude a_χ^+ of the first-order correction term for the susceptibility (related to the confluent amplitude a_ξ^+ of the correlation length by the mean universal value of the ratio $a_\xi^+/a_\chi^+=0.679\,19$ [9]). The resulting amplitude set $\{\xi_0^+,\Gamma^+,a_\chi^+\}$ defines the complete asymptotic crossover of each one-component fluid within the Ising-like preasymptotic domain. Obviously, when an explicit length scale unit denoted $\alpha_c=k_BT_c/p_c$ (see below) is used in addition to the energy scale unit $(\beta_c)^{-1}=k_BT_c$ to provide the dimensionless description of the fluid thermodynamic properties, the corresponding dimensionless form of the correlation length is $\xi^*(\Delta\tau^*)=\xi(\Delta\tau^*)/\alpha_c$, with a resulting dimensionless leading amplitude $\xi^+=\xi_0^+/\alpha_c$. The three dimensionless-amplitude set $S_A=\{\xi^+,\Gamma^+,a_\chi^+\}$ is Ising-like equivalent (in quantity and nature) to the previous scale factor set $S_{\mathrm{SF}}^{\{MR\}}=\{u_f^*,\vartheta,\psi\}$ used to characterize the Ising-like critical behavior within the preasymptotic domain of each Ising-like system. For an unambiguous understanding of the notation, we recall that a quantity denoted with an open-font capital letter refers to the mean crossover functions, a quantity denoted with a script capital letter refers to the master crossover functions, and a quantity with an italic capital letter refers to the physical functions, while the subscripts A or SF in a parameter set notation distinguish the nature (amplitude or scale factor) of the parameters. In a complementary manner, the superscripts $\{MR\}$ or $\{1f\}$ in a set notation distinguish the parameters which are needed for use of the mean crossover functions or the master crossover functions, respectively. By convenient extension to simplify our presentation, the complete Ising-like universality class with $d=3$ and $n=1$ is labeled the $\{\Phi_3(1)\}$ class, while the one-component fluid "subclass" made of all one-component fluids is labeled the $\{1f\}$ subclass, with the obvious relation $\{1f\}$ subclass $\subset\{\Phi_3(1)\}$ class.

As recalled in I, this three-dimensionless-parameter description of the asymptotic singular behavior of the correlation length and the susceptibility of xenon was studied [16] using the crossover functions initially derived by Bagnuls and Bervillier [15] from the massive renormalization scheme. The dimensionless parameter u_f^*, such that $u_f^*=g_0\alpha_c$, was obtained precisely, introducing $\alpha_c=(k_BT_c/p_c)^{1/d}$ as a length unit. In this pioneering study of the crossover, for the first time the minimal quantity (three) of Ising-like nonuniversal dimensionless parameters of xenon was introduced as a set of a single dimensionless wavelength (defined at the critical point), and two dimensionless scale factors expressing the analytical approximation between the two relevant scaling (thermal-like t and magneticlike h) fields and the corresponding physical ($\Delta\tau^*$ and Δh^*) fields [3,4]. Subsequently, theoretical and numerical approaches applied to the asymptotic crossover description of the singular behavior observed in pure fluids have confirmed this characterization with three dimensionless parameters (see, for example, Refs. [20–25] and the review in Ref. [7]). For example, after the appropriate introduction of two fluid-dependent factors in conformity with the two-scale-factor universality, it is now well established that any theoretical function which fits the temperature dependence of the effective critical exponent along the critical isochore may be made universal by simply rescaling the temperature distance to the critical temperature, as initially proposed by Kouvel and Fisher [26] who introduced a single crossover temperature scale $\Delta\tau_X^*$. Today, the unsolved problems in these theoretical approaches remain the validity of the linear approximations of two relevant fields (which correctly introduce the two system-dependent scale factors), the importance of the neglected analytical and nonanalytical corrections, and, more generally, the estimation of the extension range in temperature and densities around the liquid gas critical point where the Ising-like universal features should be observed.

To complete the above introduction of the nonuniversal character of the asymptotic crossover in pure fluids, we also recall that, at the beginning of the 1980s, the description of the behavior of the singular thermodynamic properties at *finite* distance from the liquid-gas critical point was also made using the theoretical formulation of the nonasymptotic crossover from a regime of Ising-like scaled behavior to another regime in which the critical anomalies due to large fluctuations are ignored [7,27–31]. The common attempt to address this problem was based on the classical-to-critical crossover description of the free energy density. Indeed, this approach is useful for better understanding of crossover critical phenomena in *complex* fluids where the character of the crossover reflects an interplay between Ising-like universality caused by long-range fluctuations and a specific supramolecular structure characterized by an additional nanoscopic or mesoscopic length scale [which can then differ significantly from $(\Lambda_0)^{-1}$]. Therefore, while the Ising-like two-scale-factor universality was similarly accounted for by introducing the two dimensionless parameters of proportionality between the respective relevant (physical and renormalized) fields [for example c_t ($\sim\vartheta$) and c_ρ ($\sim\psi$) in the notation of Refs. [27,28]], the fundamental difference with the asymptotic crossover description comes from the introduction of two independent dimensionless parameters (for example, $\bar{u}$ and Λ in the notation of Ref. [30]), in order to control this nonasymptotic crossover character in complex fluids. However, in an application related to the pure fluid case, it is not necessary to introduce an additional mesoscopic length scale to account for the realistic microscopic situation in one-component fluids [32]. Such pure fluids can then be assimilated to Lennard-Jones-like fluids when they are made of atoms or highly centrosymmetrical molecules, or to short-distance associating fluids when they include more sophisticated short-range molecular interactions between unsymmetrical molecules, polar molecules, bondinglike molecules, etc. Moreover, the representation of the experimental phase surface of any pure fluid by a van der Waals–like equation of state is not accurate either close to or far away from the critical point (since the van der Waals equation of state is theoretically justified only for infinite range of the molecular interaction). As a final result, the fluid-dependent parameters needed to describe the classical behavior of the free energy density have no quantitative significance. The nonuniversal complexity of each pure fluid was then accounted for by introducing a significant number of adjustable parameters whose coupling with the two dimensionless crossover parameters $\bar{u}$ and Λ cannot be completely defined. Therefore, in spite of the correct introduction of a crossover function in the definition of variables and thermodynamic potentials, the only well-founded theoretical challenge of the nonasymptotic crossover applied to one-component fluids remains to account for the correct Ising-like universal features with a single crossover scale. The uniqueness of the crossover scale can thus be defined by introducing an arbitrary fixed value of the product $\bar{u}\Lambda$ [31]. The set $\{\bar{u}\,(\text{or }\Lambda), c_t, c_\rho\}$ appears "Ising-like" equivalent to the set $\{u_f^*, \vartheta, \psi\}$. This nonasymptotic crossover (which thus must match the asymptotic critical crossover close to the Wilson-Fisher fixed point [33]) does not have to be completely solved in regard to the most recent theoretical predictions of universal exponents [9,34] and universal amplitude ratios [9,10]. Moreover, the introduction of the single crossover parameter, which is then related to the mean-field concept of the Ginzburg number [29], adds conceptual difficulties to understanding the role of real microscopic parameters in controlling a true rescaled universal behavior in the whole crossover region [2,6,35].

Finally, since the van der Waals doctoral dissertation in 1873, the real difficulty for scientists interested in liquid-gas critical phenomena in pure fluids comes from the nonclassical (i.e., renormalizable) theories, which are not able to predict the location of the critical point, while the classical theories provide an incorrect location. This difficulty has generated a crucial experimental challenge where the determination of the two characteristic leading amplitudes and the characteristic crossover parameter of each pure fluid, and alternatively, but equivalently, the localization of its liquid-gas critical point on the p,V,T phase surface, remains mandatory.

Based on this recurrent situation, an alternative phenomenological way to characterize the asymptotic singular behavior of the one-component fluids was also formulated by Garrabos [12] as follows: "If you are able to locate a single liquid-gas critical point on the experimental $p,v_{\bar{p}},T$ phase surface of a fluid particle of mass $m_{\bar{p}}$, then you are also able to describe the asymptotic crossover around this isolated point." p is the pressure, T is the temperature, $v_{\bar{p}}=V/N=m_{\bar{p}}/\rho$ is the volume of the particle, and ρ is the (mass) density. Accordingly, a minimal set $Q_{c,a_{\bar{p}}}^{\min}$ made of four critical coordinates [12] [see below, Eq. (1)], provides unequivocal determination of four (two dimensional and two dimensionless) scale factors [see below, Eqs. (3)–(7)]. Then a scale dilatation method of the physical fields can be used to observe and quantify the master (i.e., unique) asymptotic crossover behavior of the $\{1f\}$ subclass [13,14]. The two dimensional critical parameters, i.e., $(\beta_c)^{-1}$ and α_c, take appropriate energy and length dimensions, respectively, to reduce the physical variables, the thermodynamic functions, and the correlation functions. The two dimensionless critical numbers, denoted Y_c and Z_c, are well-defined characteristic parameters of the critical interaction cell of volume $(\alpha_c)^d$. An additional adjustable parameter, denoted Λ_{qe}^*, accounts for quantum effects in light fluids at the critical temperature [36]. Conversely, when $Q_{c,a_{\bar{p}}}^{\min}$ and Λ_{qe}^* are known for the selected fluid, the asymptotic master behavior characterized by three master (i.e., constant) amplitudes is used to calculate the amplitude set $S_{\mathrm{A}}=\{\xi^+, \Gamma^+, a_\chi^+\}$ which characterizes the asymptotic singular behavior of this fluid. In addition to this intrinsic predictive power, another important characteristic attached to the scale dilatation method was the Ising-like analogy in its formal introduction of the two dimensionless scale factors Y_c and Z_c and the corresponding ones ϑ and ψ introduced by linear approximations in the massive renormalization scheme. The resulting explicit links between Y_c and ϑ on the one hand, and between Z_c and ψ on the other hand, are the basic tools expected for easy experimental analysis.

As a matter of fact, for each selected fluid belonging to the $\{1f\}$ subclass, this analogy can be useful to provide explicit estimation of the unknown scale factor set $S_{\mathrm{A}}^{\{MR\}}=\{u_f^*, \vartheta, \psi\}$ [or $\{(g_0)^{-1}, \vartheta, \psi\}$] of the theoretical crossover functions (then using the thermodynamic length scale unit α_c of the selected one-component fluid as a reference length a_0). Especially in the case of the unique form of the *mean* theoretical functions estimated in I, which incorporates the error-bar propagation of the minimum and maximum crossover functions reconsidered in [10], we can formulate an unambiguous modification of each theoretical crossover function valid for the $\{\Phi_3(1)\}$ class to exactly match the master two-term Wegner-like expansion valid within the Ising-like preasymptotic domain of the $\{1f\}$ subclass. Such a modified formulation was used to study the correlation length in the homogeneous domain of seven one-component fluids [37], the squared capillary length in the nonhomogeneous domain of 20 one-component fluids [38], and practical parachor correlations (i.e., equations expressing surface tension as a power law of the density difference between coexisting gas and liquid phases) [39].

The main objective of the present paper is to achieve the complete formulation of these master crossover functions that describe the master singular behavior of the $\{1f\}$ subclass, introducing only three well-defined dimensionless numbers to modify the mean crossover functions given in I. For this study, the analytical relations between the relevant scaling fields of the theoretical and master descriptions must be defined, demonstrating an unambiguous link between the experimental critical coordinate set $Q_{c,a_{\bar{p}}}^{\min}=\{T_c, p_c, v_{\bar{p},c}, \gamma_c'\}$ (or $Q_c^{\min}=\{(\beta_c)^{-1}, \alpha_c, Z_c, Y_c\}$), and the "free" parameter set $S_{\mathrm{A}}^{\{MR\}}=\{u_f^*, \vartheta, \psi\}$ obtained when the energy and length units are $(\beta_c)^{-1}$ and α_c, respectively, i.e., $Q_{c,\mathrm{th}}=\{T_c, \alpha_c, u_f^*, \vartheta, \psi\}$. As a final result of predictive interest, when the experimental critical set $Q_{c,a_{\bar{p}}}^{\min}$ is known, the asymptotic behavior of any singular part of the thermodynamic properties must be calculated using these master crossover functions without any adjustable parameter, in conformity with the two-scale-factor universality predicted by the massive renormalization scheme and in agreement with the phenomenological expectation of Garrabos.

The paper is organized as follows. In Sec. II the master description of the universal features within the Ising-like preasymptotic domain is recalled. In Sec. III, a brief presentation of the theoretical crossover functions for the correlation length and the susceptibility in the homogeneous phase is given to demonstrate the analytical matching with the master singular behavior provided by the scale dilatation method. Introducing a set (denoted $\mathcal{S}_{SF}^{\{1f\}}=\{\Theta^{\{1f\}},\mathbb{L}^{\{1f\}},\Psi^{\{1f\}}\}$) of three well-defined dimensionless scale factors which characterizes the $\{1f\}$ subclass, the unequivocal link between the set $\mathbb{S}_{\mathrm{A}}^{\{MR\}}=\{(\mathbb{Z}_{\xi}^{+})^{-1},(\mathbb{Z}_{\chi}^{+})^{-1},\mathbb{Z}_{\chi}^{1,+}\}$ of three theoretical amplitudes, which characterizes the $\{\Phi_3(1)\}$ universality class, and the set $\mathcal{S}_{\mathrm{A}}^{\{1f\}}=\{\mathcal{Z}_{\xi}^{+},\mathcal{Z}_{\chi}^{+},\mathcal{Z}_{\chi}^{1,+}\}$ of three master amplitudes, which characterizes the $\{1f\}$ subclass, is given before concluding in Sec. IV. Two appendixes deal with, first, a similar result obtained with other one-parameter crossover models, and, second, the determination of the crossover parameter beyond the preasymptotic domain using the well-known linear model of the parametric equation of state with effective exponents.

II. MASTER SINGULAR DESCRIPTION OF THE ONE-COMPONENT FLUID SUBCLASS

We recall hereafter the main information given in Refs. [12,13], where it was hypothesized that the raw materials needed to characterize nonquantum fluid critical phenomena are "localized" at the liquid-gas critical point [the supplementary information to characterize quantum fluids is given in Ref. [36]; see also below, Eqs. (9) and (10)]. Starting from the four critical coordinates of this critical point, we define the four scale factors which are needed to unambiguously determine the three dimensionless amplitudes that characterize the Ising-like preasymptotic domain of each one-component fluid. In the following, we show the master singular behavior of the isothermal compressibility, applying the scale dilatation method to the related physical quantities. That completes the master singular behavior of the correlation length [37] and closes the master singular description of the $\{1f\}$ subclass in conformity with the two-scale-factor universality of the $\{\Phi_3(1)\}$ universality class.

A. The minimal set of critical parameters

The minimal set of four critical coordinates needed to localize the single liquid-gas critical point and its tangent plane on the experimental phase surface of the normalized equation of state $\Phi_{a_{\bar{p}}}^{p}(p,v_{\bar{p}},T)=0$ reads as follows:

$$Q_{c,a_{\bar{p}}}^{\min}=\{T_c,p_c,v_{\bar{p},c},\gamma_c'\},\tag{1}$$

where $v_{\bar{p},c}=V/N_c=m_{\bar{p}}/\rho_c$ is the critical volume per particle (V is the total volume, N_c is the total critical number of particles, and ρ_c is the critical density), and

$$\gamma_c'=\left(\frac{\partial p}{\partial T}\right)_{v_{\bar{p}}=v_{\bar{p},c};T=T_c}=\left(\frac{dp_{\mathrm{sat}}}{dT}\right)_{T=T_c}\tag{2}$$

is the common critical direction at the critical point, of the critical isochore and the saturation pressure curve of equation $P_{\mathrm{sat}}(T)$ in the $p;T$ diagram. γ_c' is related to the Riedel factor [40], $\alpha_{R,c}=(d\log p_{\mathrm{sat}}/d\log T)_{T=T_c}$, through the relation $\alpha_{R,c}=(T_c/p_c)\gamma_c'$. The subscript c refers to a critical quantity. From Eq. (1), we can construct a more convenient set,

$$Q_c^{\min}=\{(\beta_c)^{-1},\alpha_c,Y_c,Z_c\},\tag{3}$$

making use of the following four scale factors:

$$(\beta_c)^{-1}=k_BT_c\sim[\text{energy}],\tag{4}$$

$$\alpha_c=\left(\frac{k_BT_c}{p_c}\right)^{1/d}\sim[\text{length}],\tag{5}$$

$$Z_c=\frac{p_cv_{\bar{p},c}}{k_BT_c}\sim[\text{dimensionless}],\tag{6}$$

$$Y_c=\left(\gamma_c'\frac{T_c}{P_c}\right)-1\sim[\text{dimensionless}].\tag{7}$$

$(\beta_c)^{-1}$ and α_c are used to express dimensionless quantities. α_c is a measure of the effective range of the microscopic short-range molecular interaction (Lennard-Jones-like in nature) [32]. Z_c is the critical compression factor, while $Y_c=\alpha_{R,c}-1$. In the above dimensionless form of the thermodynamic functions normalized per particle, $1/Z_c$ is the number of particles in the volume

$$v_{c,I}=(\alpha_c)^d,\tag{8}$$

which corresponds to the volume of the *critical interaction cell* [12].

This actual set $Q_c^{\min}$ (made of measured critical parameters) refers to the characteristic range of the microscopic molecular interaction in "classical" (i.e., nonquantum) fluids [here the molecular interaction range is measured by α_c of Eq. (5)]. To include quantum fluids in the one-component fluid subclass [36], we need the phenomenological introduction of a supplementary adjustable parameter, denoted Λ_{qe}^{*}, which accounts for the quantum effects at this microscopic length scale of the effective molecular interaction. The (dimensionless) parameter Λ_{qe}^{*} [36] is given by

$$\Lambda_{qe}^{*}=1+\lambda_c\tag{9}$$

with

$$\lambda_c=\lambda_{q,f}\frac{\Lambda_{T,c}}{\alpha_c}.\tag{10}$$

$\lambda_{q,f}$ (with $\lambda_{q,f}>0$), is thus a nonuniversal adjustable number which accounts for statistical contributions due to the nature (boson, fermion, etc.) of the quantum particle. $\Lambda_{T,c}=h_P/(2\pi m_{\bar{p}}k_BT_c)^{1/2}$ is the de Broglie thermal wave vector at $T=T_c$, and h_P is the Planck constant (the subscript P is here added to make a distinction from the theoretical ordering field noted h).

B. Thermodynamic characterization of the critical interaction cell

We introduce the (mass) density variable $\rho=Nm_{\bar{p}}/V=m_{\bar{p}}/v_{\bar{p}}$ and we consider the usual compression factor

$$Z = \frac{pV}{Nk_BT} = \frac{pm_{\bar{p}}}{\rho k_BT} \tag{11}$$

generally expressed in thermodynamic textbooks [41] as a function $Z(T^*,\tilde{\rho})$ of the two dimensionless variables $T^*=T/T_c$ and $\tilde{\rho}=\rho/\rho_c$. Here we note the distinction underlined by using a superscript asterisk for a dimensionless quantity obtained only from $(\beta_c)^{-1}$ and α_c units, and a tilde for a dimensionless quantity which can refer to a specific amount of matter, by introducing then the critical density $\rho_c=m_{\bar{p}}/v_{\bar{p},c}$. In practice, the two dimensionless critical parameters

$$y^*_{\bar{p},c} = \left[\left(\frac{\partial Z}{\partial T^*}\right)_{\tilde{\rho}=\tilde{\rho}_c}\right]_{\rm CP} = Y_cZ_c, \tag{12}$$

$$z^*_{\bar{p},c} = \left[\left(\frac{\partial Z}{\partial \tilde{\rho}}\right)_{T^*=T^*_c}\right]_{\rm CP} = -Z_c \tag{13}$$

are the two preferred directions [42] of the characteristic surface related to the total grand potential $J(T,V,\mu_{\bar{p}})$, expressed per particle. $\mu_{\bar{p}}$ is the chemical potential per particle related to the specific (i.e., per mass unit) chemical potential μ_ρ by $\mu_\rho=\mu_{\bar{p}}/m_{\bar{p}}$ (the subscript ρ refers to a specific property). Therefore, it is essential to note that $y^*_{\bar{p},c}=Y_cZ_c$ and $z^*_{\bar{p},c}=-Z_c$ are the dimensionless forms of two characteristic molecular (i.e., per particle) quantities.

As a matter of fact, when we consider the thermodynamic description of a one-component fluid at constant volume of matter, the total grand potential $J(T,V,\mu_{\bar{p}})=-p(T,\mu_{\bar{p}})V$ takes, alternatively but equivalently, the role of the total Gibbs free energy $G(T,p,N)=\mu_{\bar{p}}(T,p)N$ usually considered in the thermodynamic description of a one-component fluid of a constant amount of matter. The external pressure $p(T,\mu_{\bar{p}})=-J/V$ of the container maintained at constant volume, in contact with a particle reservoir, is then the thermodynamic potential equivalent to the molecular chemical potential $\mu_{\bar{p}}(T,p)=G/N$ of the fluid at constant amount of matter, in contact with a volume reservoir. Therefore, considering the normalization per particle of the thermodynamic description of a one-component fluid at constant volume (i.e., $v_{\bar{p}}$=const), the molecular (i.e., per particle) grand potential reads $j_{\bar{p},v_{\bar{p}}=\rm const}(T)=-p(T,\mu_{\bar{p}})v_{\bar{p}}$. Using the associated opposite Massieu form, $z_{\bar{p},v_{\bar{p}}=\rm const}=-[(j_{\bar{p},v_{\bar{p}}=\rm const})/T]$, and the universal Boltzmann constant k_B as unique unit, we obtain the following dimensionless form:

$$z^*_{\bar{p},v_{\bar{p}}=\rm const} = \frac{z_{\bar{p},v_{\bar{p}}=\rm const}}{k_B} = \frac{p(T,\mu_{\bar{p}})v_{\bar{p}}}{k_BT} \equiv Z, \tag{14}$$

which demonstrates that the compression factor Z of a constant amount of fluid matter maintained at constant volume (i.e., a one-component fluid monitored by the temperature along an isochore) is indeed a dimensionless molecular potential [43]. For the critical filling $N=N_c$ of this isochoric container, we obtain $-[(j_{\bar{p},v_{\bar{p}}=\rm const})/T]^*_{N=N_c}=Z_{\tilde{\rho}=1}\equiv(p^*/T^*)v^*_{\bar{p},c}=(T_c/P_c)[p(T)/T]_{\rho=\rho_c}Z_c$. Here, $[p(T)/T]_{\rho=\rho_c}$ acts as a first characteristic (i.e., independent) equation of state for a critical *isochoric* fluid, where the two extensive variables V and N_c are fixed [i.e., a critical fluid at $\tilde{\rho}=1$ in contact with a thermostat (i.e., an energy reservoir) of constant energy k_BT]. Multiplying the particle property $y^*_{\bar{p},c}$ by the number of particles $1/Z_c$ in the critical interaction cell, it appears that the critical quantity $Y_c=\{[\partial(p^*/T^*)/\partial T^*]_{v_{\bar{p}}=v_{\bar{p},c}}\}_{\rm CP}$ is readily a characteristic parameter of the critical interaction cell.

Now considering a critical *isothermal* fluid where the two variables V and T_c are fixed (i.e., a critical fluid at $T^*=1$, filling a constant total volume V=const, thermostated at constant critical energy k_BT_c, in contact with a particle reservoir), we obtain $-[(j_{\bar{p},V=\rm const}/T)]^*_{T=T_c}=Z_{T^*=1}\equiv(p^*/1)v^*_{\bar{p}}=(1/k_B)\{[p(\mu_{\bar{p}})]_{T=T_c}/T_c\}v_{\bar{p}}$. Here, $[p(\mu_{\bar{p}})/T]_{T=T_c}$ acts as a second characteristic (i.e., independent) equation of state for a critical isothermal one-component fluid. In such a thermostated container at fixed total volume, we underline the fact that the only independent extensive variable to monitor the thermodynamic fluid state is the number of particles N which fixes the equilibrium mean value of the molecular chemical potential $\mu_{\bar{p}}$. For $N=N_c$, at $T^*=1$ (i.e., the critical point condition), the critical chemical potential per particle takes the value $\mu_{\bar{p},c}$, such that $(z^*_{\bar{p},v_{\bar{p}}=v_{\bar{p},c}})_{T=T_c}=p_c(\mu_{\bar{p},c})v_{\bar{p},c}/k_BT_c=Z_c$. Within the critical interaction cell filled with $1/Z_c$ particles, the normalized grand potential takes the master critical value $(1/Z)_c(z^*_{\bar{p},v_{\bar{p}}=v_{\bar{p},c}})_{T=T_c}=1$.

Therefore, as an essential microscopic meaning related to Eq. (8), we note that the critical set $Q_c^{\rm min}$ of Eq. (3) characterizes the master thermodynamic information contained in the critical interaction cell volume of each one-component fluid at the critical point.

Finally, we summarize the two main constraints for the thermodynamic description of a one-component fluid near its gas-liquid critical point.

(i) The dimensionless reduction of the variables is mandatorily made by using the two dimensional factors $(\beta_c)^{-1}$ and α_c of Eqs. (4) and (5), respectively (see also Ref. [11]).

(ii) The thermodynamic properties expressed per particle are better suited to understand the microscopic nature of the two dimensionless numbers Y_c and Z_c. That permits also to estimate easily a dimensionless property which refers to the corresponding dimensionless property of the critical interaction cell. Then the thermodynamic origin of the corresponding dimensionless master (i.e., unique) constants is well identified.

C. The relevant physical fields crossing the liquid-gas critical point

Such a constrained dimensionless thermodynamic description is appropriately obtained from the grand canonical statistical distribution, considering a one-component fluid in contact with a "particle-energy" reservoir maintained at constant total volume V. Selecting the thermodynamic nature (fixing either the energy level k_BT or the particle amount N) of the reservoir reaching the critical point (at either constant critical density, or constant critical temperature), the normalized thermodynamic potential is then related to the intensive quantities $[p(T)/T]_{\rho=\rho_c}$ or $[p(\mu_{\bar{p}})/T]_{T=T_c}$. In addition to the temperature variable conjugated to the total entropy, the other natural (intensive) variable is the chemical potential per particle $\mu_{\bar{p}}$, conjugated to the natural fluctuating total number of particles N (leading to the fluctuating number density $n=N/V$). Therefore, the two relevant physical fields, either to express the finite distance to the critical point, or to cross it, along the critical isochore and along the critical isotherm, are

$$\Delta\tau^* = k_B\beta_c(T-T_c) \tag{15}$$

and

$$\Delta h^* = \beta_c(\mu_{\bar{p}} - \mu_{\bar{p},c}), \tag{16}$$

respectively. Using the thermodynamic description per particle, the order parameter density is then proportional to the critical number density difference $n-n_c$ ($n_c=N_c/V$ is the critical number density), and the associated dimensionless order parameter density is given by [13,44]

$$\Delta m^* = (n-n_c)(\alpha_c)^d. \tag{17}$$

We retrieve the distinction (using asterisk or tilde), either between Δh^* [see Eq. (16)] and the reduced specific chemical potential difference

$$\Delta\tilde{\mu} = (\mu_\rho - \mu_{\rho,c})\frac{\rho_c}{p_c}, \tag{18}$$

or between Δm^* [see Eq. (17)] and the reduced density difference:

$$\Delta\tilde{\rho} = \frac{\rho-\rho_c}{\rho_c}, \tag{19}$$

where $\Delta\tilde{\mu}$ and $\Delta\tilde{\rho}$ were customarily defined in a critical fluid description using specific properties and practical dimensionless variables $\tilde{x}=x/x_c$ (see, for example, Refs. [7,17]. The corresponding relations can be expressed as follows:

$$\Delta h^* = Z_c\Delta\tilde{\mu}, \tag{20}$$

$$\Delta m^* = \frac{1}{Z_c}\Delta\tilde{\rho}, \tag{21}$$

which show that the dimensionless isothermal susceptibilities $\chi_T^*=[\partial(\Delta m^*)/\partial(\Delta h^*)]_{T^*}$ and $\tilde{\chi}_T=[\partial(\Delta\tilde{\rho})/\partial(\Delta\tilde{\mu})]_{\tilde{T}}$ differ by a factor $(1/Z_c)^2$. Equations (20) and (21) illustrate the primary role of Z_c in the dimensionless form of thermodynamics, due to the fact that $(Z_c)^{-1}$, i.e., the particle number within the critical interaction cell volume, is then equivalent to n_0 and accounts for extensivity of the critical fluid.

D. The scale dilatation method for the {1*f*} subclass

The scale dilatation method [13,14,36] uses explicit analytical transformations of each physical field $\Delta\tau^*$ and Δh^* given by the equations

$$\mathcal{T}_{\rm qf}^* \equiv \mathcal{T}^* = Y_c|\Delta\tau^*|, \tag{22}$$

$$\mathcal{H}_{\rm qf}^* = (\Lambda_{qe}^*)^2\mathcal{H}^* = (\Lambda_{qe}^*)^2(Z_c)^{-d/2}|\Delta h^*|, \tag{23}$$

where $\mathcal{T}_{\rm qf}^* \equiv \mathcal{T}^*$ is the renormalized thermal field, and $\mathcal{H}_{\rm qf}^*$ is the renormalized ordering field. The subscript "qf" distinguishes between a quantity that refers to a quantum fluid (i.e., $\Lambda_{qe}^* \neq 1$) and one that refers to a nonquantum fluid (i.e., $\Lambda_{qe}^*=1$) [36]. Accordingly, the analytic transformation between the physical order parameter density Δm^* and the renormalized order parameter density $\mathcal{M}_{\rm qf}^*$, reads as follows [13,36,44]:

$$\mathcal{M}_{\rm qf}^* = \Lambda_{qe}^*\mathcal{M}^* = \Lambda_{qe}^*(Z_c)^{d/2}|\Delta m^*|. \tag{24}$$

Introducing then the dimensionless correlation length $\xi^* = \xi/\alpha_c$, the renormalized correlation length $\ell_{\rm qf}^*$ is given by the equation

$$\ell_{\rm qf}^* = (\Lambda_{qe}^*)^{-1}\ell^* = (\Lambda_{qe}^*)^{-1}\xi^*, \tag{25}$$

which preserves the same length unit for thermodynamic and correlation functions (with $\ell^* \equiv \xi^*$ for the nonquantum fluid case).

The master asymptotic singular behavior of $\ell_{\rm qf}^*(\mathcal{T}^*)$ was studied in [37]. Specifically, when $\mathcal{T}_{\rm qf}^* \to 0$ within the preasymptotic domain, the observed asymptotic divergence of $\ell_{\rm qf}^*$ was represented by the following (two-term) Wegner expansion:

$$\ell_{\rm qf}^* = \mathcal{Z}_\xi^+(\mathcal{T}^*)^{-\nu}[1+\mathcal{Z}_\xi^{1,+}(\mathcal{T}^*)^\Delta] \tag{26}$$

where $\nu=0.630\,387\,5$ and $\Delta=0.501\,89$ [9]. The leading amplitude $\mathcal{Z}_\xi^+=0.5729$ and the first confluent amplitude $\mathcal{Z}_\xi^{1,+}=0.38$ have master (i.e., unique) values for the {1*f*} subclass. The associated asymptotic singular behavior of the physical correlation length was given by

$$\xi_{\rm expt}(\Delta\tau^*) = \xi_0^+(\Delta\tau^*)^{-\nu}[1+a_\xi^+(\Delta\tau^*)^\Delta]. \tag{27}$$

The term-to-term comparison of (master) Eq. (26) and (physical) Eq. (27), results in the following amplitude combinations:

$$\frac{\xi_0^+}{\alpha_c} = \xi^+ = \Lambda_{qe}^*(Y_c)^{-\nu}\mathcal{Z}_\xi^+, \tag{28}$$

$$a_\xi^+ = \mathcal{Z}_\xi^{1,+}(Y_c)^\Delta. \tag{29}$$

Applying now the scale dilatation method to any physical (thermodynamic) property $P(\Delta\tau^*)$, the master singular behavior for the renormalized (thermodynamic) property $\mathcal{P}_{\rm qf}^*(\mathcal{T}^*)$ can also be observed and represented within the preasymptotic domain by the restricted expansion

$$\mathcal{P}_{\rm qf}^* = \mathcal{Z}_P^\pm(\mathcal{T}^*)^{-e_P}[1+\mathcal{Z}_P^{1,\pm}(\mathcal{T}^*)^\Delta], \tag{30}$$

where $\mathcal{Z}_P^\pm$ and $\mathcal{Z}_P^{1\pm}$ are two master constants for any one-component fluid. Therefore, to close the master description in conformity with the universal features estimated within this Ising-like preasymptotic domain, i.e., to have only three independent master amplitudes among $\mathcal{Z}_\xi^+$, $\mathcal{Z}_\xi^{1,+}$, $\mathcal{Z}_P^\pm$, and $\mathcal{Z}_P^{1\pm}$, we complete the representation of the master correlation length with the one of the master susceptibility $\mathcal{X}_{\rm qf}^*$ obtained from the master order parameter density $\mathcal{M}_{\rm qf}^*$ and master ordering field $\mathcal{H}_{\text{qf}}^{*}$, using the thermodynamic definition $\mathcal{X}_{\text{qf}}^{*}=(\partial\mathcal{M}_{\text{qf}}^{*}/\partial\mathcal{H}_{\text{qf}}^{*})_{\mathcal{T}^{*}}$. $\mathcal{X}_{\text{qf}}^{*}$ is related to the dimensionless isothermal susceptibility $\chi_{T}^{*}=[\partial(\Delta m^{*})/\partial(\Delta h^{*})]_{\Delta\tau^{*}}$ by the following equations:

$$\mathcal{X}_{\text{qf}}^{*}=(\Lambda_{qe}^{*})^{2-d}\varkappa_{T}^{*}=(\Lambda_{qe}^{*})^{2-d}(Z_c)^{d}\chi_{T}^{*}. \quad (31)$$

As previously mentioned for the critical isochore case, $\chi_{T}^{*}(n_c^{*})=\tilde{\chi}_{T}(\tilde{\rho}=1)/(Z_c)^2$, while $\tilde{\chi}_{T}(\tilde{\rho}=1)\equiv\kappa_{T}^{*}(\tilde{\rho}=1)$ {with $\tilde{\chi}_{T}=[\partial(\Delta\tilde{\rho})/\partial(\Delta\tilde{\mu})]_{\Delta\tau^{*}}=(\tilde{\rho})^{2}\kappa_{T}^{*}$}, where κ_{T}^{*} is the dimensionless isothermal compressibility $\kappa_{T}^{*}=[1/\beta_c(\alpha_c)^{d}][(1/\rho)(\partial\rho/\partial p)_T]=p_c\kappa_T$ [with $\kappa_T=(1/\rho)(\partial\rho/\partial p)_T$]. Therefore, the master susceptibility can also be related to the dimensionless isothermal compressibility by

$$\mathcal{X}_{\text{qf}}^{*}=(\Lambda_{qe}^{*})^{2-d}Z_c\kappa_{T}^{*}. \quad (32)$$

The master asymptotic singular behavior of $\mathcal{X}_{\text{qf}}^{*}$ reads as follows:

$$\mathcal{X}_{\text{qf}}^{*}=\mathcal{Z}_{\chi}^{+}(\mathcal{T}^{*})^{-\gamma}[1+\mathcal{Z}_{\chi}^{1,+}(\mathcal{T}^{*})^{\Delta}] \quad (33)$$

where $\gamma=1.239\,593\,5$ [9]. The master values of the leading and confluent amplitudes are $\mathcal{Z}_{\chi}^{+}=0.119\,75$ and $\mathcal{Z}_{\chi}^{1,+}=0.555$, respectively, where the universal value of the confluent amplitude ratio $\mathcal{Z}_{\xi}^{1,+}/\mathcal{Z}_{\chi}^{1,+}=0.679\,19$ is given in Ref. [10]. The associated asymptotic singular behavior of the isothermal compressibility reads as follows:

$$\kappa_{T,\text{expt}}^{*}(\Delta\tau^{*})=\Gamma^{+}(\Delta\tau^{*})^{-\gamma}[1+a_{\chi}^{+}(\Delta\tau^{*})^{\Delta}]. \quad (34)$$

The term-to-term comparison of (master) Eq. (33) and (physical) Eq. (34) leads to the following amplitude estimations:

$$\Gamma^{+}=(\Lambda_{qe}^{*})^{d-2}(Z_c)^{-1}(Y_c)^{-\gamma}\mathcal{Z}_{\chi}^{+}, \quad (35)$$

$$a_{\chi}^{+}=\mathcal{Z}_{\chi}^{1,+}(Y_c)^{\Delta}, \quad (36)$$

with

$$\frac{\mathcal{Z}_{\xi}^{1,+}}{\mathcal{Z}_{\chi}^{1,+}}=\frac{a_{\xi}^{+}}{a_{\chi}^{+}}=0.679\,19. \quad (37)$$

The expected asymptotic collapse of the fluid susceptibility on a master curve due to the scale dilatation method is illustrated in Fig. 1 (log-log scale). The raw data are reported in Fig. 1(a) to easily distinguish between singular behavior of κ_T (expressed in Pa^{-1}) as a function of $T-T_c$ (expressed in K) for each one-component fluid [see the fluid color key inserted in Fig. 1(c)]. Figure 1(b) illustrates the differences between the corresponding dimensionless behaviors $\kappa_{T}^{*}(\Delta\tau^{*})$ and confirms the failure of results provided by the two-parameter corresponding state principle. This figure also shows the failure of mean-field-like behavior predicted from the van der Waals (vdW) equation of state, which is here represented by the black full curve of equation $\kappa_{T,\text{vdW}}^{*}(\Delta\tau^{*})^{\gamma_{\text{vdW}}}=\Gamma_{\text{vdW}}^{+}=\frac{1}{6}$, with $\gamma_{\text{vdW}}=\gamma_{\text{MF}}=1$. On the other hand, Fig. 1(c) demonstrates the collapse of $\mathcal{X}_{\text{qf}}^{*}(\mathcal{T}^{*})$ on a master curve where the scatter corresponds to the estimated κ_T precision (5–10 %) for each fluid. We underline the combination of the "scaling" and "extensive" roles of the characteristic factor Z_c in the renormalization [see Eqs. (31) and (32)] of the ordinate axis of Fig. 1(c) (compare, for example, with Fig. 3 of Ref. [21] or with Fig. 2 of Ref. [23]). The complementary materials for complete analysis of this Fig. 1(c) will be given below and in Appendix B.

Adding the correlation length results given in Fig. 1c of Ref. [37] to the present isothermal susceptibility results, we have closed the asymptotic master behavior generated by the scale dilatation method, in conformity with the universal features of the Ising-like systems, as illustrated by the results given in Table I. As a matter of fact, knowing $\mathcal{Z}_{\xi}^{+}=0.5729$, $\mathcal{Z}_{\chi}^{+}=0.11975$, and $\mathcal{Z}_{\chi}^{1,+}=0.555$, all the values of the master amplitudes given in columns 3 and 5 of Table I have been calculated only using universal values of amplitude ratios and amplitude combinations estimated by the renormalization scheme (for their numerical values, see the related Table II below). In addition, knowing the four critical point coordinates of any pure fluid, the values of the physical amplitudes of any two-term Wegner-like expansion can be calculated from the relations given in columns 7 and 8 of Table I.

We summarize the main interest of this Ising-like master description of the $\{1f\}$ subclass in relation to our Introduction (and our selected set $\{\Delta;\nu;\gamma\}$ of three independent universal exponents). We have successively introduced the following sets.

(i) The physical dimensionless amplitude set

$$S_{\text{A}}=\{a_{\chi}^{+};\xi^{+};\Gamma^{+}\} \quad (38)$$

which characterizes the physical Ising-like universal features of each selected pure fluid having the critical set $\{Q_c^{\min};\Lambda_{qe}^{*}\}$.

(ii) The corresponding dimensionless scale factor set

$$S_{\text{SF}}^{\{1f\}}=\{Y_c;Z_c;\Lambda_{qe}^{*}\} \quad (39)$$

which characterizes the master (i.e., unique) features of the critical interaction cell of each selected pure fluid having $(\beta_c)^{-1}$ and α_c as energy and length units, respectively.

(iii) The master dimensionless amplitude set

$$\mathcal{S}_{\text{A}}^{\{1f\}}=\left\{\begin{array}{ccc}\mathcal{Z}_{\chi}^{1,+} & = & 0.555\\ \mathcal{Z}_{\xi}^{+} & = & 0.5729\\ \mathcal{Z}_{\chi}^{+} & = & 0.11975\end{array}\right\} \quad (40)$$

which characterizes the master Ising-like universal features of the $\{1f\}$ subclass. The three independent relations. i.e., Eqs. (28), (35), and (36), connecting these three previous sets, can be written in the following condensed functional form:

$$\mathcal{S}_{\text{A}}^{\{1f\}}=\{S_{\text{A}}\times\mathcal{U}(S_{\text{SF}}^{\{1f\}})\}_{(\beta_c)^{-1},\alpha_c,\Lambda_{qe}^{*}}, \quad (41)$$

where the function $\mathcal{U}(S_{\text{SF}}^{\{1f\}})$ takes a universal scaling form of the two (fluid-dependent) scale factors Y_c and Z_c. As shown in Table I(a), all the master amplitudes of the two-term master crossover functions can be estimated from these three independent master amplitudes, using the universal mean values of their ratios and combinations estimated in Ref. [10] [which are also explicitly given in Table II(a) below].

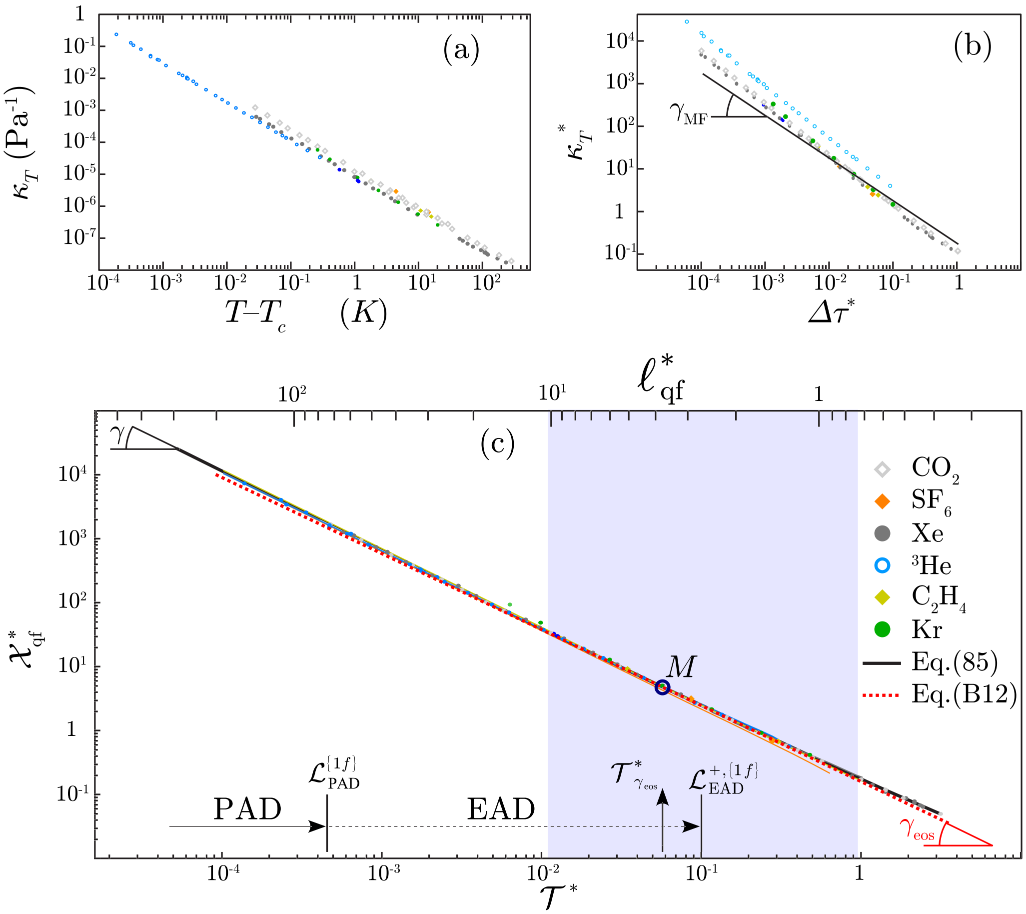

FIG. 1. (Color online) Singular behavior of the isothermal compressibility of the one-component fluids. (a) κ_T as a function of $T-T_c>0$ (log-log scale), along the critical isochore for Xe, Kr, ^{3}He, SF_6, CO_2, and C_2H_4 (see inserted key for fluid color index). (b) log-log plot of $\kappa_T^*(\Delta\tau^*)$: black full curve, mean-field behavior of equation $\kappa^*_{T,\mathrm{vdW}}=(1/6)(\Delta\tau^*)^{-\gamma_{\mathrm{MF}}}$ with $\gamma_{\mathrm{MF}}=1$. (c) Matched master behavior (log-log scale) of the renormalized susceptibility $\mathcal{X}^*_{\mathrm{qf}}=(\Lambda^*_{qe})^{2-d}Z_c\kappa^*_T$ [see Eqs. (32)], as a function of the renormalized thermal field $\mathcal{T}^*$ [see Eqs. (22)]: black full curve, Eq. (85); red dashed curve with slope $\gamma_{\mathrm{EOS}}=1.19$, tangent of Eq. (B12) at the point M (see text and Appendix B); (full) arrow (label PAD), master extension of the Ising-like preasymptotic domain of Eq. (102); (dashed) arrow (label EAD), effective extension of the extended asymptotic domain of Eq. (114) corresponding to $\ell^*_{\mathrm{qf}}=(\Lambda^*_{qe})^{-1}\xi/\alpha_c\gtrsim 3$ (see Ref. [37]); blued area, master correlation length range $10.5\lesssim\ell^*_{\mathrm{qf}}\lesssim 0.73$ (thermal field range $1.9\times10^{-2}\lesssim\mathcal{T}^*\lesssim 1$) discussed in Appendix B.

Accordingly, when the four critical coordinates of a one-component fluid are known, all the related physical amplitudes can be estimated from the equations given in Table I(b), satisfying then to the two-scale-factor universality of the $\{\Phi_3(1)\}$ class. We note that, to provide this predictive nature, we have used xenon as a standard critical fluid, selecting its three "measured" physical amplitudes (namely, Γ^+, B, and a_χ^+ in the usual notation) to estimate the corresponding master amplitudes ($\mathcal{Z}_\chi^+$, $\mathcal{Z}_M$, and $\mathcal{Z}_\chi^{1,+}$), labeled with an asterisk in Table I(a) (see Refs. [13,14,46] for details). However, the effective extension range where the master behavior is observed, as an explicit criterion which defines the preasymptotic range where the two-term Wegner-like expansion is valid, remains not easy to estimate precisely using only the scale dilatation method. These two problems can be solved using a master modification of the mean crossover functions [8] obtained from the massive renormalization scheme, as shown in the next section.

III. MASTER MODIFICATIONS OF THE MEAN CROSSOVER FUNCTIONS

A. System-dependent parameters of the mean crossover functions

For the $\{\Phi_3(1)\}$ class, the mean crossover functions $F_P(t,h=0)$ describing the crossover behavior of the theoretical properties $P_{\mathrm{th}}(t)$ as a function of the relevant (temperaturelike) field t, for zero value of the external ordering (magneticlike) field h, are given in detail in I. All the theoretical *ad hoc* functions $F_P(t)$ have the same following functional form whatever P_{th}:

$$F_P(|t|,h=0)=\mathbb{Z}_P^{\pm}|t|^{e_P}\Pi_P(|t|^{D(|t|)}), \tag{42}$$

where e_P and $\mathbb{Z}_P^{\pm}$ are the respective critical exponent and leading amplitude which characterize the asymptotic Ising-like behavior of P_{th} when $|t|\to0^{\pm}$ [the superscript $\pm$ indicates the possible homogeneous (+, i.e., $t>0$) and nonhomo-

TABLE I. Amplitude values of the two-term Wegner-like expansion for (a) master property ($\mathcal{P}^*_{\mathrm{qf}}$) and (b) corresponding physical property (P^*_{expt}) along the critical isochore of any one-component fluid. Rows 2 and 3, correlation length; rows 4 and 5, susceptibility; rows 6 and 7, specific heat; row 8, order parameter density. Columns 2 and 4, independent master amplitudes [see Eq. (40); columns 3 and 5, amplitude values calculated in conformity with the theoretical universal features estimated within the Ising-like preasymptotic domain [9,10] [see also Table II(a) below]; asterisk indicates an "experimental" master value estimated using xenon as a standard critical fluid (see text and Refs. [13,14,46]); columns 7 and 8, related physical amplitudes when $Q_c^{\min}=\{(\beta_c)^{-1},\alpha_c,Y_c,Z_c\}$ and Λ^*_{qe} are known for a selected one-component fluid.

(a) $\mathcal{P}^*_{\mathrm{qf}}$	$\{\mathcal{Z}^+_\chi;\mathcal{Z}^+_\xi\}$	$\mathcal{Z}^\pm_P$	$\{\mathcal{Z}^{1,+}_\chi\}$	$\mathcal{Z}^{1,\pm}_P$	(b) P^*_{expt}	$P^{0,\pm}$	$P^{1,\pm}$
ℓ^*_{qf}	$\mathcal{Z}^+_\xi=0.5729$			$\mathcal{Z}^{1,+}_\xi=0.37695$	ξ^*	$\xi^\pm=\xi_0^\pm/\alpha_c=\Lambda^*_{qe}(Y_c)^{-\nu}\mathcal{Z}^\pm_\xi$	$a^\pm_\xi=\mathcal{Z}^{1,\pm}_\xi(Y_c)^\Delta$
		$\mathcal{Z}^-_\xi=0.292297$		$\mathcal{Z}^{1,-}_\xi=0.34268$			
$\mathcal{X}^*_{\mathrm{qf}}$	$\mathcal{Z}^+_\chi=0.11975^*$		$\mathcal{Z}^{1,+}_\chi=0.555^*$		κ^*_T	$\Gamma^\pm=(\Lambda^*_{qe})^{d-2}(Z_c)^{-1}(Y_c)^{-\gamma}\mathcal{Z}^\pm_\chi$	$a^\pm_\chi=\mathcal{Z}^{1,\pm}_\chi(Y_c)^\Delta$
		$\mathcal{Z}^-_\chi=0.0250$		$\mathcal{Z}^{1,-}_\chi=2.58741$			
$\mathcal{C}^*_{\mathrm{qf}}$		$\mathcal{Z}^+_C=0.104324$		$\mathcal{Z}^{1,+}_C=0.522743$	c^*_V	$\frac{A^\pm}{\alpha}=(\Lambda^*_{qe})^{-d}(Y_c)^{2-\alpha}\mathcal{Z}^\pm_C$	$a^\pm_C=\mathcal{Z}^{1,\pm}_C(Y_c)^\Delta$
		$\mathcal{Z}^-_C=0.194344$		$\mathcal{Z}^{1,-}_C=0.384936$			
$\mathcal{M}^*_{\mathrm{qf}}$		$\mathcal{Z}_M=0.4665^*$		$\mathcal{Z}^1_M=0.4995$	$\Delta\tilde{\rho}_{LV}$	$B=(\Lambda^*_{qe})^{-1}(Z_c)^{-\frac{1}{2}}(Y_c)^\beta\mathcal{Z}_M$	$a_M=\mathcal{Z}^1_M(Y_c)^\Delta$

geneous (–, i.e., $t<0$) domains]. The (confluent) function Π_P, which reaches the Ising-like limit $\Pi_P\to 1$ when $|t|\to 0^\pm$, is a simplified three-term product of the variable $|t|^{D(|t|)}$, accounting for the required fit quality of each theoretical calculation of the Wegner-like expansion done point by point in the discretized complete $|t|=\{\infty,0\}$ range (see I for details). $D(t)$ is a universal mean crossover function for the confluent exponents Δ and Δ_{MF} which reads

$$D(t)=\frac{\Delta_{\mathrm{MF}}S_2\sqrt{t}+\Delta}{S_2\sqrt{t}+1}, \qquad (43)$$

such that $D(t=1/(S_2)^2)=(\Delta_{\mathrm{MF}}+\Delta)/2$ (independently of the phase domain and the singular property). We recall that the theoretical singular behavior of the specific heat is particular since it involves and additive critical constant to the corresponding Eq. (42). However, as already noted, this critical constant is nonessential for our present purpose.

Within the Ising-like preasymptotic domain, only the contribution of the first confluent term of the Wegner-like expansion is significant and any function Π_P can be approximated by $\Pi_P\simeq 1+\mathbb{Z}_P^{1,\pm}|t|^\Delta$, where the amplitudes $\mathbb{Z}_P^{1,\pm}$ are defined in I. Each crossover function of Eq. (42) is then Ising-like equivalent to its restricted (two-term) Wegner expansion and, correspondingly, conforms to the universal features estimated by the massive renormalization scheme within this Ising-like preasymptotic domain. That means that only two leading amplitudes among all $\mathbb{Z}_P^\pm$, and one confluent amplitude among all $\mathbb{Z}_P^{1,\pm}$, are independent and characteristic of the complete set of crossover functions, as illustrated in

TABLE II. Amplitude values of the two-term Wegner-like expansion for (a) theoretical property (P_{th}) and (b) physical (fluid) property (P^*_{expt}) along the critical isochore. Rows 2 and 3, correlation length; rows 4 and 5, susceptibility; rows 6 and 7, the specific heat; row 8, order parameter density. All these amplitude values are in conformity with the theoretical universal values given in columns 3 and 5 (see text and Ref. [10]). The physical amplitudes ψ_ρ are related to the theoretical amplitudes through the (fluid-dependent) scale factor set $S_{\mathrm{SF}}^{\{MR\}}=\{\vartheta;\mathbb{L}^{\{1f\}};\psi_\rho\}$ [see Eq. (52)] when the fluid-dependent parameters $(\beta_c)^{-1}$ [see Eq. (4)], α_c [see Eq. (5)], and Λ^*_{qe} [see Eq. (9)], are known.

(a) P_{th}	$\{\mathbb{Z}^+_\chi;\mathbb{Z}^+_\xi\}$	$\mathbb{Z}^\pm_P$	$\{\mathbb{Z}^{1,+}_\chi\}$	$\mathbb{Z}^{1,\pm}_P$	(b) P^*_{expt}	$P^{0,\pm}$	$P^{1,\pm}$
ℓ_{th}	$(\mathbb{Z}^+_\xi)^{-1}=0.471474$			$\mathbb{Z}^{1,+}_\xi=0.68\mathbb{Z}^{1,+}_\chi$	ξ^*	$\xi^\pm=\xi_0^\pm/\alpha_c=\Lambda^*_{qe}(\mathbb{L}^{\{1f\}})^{-1}\vartheta^{-\nu}(\mathbb{Z}^\pm_\xi)^{-1}$	$a^\pm_\xi=\mathbb{Z}^{1,\pm}_\xi\vartheta^\Delta$
		$(\mathbb{Z}^-_\xi)^{-1}=\frac{(\mathbb{Z}^+_\xi)^{-1}}{1.96}$		$\mathbb{Z}^{1,-}_\xi=\frac{0.68}{1.1}\mathbb{Z}^{1,+}_\chi$			
$\mathcal{X}_{\mathrm{th}}$	$(\mathbb{Z}^+_\chi)^{-1}=0.269571$		$\mathbb{Z}^{1,+}_\chi=8.56347$		κ^*_T	$\Gamma^\pm=(\Lambda^*_{qe})^{d-2}(L^{\{1f\}})^d\psi_\rho\vartheta^{-\gamma}(\mathbb{Z}^\pm_\chi)^{-1}$	$a^\pm_\chi=\mathbb{Z}^{1,\pm}_\chi\vartheta^\Delta$
		$(\mathbb{Z}^-_\chi)^{-1}=\frac{(\mathbb{Z}^+_\chi)^{-1}}{4.788}$		$\mathbb{Z}^{1,-}_\chi=\frac{\mathbb{Z}^{1,+}_\chi}{0.215}$			
$\mathcal{C}_{\mathrm{th}}$		$\alpha\mathbb{Z}^+_C=(0.2696\mathbb{Z}^+_\xi)^d$		$\mathbb{Z}^{1,+}_C=8.68\mathbb{Z}^{1,+}_\chi$	c^*_V	$\frac{A^\pm}{\alpha}=(\Lambda^*_{qe})^{-d}(L^{\{1f\}})^d\vartheta^{2-\alpha}\mathbb{Z}^\pm_C$	$a^\pm_C=\mathbb{Z}^{1,\pm}_C\vartheta^\Delta$
		$\alpha\mathbb{Z}^-_C=\frac{(0.2696\mathbb{Z}^+_\xi)^d}{0.537}$		$\mathbb{Z}^{1,-}_C=\frac{8.68}{1.36}\mathbb{Z}^{1,+}_\chi$			
m_{th}		$\mathbb{Z}_M=\frac{(0.2696\mathbb{Z}^+_\xi)^{d/2}}{(0.0574\mathbb{Z}^+_\chi)^{1/2}}$		$\mathbb{Z}^1_M=0.90\mathbb{Z}^{1,+}_\chi$	$\Delta\tilde{\rho}_{LV}$	$B=(\Lambda^*_{qe})^{2-d}(L^{\{1f\}})^d(\psi_\rho)^{\frac{1}{2}}\vartheta^\beta\mathbb{Z}_M$	$a_M=\mathbb{Z}^1_M\vartheta^\Delta$

Table II (see also the discussion in Sec. III B below).

As noted in I for the selected set $\{\Delta;\nu;\gamma\}$ of three independent universal exponents, a closed presentation of these Ising-like universal features only needs to use the mean crossover functions $F_\ell(t)=1/\ell_{\text{th}}(t)$ for the inverse correlation length, and $F_\chi(t)=1/\chi_{\text{th}}(t)$ for the inverse susceptibility, at $h=0$, in the homogeneous phase $t>0$. Making explicit the respective three-term products of Π_ℓ and Π_χ, these two theoretical crossover functions read as follows:

$$[\ell_{\text{th}}(t)]^{-1}=\mathbb{Z}_\xi^+ t^\nu \prod_{i=1}^{3}(1+X_{\xi,i}^+ t^{D(t)})^{Y_{\xi,i}^+}, \tag{44}$$

$$[\mathcal{X}_{\text{th}}(t)]^{-1}=\mathbb{Z}_\chi^+ t^\gamma \prod_{i=1}^{3}(1+X_{\chi,i}^+ t^{D(t)})^{Y_{\chi,i}^+} \tag{45}$$

(for the numerical values of exponents ν, γ, Δ, and Δ_{MF}, and amplitudes $\mathbb{Z}_\xi^+$, $X_{\xi,i}^+$, $Y_{\xi,i}^+$, $\mathbb{Z}_\chi^+$, $X_{\chi,i}^+$, $Y_{\chi,i}^+$, and S_2, see I).

The temperaturelike field t is analytically related to the physical dimensionless temperature distance

$$\Delta\tau^*=\frac{T-T_c}{T_c} \tag{46}$$

by the linear approximation

$$t=\vartheta\Delta\tau^*, \tag{47}$$

which introduces ϑ as an adjustable (system-dependent) parameter. Here ϑ is a scale factor for the temperature field. It is also important to note that the definition of $\Delta\tau^*$ [see Eq. (46)] introduces the critical temperature T_c as a system-dependent parameter. Then the relation between the dimensionless thermodynamic free energies of the Φ^4 model and the physical (one-component fluid) system involves only the energy unit $(\beta_c)^{-1}=k_BT_c$.

Similarly, the orderinglike field h is analytically related to the corresponding physical dimensionless variables $\Delta\tilde{\mu}$ (or Δh^*) by the following linear approximations (including quantum effects):

$$h=\psi_\rho(\Lambda_{qe}^*)^2\Delta\tilde{\mu}$$

$$\text{or}\quad h=\psi(\Lambda_{qe}^*)^2\Delta h^*, \tag{48}$$

which introduce ψ_ρ (or ψ) as adjustable (system-dependent) parameters. ψ_ρ [$\psi=(Z_c)^{-1}\psi_\rho$] is a scale factor for the ordering field $\Delta\tilde{\mu}$ [$\Delta h^*=Z_c\Delta\tilde{\mu}$].

Accordingly, the dimensional analysis of each term of the dimensionless Hamiltonian of the Φ^4 model leads to the introduction of a finite arbitrary wave vector Λ_0, the so-called cutoff parameter, which is related to the finite short range of the microscopic interaction (see, for example, Ref. [15]). Since the value of the cutoff parameter of a selected physical system is generally unknown, a convenient method at $d=3$ consists in replacing Λ_0 by g_0 [8,16], which is the critical coupling constant having the correct wave number dimension (see our Introduction). This system-dependent wave number g_0 provides the practical "adjustable" link between the theoretical dimensionless correlation length (ℓ_{th}) and the measured physical correlation length (ξ_{expt}) of the system at $d=3$, through the fitting equation

$$(\Lambda_{qe}^*)^{-1}\xi_{\text{expt}}(\Delta\tau^*)=(g_0)^{-1}\ell_{\text{th}}(t). \tag{49}$$

In Eq. (49), $(g_0)^{-1}$ appears as a metric prefactor for the theoretical correlation length function. From Eqs. (47)–(49), the asymptotic nonuniversal nature of each physical system is then characterized by the scale factor set $\{\vartheta;(g_0)^{-1};\psi_\rho \text{ (or } \psi)\}$ (with implicit knowledge of T_c and Λ_{qe}^*). However, for the present fluid study where the thermodynamic length unit $a_0=\alpha_c$ is already fixed by Eq. (5), the above fitting Eq. (49) introduces one supplementary dimensionless number defined such that

$$\mathbb{L}^{\{1f\}}=g_0\alpha_c, \tag{50}$$

where the notation $\mathbb{L}^{\{1f\}}$ replaces our introductory notation u_f^* (with $u_f^*=g_0a_0$), anticipating a master (i.e., unique) nature of this product which we will demonstrate below [see Eq. (94)]. More generally, in order to maintain uniqueness of the length unit in the dimensionless description of the singular behavior, any theroretical density property [which implicitly refers to the length scale unit $(g_0)^{-1}$] needs to introduce the proportionality factor $(\mathbb{L}^{\{1f\}})^{-d}$ to the corresponding dimensionless physical density which refers to the length scale unit α_c. As a direct consequence of the fitting Eq. (49) for the correlation length ℓ_{th}, the order parameter density m_{th} must be analytically related to the corresponding physical dimensionless variables $\Delta\tilde{\rho}$ (or Δm^*) by the following linear approximation (including quantum effects):

$$m_{\text{th}}=(\mathbb{L}^{\{1f\}})^{-d}(\psi_\rho)^{-1}[\Lambda_{qe}^*\Delta\tilde{\rho}]$$

$$\text{or}\quad m_{\text{th}}=(\mathbb{L}^{\{1f\}})^{-d}\psi^{-1}[\Lambda_{qe}^*\Delta m^*]. \tag{51}$$

For simplification of the following presentation, we use only ψ_ρ related to the practical dimensionless form of the variables (see Sec. II C).

Finally, adding the knowledge of the energy unit and the length unit for each pure fluid to the theoretical results obtained from the massive renormalization scheme, the dimensionless singular behaviors of the fluid properties are now characterized by the set

$$S_{\text{SF}}^{\{MR\}}=\{\vartheta;\mathbb{L}^{\{1f\}};\psi_\rho\} \tag{52}$$

made of three dimensionless scale factors [admitting that $(\beta_c)^{-1}$, α_c, and Λ_{qe}^* are known]. Therefore, it is easy to analytically define these three dimensionless parameters which characterize each Ising-like fluid, thanks to the exact values of the mean crossover functions within this preasymptotic domain.

B. Three-scale-factor characterization within the Ising-like preasymptotic domain

As previously mentioned and discussed in a detailed manner in I, this asymptotic characterization is valid within the Ising-like preasymptotic domain where the complete crossover functions of Eqs. (44) and (45) can be approximated by the following restricted (two-term) Wegner-like expansions [5]:

$$\ell_{\mathrm{PAD,th}}(t) = (\mathbb{Z}_\xi^+)^{-1} t^{-\nu}[1 + \mathbb{Z}_\xi^{1,+} t^{\Delta}], \quad (53)$$

$$\mathcal{X}_{\mathrm{PAD,th}}(t) = (\mathbb{Z}_\chi^+)^{-1} t^{-\gamma}[1 + \mathbb{Z}_\chi^{1,+} t^{\Delta}]. \quad (54)$$

In Eqs. (53) and (54), $\mathbb{Z}_\xi^{1,+}$ [see below Eq. (55)], is the amplitude of the first confluent correction to scaling for the correlation length, which is related to the one for the susceptibility $\mathbb{Z}_\chi^{1,+}$ [see below Eq. (56)], by the universal ratio $\mathbb{Z}_\xi^{1,+}/\mathbb{Z}_\chi^{1,+}=0.679\,19$ [10], with

$$\mathbb{Z}_\xi^{1,+} = -\sum_{i=1}^{3} X_{\xi,i}^+ Y_{\xi,i}^+, \quad (55)$$

$$\mathbb{Z}_\chi^{1,+} = -\sum_{i=1}^{3} X_{\chi,i}^+ Y_{\chi,i}^+. \quad (56)$$

The theoretical field extension $t \lesssim \mathcal{L}_{\mathrm{PAD}}^{\mathrm{Ising}}$ of the Ising-like preasymptotic domain where the restricted Eqs. (53) and (54) are valid is defined in I, such that

$$\mathcal{L}_{\mathrm{PAD}}^{\mathrm{Ising}} = \frac{10^{-3}}{(S_2)^2} \approx 1.9 \times 10^{-6}. \quad (57)$$

Now considering all the theoretical functions estimated for all the singular properties of the Ising-like systems (see I), we can note that the universal features within the Ising-like preasymptotic domain are characterized by the set

$$\mathbb{S}_{\mathrm{A}}^{\{MR\}} = \begin{Bmatrix} \mathbb{Z}_\chi^{1,+} & = & 8.563\,47 \\ (\mathbb{Z}_\xi^+)^{-1} & = & 0.471\,474 \\ (\mathbb{Z}_\chi^+)^{-1} & = & 0.269\,571 \end{Bmatrix} \quad (58)$$

of three theoretical amplitudes associated with the set $\{\Delta;\nu;\gamma\}$ of three universal exponents selected as independent. As mentioned above, these universal features are illustrated in Table II(a) [which is similar to Table I(a) for the master amplitudes], where any other theoretical amplitude among $\mathbb{Z}_P^{\pm}$ and $\mathbb{Z}_P^{1,\pm}$ given in columns 3 and 5 is calculated from $\mathbb{S}_{\mathrm{A}}^{\{MR\}}$, using the universal mean values of amplitude ratios and/or amplitude combinations [i.e., $R_\xi^+=(\mathbb{Z}_\xi^+)^{-1}(\mathbb{Z}_C^+)^{1/d}=0.2696$, $R_C^+=\mathbb{Z}_C^+(\mathbb{Z}_\chi^+)^{-1}/(\mathbb{Z}_M)^2=0.0574$, $\mathbb{Z}_C^+/\mathbb{Z}_C^-=0.537$, $\mathbb{Z}_\xi^{1,+}/\mathbb{Z}_\xi^{1,-}=1.1$, and $\mathbb{Z}_C^{1,+}/\mathbb{Z}_C^{1,-}=1.36$, according to I and Ref. [10]). The restricted forms of two independent fitting equations valid within the Ising-like preasymptotic domain are

$$(\Lambda_{qe}^*)^{-1}\frac{\xi_{\mathrm{expt}}}{\alpha_c} = (\mathbb{L}^{\{1f\}})^{-1}(\mathbb{Z}_\xi^+)^{-1}(\vartheta\Delta\tau^*)^{-\nu}[1 + \mathbb{Z}_\xi^{1,+}(\vartheta\Delta\tau^*)^{\Delta}], \quad (59)$$

$$(\Lambda_{qe}^*)^2\kappa_{T,\mathrm{expt}}^* = (\mathbb{L}^{\{1f\}})^d(\psi_\rho)^2(\mathbb{Z}_\chi^+)^{-1}(\vartheta\Delta\tau^*)^{-\gamma} \times[1 + \mathbb{Z}_\chi^{1,+}(\vartheta\Delta\tau^*)^{\Delta}], \quad (60)$$

where ξ_{expt} and $\kappa_{T,\mathrm{expt}}^*$ are given by the restricted Wegner-like expansions of Eqs. (27) and (34), respectively. That provides the following *hierarchical* relations:

$$a_\chi^+ = \mathbb{Z}_\chi^{1,+}\vartheta^{\Delta}, \quad (61)$$

$$\frac{\xi_0^+}{\alpha_c} = \xi^+ = (\mathbb{Z}_\xi^+)^{-1}(\mathbb{L}^{\{1f\}})^{-1}\Lambda_{qe}^*\vartheta^{-\nu}, \quad (62)$$

$$\Gamma^+ = (\mathbb{Z}_\chi^+)^{-1}(\mathbb{L}^{\{1f\}})^d(\Lambda_{qe}^*)^{d-2}(\psi_\rho)^2\vartheta^{-\gamma}, \quad (63)$$

with $a_\xi^+/a_\chi^+=\mathbb{Z}_\xi^{1,+}/\mathbb{Z}_\chi^{1,+}=0.679\,19$ [9,10]. We underline the fact that Eq. (61) (or equivalently the equation $a_\xi^+=\mathbb{Z}_\xi^{1,+}\vartheta^{\Delta}$ in the correlation length case) is to be first validated [to confer unequivocal Ising-like equivalence between the first (system-dependent) scale factors ϑ and a_χ^+]. Then Eq. (62) fixes the asymptotic amplitude of the dimensionless correlation length and generates a single scale factor attached to the selected (physical) length unit, which is then mandatorily common to the thermodynamic and correlation functions. Finally, the validation of Eq. (63) provides unequivocal Ising-like equivalence between the second (system-dependent) scale factors ψ_ρ and Γ^+ (accounting for the "critical" and "extensive" nature of the susceptibility).

Equations (61)–(63) satisfy the following condensed functional form:

$$\mathbb{S}_{\mathrm{A}}^{\{MR\}} = \{S_{\mathrm{A}} \times \mathbb{U}(S_{\mathrm{SF}}^{\{MR\}})\}_{(\beta_c)^{-1},\alpha_c,\Lambda_{qe}^*}, \quad (64)$$

where the function $\mathbb{U}$ takes a universal scaling form of the dimensionless asymptotic scale factors ϑ and ψ_ρ. Obviously, when the scale factor set $S_{\mathrm{SF}}^{\{MR\}}$ of a one-component fluid is known in addition to $(\beta_c)^{-1}$, α_c, and Λ_{qe}^*, all the related physical amplitudes can be estimated from the equations given in Table II(b), satisfying then the two-scale-factor universality of the $\{\Phi_3(1)\}$ class.

We note that the universal character of Eq. (64) occurs for any one-parameter crossover modeling. That implies Ising-like equivalence between all estimated crossover functions using only three (model-dependent) characteristic numbers. This result is shown in Appendix A, considering the asymptotic crossover implied by the minimal-subtraction renormalization scheme [19,25] and the phenomenological approach given by a parametric model of the equation of state [31].

Similarly, from Eqs. (47) and (57), it is also easy to define the extension range

$$\Delta\tau^* < \mathcal{L}_{\mathrm{PAD}}^{f} = \frac{\mathcal{L}_{\mathrm{PAD}}^{\mathrm{Ising}}}{\vartheta} \simeq \frac{1.9 \times 10^{-6}}{\vartheta} \quad (65)$$

of the Ising-like preasymptotic domain of the selected fluid (labeled with superscript f). Now, for each one-component fluid for which ϑ (or equivalently one confluent amplitude among a_χ^+ or a_ξ^+) is an unknown parameter, the remaining question of concern is how to define the validity range $\Delta\tau^* < \mathcal{L}_{\mathrm{PAD}}^{f}$ where the theoretical Ising-like characterization by three scale factors can replace the experimental characterization by three asymptotic amplitudes.

C. Three-free-parameter characterization beyond the Ising-like preasymptotic domain

As noted in Ref. [10], in the absence of information concerning the true extension of the Ising-like behavior for a real system belonging to the three-dimensional (3D) Ising-like universality class, the introduction of the scale factors ϑ, ψ_ρ, and the wavelength unit g_0 throughout Eqs. (47)–(49) cannot be easily controlled. Alternatively, it was proposed to introduce three adjustable dimensionless parameters $\mathbb{L}^*_{0,\mathcal{L}}$, $\mathbb{X}^*_{0,\mathcal{L}}$, and $\vartheta_\mathcal{L}$, using the following fitting equations:

$$\frac{\alpha_c}{\xi^*_{\mathrm{expt}}(\Delta\tau^*)} = (\mathbb{L}^*_{0,\mathcal{L}})^{-1}\mathbb{Z}^+_\xi(\Delta\tau^*)^\nu \prod_{i=1}^{3}(1+X^+_{\xi,i}(t)^{D(t)})^{Y^+_{\xi,i}} \tag{66}$$

$$\frac{1}{\kappa^*_{T,\mathrm{expt}}(\Delta\tau^*)} = (\mathbb{X}^*_{0,\mathcal{L}})^{-1}\mathbb{Z}^+_\chi(\Delta\tau^*)^\gamma \prod_{i=1}^{3}(1+X^+_{\chi,i}(t)^{D(t)})^{Y^+_{X,i}}, \tag{67}$$

with

$$t = \vartheta_\mathcal{L}\Delta\tau^*. \tag{68}$$

$\mathbb{L}^*_{0,\mathcal{L}}$ and $\mathbb{X}^*_{0,\mathcal{L}}$ are two adjustable metric prefactors (with the same value above and below T_c) for the "physical" pure power law behavior, while, for the confluent correction terms to this asymptotic scaling, the three-term products are the same as the ones of Eqs. (44) and (45), respectively. Then $\vartheta_\mathcal{L}$ is a global crossover parameter in the sense where it is attached to an unknown effective parameter $\mathcal{L}^f$ which measures the extent of the fitting agreement involving an undefined number of terms in the Wegner-like expansion (see I for details). The determination of $\vartheta_\mathcal{L}$ is then equivalent to the determination of $\mathcal{L}^f$. Within the Ising-like preasymptotic domain, the restricted forms of the fitting equations (66) and (67) are

$$\frac{\xi_{\mathrm{expt}}}{\alpha_c} = \mathbb{L}^*_{0,\mathcal{L}}(\mathbb{Z}^+_\xi)^{-1}(\Delta\tau^*)^{-\nu}[1+\mathbb{Z}^{1,+}_\xi(\vartheta_\mathcal{L}\Delta\tau^*)^\Delta], \tag{69}$$

$$\kappa^*_{T,\mathrm{expt}} = \mathbb{X}^*_{0,\mathcal{L}}(\mathbb{Z}^+_\chi)^{-1}(\Delta\tau^*)^{-\gamma}[1+\mathbb{Z}^{1,+}_\chi(\vartheta_\mathcal{L}\Delta\tau^*)^\Delta]. \tag{70}$$

Therefore, the physical leading amplitudes can be calculated using the (independent) equations

$$\frac{\xi^+_0}{\alpha_c} = \mathbb{L}^*_{0,\mathcal{L}}(\mathbb{Z}^+_\xi)^{-1}, \tag{71}$$

$$\Gamma^+ = \mathbb{X}^*_{0,\mathcal{L}}(\mathbb{Z}^+_\chi)^{-1}, \tag{72}$$

i.e., without explicit reference to $\vartheta_\mathcal{L}$ (however, the subscript $\mathcal{L}$ recalls the implicit $\vartheta_\mathcal{L}$ dependence due to the fitting in the temperature range $\Delta\tau^* \leq \mathcal{L}^f$, with $\mathcal{L}^f > \mathcal{L}^f_{\mathrm{PAD}}$). A noticeable distinction occurs for the confluent corrections to scaling since the first confluent amplitudes are only $\vartheta_\mathcal{L}$ dependent and can be calculated using the equations

$$a^+_\xi = (\vartheta_\mathcal{L})^\Delta \mathbb{Z}^{1,+}_\xi, \tag{73}$$

$$a^+_\chi = (\vartheta_\mathcal{L})^\Delta \mathbb{Z}^{1,+}_\chi, \tag{74}$$

interrelated by the universal ratio $\mathbb{Z}^{1,+}_\xi/\mathbb{Z}^{1,+}_\chi = 0.679\,19$ [10].

For better understanding of the scaling nature of the analytical transformations of the physical variables [such as Eq. (47) or (68)], we select Eq. (74) as the independent equation for the critical crossover characterization. We must then rewrite the above Eqs. (71)–(74) in the following hierarchical forms:

$$(\vartheta_\mathcal{L})^{-\Delta}a^+_\chi = \mathbb{Z}^{1,+}_\chi = \text{universal constant}, \tag{75}$$

$$(\mathbb{L}^*_{0,\mathcal{L}})^{-1}\frac{\xi^+_0}{\alpha_c} = (\mathbb{Z}^+_\xi)^{-1} = \text{universal constant}, \tag{76}$$

$$(\mathbb{X}^*_{0,\mathcal{L}})^{-1}\Gamma^+ = (\mathbb{Z}^+_\chi)^{-1} = \text{universal constant}, \tag{77}$$

where the left-hand sides of the above equations contain all the system-dependent information, first for the Ising-like critical crossover [Eq. (75)], then for the asymptotic behavior of the correlation length function [Eq. (76)], and finally for the asymptotic behavior of the thermodynamic susceptibility function [Eq. (77)]. Moreover, this information is given in a *dual* form, i.e., as a product between a "physical" amplitude (a^+_χ, ξ^+, or Γ^+) and either a "crossover" factor ($\vartheta_\mathcal{L}$), which acts as a scale factor for the confluent correction contribution, or a "prefactor" ($\mathbb{L}^*_{0,\mathcal{L}}$ or $\mathbb{X}^*_{0,\mathcal{L}}$) which acts as a simple factor of proportionality for the corresponding leading amplitude (ξ^+ or Γ^+). The set

$$S^{\{MR\}}_{1\mathrm{C},\mathcal{L}} = \{\vartheta_\mathcal{L};\mathbb{L}^*_{0,\mathcal{L}};\mathbb{X}^*_{0,\mathcal{L}}\} \tag{78}$$

is equivalent to the previous set $S^{\{MR\}}_{\mathrm{SF}}$ of Eq. (52), except that the subscript $1\mathrm{C},\mathcal{L}$ recalls the single crossover parameter obtained over an extended temperature range $\mathcal{L}^f > \mathcal{L}^f_{\mathrm{PAD}}$, beyond the Ising-like preasymptotic domain. Equations (75)–(77) can be written in the condensed functional form

$$\mathbb{S}^{\{MR\}}_{\mathrm{A}} = \{S_\mathrm{A} \times \mathbb{U}_\mathcal{L}(S^{\{MR\}}_{1\mathrm{C},\mathcal{L}})\}_{(\beta_c)^{-1},\alpha_c,\Lambda^*_{qe}} \tag{79}$$

where $\mathbb{U}_\mathcal{L}$ are universal scaling functions, which can be used in an equivalent scaling manner to Eq. (64) when the crossover parameter $\vartheta_\mathcal{L}$ is unique within the $\mathcal{L}^f$ range.

To our knowledge, the singleness of the crossover parameter along the critical isochore of a one-component fluid has never been directly evidenced from the singular behavior of the correlation length or any other thermodynamic property. However, from simultaneous fitting analysis of several singular properties of xenon and helium-3, an indirect probe of a single value for one adjustable parameter related to the scale factor ϑ was obtained, using the crossover functions estimated in the massive renormalization scheme [16,36] and the minimal-subtraction renormalization scheme [24,25]. But these results were never used to accurately analyze the expected equivalence between Eqs. (64) and (79), and then to estimate the other two scale factors $\mathbb{L}^{\{1f\}}$ and ψ_ρ, which is the only correct way to verify the asymptotic condition $\vartheta = \vartheta_\mathcal{L}$ within the Ising-like preasymptotic domain [45].

An analytic determination of $\vartheta_\mathcal{L}$, made beyond the Ising-like preasymptotic domain without use of any adjustable parameter, is under investigation for the case of the isothermal compressibility of xenon [46]. The main objective is to carefully correlate the local value of this crossover parameter with the local value of the correlation length before validating its uniqueness by identification with the asymptotic scale factor ϑ, calculated by using Eq. (47). However, such a challenging demonstration of $\vartheta \equiv \vartheta_{\mathcal{L}}$ in the temperature range $\Delta\tau^* \leq \mathcal{L}_{\mathrm{EAD}}^f$, i.e., within the so-called Ising-like *extended* asymptotic domain (EAD) in the following, as a formulation of the three-parameter characterization of xenon selected as a standard one-component fluid, remain two preliminary attempts to test the equivalence between Eqs. (64) and (79). That needs to be examined using a more general approach, like the one proposed below, where we will introduce three master constants which relate unequivocally the dimensionless lengths and relevant fields of both (theoretical and master) descriptions, to identify the theoretical crossover of the $\{\Phi_3(1)\}$ class with the master crossover of the $\{1f\}$ subclass.

D. Identification of the theoretical and master asymptotic scaling within the Ising-like preasymptotic domain

Now, while reconsidering our previous analysis of the relations between physical and master properties, we must rewrite Eqs. (28), (35), and (36), in the following hierarchical forms:

$$Y_c(a_\chi^+)^{-1/\Delta} = (\mathcal{Z}_\chi^{1,+})^{1/\Delta} = \text{master constant}, \tag{80}$$

$$\frac{1}{\alpha_c}(Y_c)^\nu[(\Lambda_{qe}^*)^{-1}\xi_0^+] = \mathcal{Z}_\xi^+ = \text{master constant}, \tag{81}$$

$$Z_c(Y_c)^\gamma[(\Lambda_{qe}^*)^{2-d}\Gamma^+] = \mathcal{Z}_\chi^+ = \text{master constant}, \tag{82}$$

Comparison of Eqs. (75)–(77) with Eqs. (80)–(82) shows that their right-hand-side differences concern only the respective numerical values of the characteristic *master* set $\mathcal{S}_{\mathrm{A}}^{\{1f\}}$ of Eq. (40), and *universal* set $\mathbb{S}_{\mathrm{A}}^{\{MR\}}$ of Eq. (58). For their left-hand-side comparison, neglecting the quantum corrections in a first approach (i.e., fixing $\Lambda_{qe}^*=1$), the term-to-term identification between measurable amplitudes underlines the analogy between the explicit parameter set $\{Y_c;Z_c\}$, related to the master description, and the implicit one $\{\vartheta;\psi_\rho\}$, related to the massive renormalization description. We can then note that the $\{1f\}$ master formulation compares to the $\{MR\}$ universal formulation only if we have correctly accounted for the asymptotic scaling nature of each dimensionless number needed by the massive renormalization scheme. In order to reveal such a scaling nature, it is essential to note that the scale dilatation method replaces the renormalized fields (such as t, h, m_{th}, etc.) needed to observe the universal behavior of the $\Phi_3^4(1)$universality class, by the $\{1f\}$ fields (such as, $\mathcal{T}^*$, $\mathcal{H}_{\mathrm{qf}}^*$, $\mathcal{M}_{\mathrm{qf}}^*$, etc.) needed to observe the master behavior of the $\{1f\}$ subclass. The common physical variables are $\Delta\tau^*$, $\Delta\tilde{\mu}$, and $\Delta\tilde{\rho}$. Therefore, it remains to give explicit forms for the following exchanges between the theoretical variables and the $\{1f\}$ subclass variables:

$$t \to \mathcal{T}^*, \tag{83}$$

$$h \to \mathcal{H}_{\mathrm{qf}}^* \quad \text{or} \quad m_{\mathrm{th}} \to \mathcal{M}_{\mathrm{qf}}^* \tag{84}$$

(see Ref. [37] for the correlation length case). The next section is dedicated to the isothermal susceptibility case [which then closes the description of the $\{1f\}$ subclass along the critical isochore in conformity with the universal features estimated for the Ising-like universality class].

E. Master modification of the theoretical crossover for the isothermal susceptibility

We start with the following modification of Eq. (45):

$$\frac{1}{\mathcal{X}_{\mathrm{qf}}^*(|\mathcal{T}^*|)} = \mathbb{Z}_\chi^{\{1f\}}\mathcal{Z}_\chi^\pm t^\gamma \prod_{i=1}^{3}(1+X_{i,\chi}^\pm t^{D^\pm(t)})^{Y_{i,\chi}^\pm} \tag{85}$$

and the following modification of Eq. (47):

$$t = \Theta^{\{1f\}}|\mathcal{T}^*| \tag{86}$$

by introducing the prefactor $\mathbb{Z}_\chi^{\{1f\}}$ and the scale factor $\Theta^{\{1f\}}$ as master (i.e., unique) parameters for the $\{1f\}$ subclass. We note that $\Theta^{\{1f\}}$, characteristic of the (critical) isochoric line (with the same value above and below T_c), reads as follows:

$$\Theta^{\{1f\}} = \left(\frac{\mathcal{Z}_\chi^{1\pm}}{\mathbb{Z}_\chi^{1,\pm}}\right)^{1/\Delta} \tag{87}$$

whatever the one-component fluid. By virtue of the universal feature of confluent amplitude ratios (see Table I), the numerical value

$$\Theta^{\{1f\}} = 4.288 \times 10^{-3} \tag{88}$$

is the same whatever the property and the phase domain. However, we also note that $\Theta^{\{1f\}}$ contributes to the leading term. Thus, in addition to Eq. (85), we define $\mathbb{Z}_\chi^{\{1f\}}$ such that

$$\mathbb{Z}_\chi^{\{1f\}} = [\mathcal{Z}_\chi^\pm \mathbb{Z}_\chi^\pm(\Theta^{\{1f\}})^\gamma]^{-1}. \tag{89}$$

The numerical value

$$\mathbb{Z}_\chi^{\{1f\}} = 1938.48 \tag{90}$$

is the same in the homogeneous phase and in the nonhomogeneous phase. The curve labeled MR in Fig. 1 was obtained from Eqs. (85) and (86) using the numerical values of $\Theta^{\{1f\}}$ and $\mathbb{Z}_\chi^{\{1f\}}$ given by Eqs. (88) and (90), respectively.

We recall that our previous analysis [37] of the correlation length has introduced a similar prefactor $\mathbb{Z}_\xi^{\{1f\}}$ through the following modification of Eq. (66):

$$\frac{1}{\ell_{\mathrm{qf}}^*(|\mathcal{T}^*|)} = \mathbb{Z}_\xi^{\{1f\}}F_\ell(t) \tag{91}$$

with

$$\mathbb{Z}_\xi^{\{1f\}} = [\mathcal{Z}_\xi^\pm \mathbb{Z}_\xi^\pm(\Theta^{\{1f\}})^\nu]^{-1}, \tag{92}$$

which has the same numerical value

$$\mathbb{Z}_\xi^{\{1f\}} = 25.585 \tag{93}$$

for the homogeneous and nonhomogeneous domains. Of course, we retrieve here the previous Eq. (50),

$$\mathbb{Z}_\xi^{\{1f\}} \equiv \mathbb{L}^{\{1f\}} = (g_0\alpha_c)_{\forall\ \mathrm{fluid}}, \tag{94}$$

TABLE III. Three-parameter characterization of the crossover functions by using two leading amplitudes or prefactors (column 3) and one scale factor or crossover parameter (column 4); see text. (a) Mean crossover functions $F_P(t)$ defined in I. (b) Master crossover functions $\mathcal{P}^*_{\mathrm{qf}}(\mathcal{T}^*)$ [see Eq. (112)]; lines 4, 9, and 10, independent parameters; lines 5–8, related parameters; the two relations $(\mathbb{Z}_C^{\{1f\}})^{1/d}=\mathbb{Z}_\xi^{\{1f\}}$ and $(\mathbb{Z}_\xi^{\{1f\}})^d/\mathbb{Z}_\chi^{\{1f\}}=(\mathbb{Z}_M^{\{1f\}})^2$ are in conformity with the two-scale-factor universality. (c) Physical crossover functions $P^*_{\mathrm{expt}}(\Delta\tau^*)$ [see Eq. (113)]; the two relations $(\mathbb{C}^*_{0,\mathcal{L}}/\alpha)^{1/d}\mathbb{L}^*_{0,\mathcal{L}}=1$ and $(\mathbb{L}^*_{0,\mathcal{L}})^{-d}\mathbb{X}^*_{0,\mathcal{L}}/(\mathbb{M}^*_{0,\mathcal{L}})^2=1$ are in conformity with the two-scale-factor universality. All the values of $\mathbb{P}^*_{0,\mathcal{L}}$, $\vartheta_{\mathcal{L}}\equiv\vartheta$; and ψ_ρ can be estimated from $Q_c^{\min}=\{(\beta_c)^{-1},\alpha_c,Y_c,Z_c\}$ and Λ^*_{qe} of the selected one-component fluid.

(a)	F_P	$\mathbb{Z}_P^{\pm}$	$\mathbb{Z}_P^{1,\pm}$
$\mathbb{S}_{\mathrm{A}}^{\{MR\}}$, see Eq. (58)		$\{(\mathbb{Z}_\xi^+)^{-1}=0.471474;(\mathbb{Z}_\chi^+)^{-1}=0.269571\}$	$\mathbb{Z}_\chi^{1,+}=8.56347$
(b)	$\mathcal{P}^*_{\mathrm{qf}}$	$\mathbb{Z}_P^{\{1f\}}$	$\mathcal{Z}_P^{1,\pm}$
$\mathcal{S}^{\{1f\}}_{\mathrm{2P,1S}}$, see Eq. (96)		$\{\mathbb{Z}_\xi^{\{1f\}}=25.585;\mathbb{Z}_\chi^{\{1f\}}=1938.48\}$	$\Theta^{\{1f\}}=4.288\times10^{-3}$
	ℓ^*_{qf}	$\mathbb{Z}_\xi^{\{1f\}}=[\mathcal{Z}_\xi^\pm\mathbb{Z}_\xi^\pm(\Theta^{\{1f\}})^\nu]^{-1}=\mathbb{L}^{\{1f\}}=25.585$	$\Theta^{\{1f\}}=\left(\frac{\mathcal{Z}_\xi^{1,\pm}}{\mathbb{Z}_\xi^{1,\pm}}\right)^{1/\Delta}$
	$\varkappa^*_{\mathrm{qf}}$	$\mathbb{Z}_\chi^{\{1f\}}=[\mathcal{Z}_\chi^\pm\mathbb{Z}_\chi^\pm(\Theta^{\{1f\}})^\gamma]^{-1}=(\mathbb{L}^{\{1f\}})^{-d}(\Psi^{\{1f\}})^{-2}=1938.48$	$\Theta^{\{1f\}}=\left(\frac{\mathcal{Z}_\chi^{1,\pm}}{\mathbb{Z}_\chi^{1,\pm}}\right)^{1/\Delta}$
	$\mathcal{C}^*_{\mathrm{qf}}$	$\mathbb{Z}_C^{\{1f\}}=\frac{\mathcal{Z}_C^\pm}{\alpha\mathbb{Z}_C^\pm(\Theta^{\{1f\}})^{2-\alpha}}=(\mathbb{L}^{\{1f\}})^d=16747.8$	$\Theta^{\{1f\}}=\left(\frac{\mathcal{Z}_C^{1,\pm}}{\mathbb{Z}_C^{1,\pm}}\right)^{1/\Delta}$
	$\mathcal{M}^*_{\mathrm{qf}}$	$\mathbb{Z}_M^{\{1f\}}=\frac{\mathcal{Z}_M}{\mathbb{Z}_M(\Theta^{\{1f\}})^\beta}=(\mathbb{L}^{\{1f\}})^d\Psi^{\{1f\}}=2.93933$	$\Theta^{\{1f\}}=\left(\frac{\mathcal{Z}_M^1}{\mathbb{Z}_M^1}\right)^{1/\Delta}$
$\mathcal{S}^{\{1f\}}_{\mathrm{SF}}$, see Eq. (110)		$\{\mathbb{L}^{\{1f\}}=25.585;\Psi^{\{1f\}}=1.75505\times10^{-4}\}$	$\Theta^{\{1f\}}=4.288\times10^{-3}$
$\mathcal{S}^{\{1f\}}_{\mathrm{A}}$, see Eq. (40)		$\{\mathcal{Z}_\xi^+=0.5729;Z_\chi^+=0.11975\}$	$\mathcal{Z}_\chi^{1,+}=0.555$
(c)	P^*_{expt}	$\mathbb{P}^*_{0,\mathcal{L}}$	
$S^{\{MR\}}_{\mathrm{1C},\mathcal{L}}$, see Eq. (78)		$\{\mathbb{L}^*_{0,\mathcal{L}};\mathbb{X}^*_{0,\mathcal{L}}\}$	$\vartheta_{\mathcal{L}}\equiv\vartheta=Y_c\Theta^{\{1f\}}$
	ξ^*	$\mathbb{L}^*_{0,\mathcal{L}}=\Lambda^*_{qe}(Y_c)^{-\nu}\mathcal{Z}_\xi^\pm\mathbb{Z}_\xi^\pm=\Lambda^*_{qe}(Y_c)^{-\nu}[\mathbb{L}^{\{1f\}}(\Theta^{\{1f\}})^\nu]^{-1}$ $=1.21513\Lambda^*_{qe}(Y_c)^{-\nu}$	
	κ^*_T	$\mathbb{X}^*_{0,\mathcal{L}}=\frac{(\Lambda^*_{qe})^{d-2}}{Z_c(Y_c)^\gamma}\mathcal{Z}_\chi^\pm\mathbb{Z}_\chi^\pm=\frac{(\Lambda^*_{qe})^{d-2}}{Z_c(Y_c)^\gamma}(\mathbb{L}^{\{1f\}})^d(\Psi^{\{1f\}})^2(\Theta^{\{1f\}})^{-\gamma}$ $=0.444225\frac{(\Lambda^*_{qe})^{d-2}}{Z_c(Y_c)^\gamma}$	
	c^*_V	$\frac{\mathbb{C}^*_{0,\mathcal{L}}}{\alpha}=(\Lambda^*_{qe})^{-d}(Y_c)^{2-\alpha}\frac{\mathcal{Z}_C^\pm}{\alpha\mathbb{Z}_C^\pm}=(\Lambda^*_{qe})^{-d}(Y_c)^{2-\alpha}(\mathbb{L}^{\{1f\}})^d(\Theta^{\{1f\}})^{2-\alpha}$ $=0.5557355(\Lambda^*_{qe})^{-d}(Y_c)^{2-\alpha}$	
	$\Delta\tilde{\rho}_{LV}$	$\mathbb{M}^*_{0,\mathcal{L}}=\frac{(Y_c)^\beta}{\Lambda^*_{qe}(Z_c)^{1/2}}\frac{\mathcal{Z}_M}{\mathbb{Z}_M}=\frac{(Y_c)^\beta}{\Lambda^*_{qe}(Z_c)^{1/2}}(\mathbb{L}^{\{1f\}})^d\Psi^{\{1f\}}(\Theta^{\{1f\}})^\beta$ $=0.497585\frac{(Y_c)^\beta}{\Lambda^*_{qe}(Z_c)^{1/2}}$	
$S^{\{MR\}}_{\mathrm{SF}}$, see Eq. (52)		$\{\mathbb{L}^{\{1f\}}=25.585;\psi_\rho=(Z_c)^{-1/2}\Psi^{\{1f\}}\}$	$\vartheta=Y_c\Theta^{\{1f\}}$
$S^{\{1f\}}_{\mathrm{SF}}$, see Eq. (39)		$\{Y_c;Z_c;\mathbb{L}^{\{1f\}}=25.585;\Lambda^*_{qe}\}$	
S_{A}, see Eq. (38)		$\{\xi^+;\Gamma^+\}$	a_χ^+

which now is valid whatever the fluid under consideration. As is now well understood from our previous results, Eqs. (87), (89), and (92) satisfy the following condensed functional form:

$$\mathbb{S}_{\mathrm{A}}^{\{MR\}}=\{\mathcal{S}_{\mathrm{A}}^{\{1f\}}\times\mathbb{U}_P(\mathcal{S}^{\{1f\}}_{\mathrm{2P,1S}})\} \quad (95)$$

where $\mathbb{U}_P$ are universal scaling functions of the following set of master (two pre- + one scale) factors

$$\mathcal{S}_{2P,1S}^{\{1f\}} = \begin{Bmatrix} \Theta^{\{1f\}} = 4.288 \times 10^{-3} \\ \mathbb{Z}_{\xi}^{\{1f\}} = 25.585 \\ \mathbb{Z}_{\chi}^{\{1f\}} = 1938.48 \end{Bmatrix}, \tag{96}$$

which closes the universal behavior of the $\{1f\}$ subclass, as shown by the results reported in Table III for all the properties calculated along the critical isochore (for notations see below and Refs. [36,37,46]). Equation (50) [or Eq. (94)] appears then as the basic hypothesis which defines the critical length uniqueness [11] between correlation functions and thermodynamic functions of the one component fluid subclass. $\mathbb{L}^{\{1f\}}$ takes an equivalent nature to the length reference used in the renormalization scheme applied to the $\Phi_3(1)$ class, whatever the selected physical system.

The major interest of Eqs. (89) and (92) is that they introduce the needed cross relation between pure asymptotic scaling description and the first confluent correction to scaling, in order to obtain only two independent leading amplitudes within the Ising-like preasymptotic domain. Such a cross relation occurs if the nonuniversal scale factor associated with the irrelevant field which induces the correction-to-scaling term of lowest relative order $(\Delta\tau^*)^{\Delta}$ in a Wegner-like expansion is the same as the nonuniversal scale factor associated with the relevant (thermal) field which gives the leading scaling term $(\Delta\tau^*)^{-e_P}$.

In that universal description of the confluent corrections to scaling, each crossover function includes the (two-term) master behavior expected using the scale dilatation method. By comparing the leading terms on each member of Eqs. (85), (33), and (70), we obtain the relations

$$\Gamma^{\pm} = (\mathbb{Z}_{\chi}^{\{1f\}}\mathbb{Z}_{\chi}^{\pm})^{-1}\vartheta^{-\gamma} = \mathbb{X}_{0,\mathcal{L}}^{*}(\mathbb{Z}_{\chi}^{\pm})^{-1} \tag{97}$$

where the fluid-dependent metric prefactor $\mathbb{X}_{0,\mathcal{L}}^{*}$ of Eq. (70) now reads as follows:

$$\mathbb{X}_{0,\mathcal{L}}^{*} = \mathcal{Z}_{\chi}^{\pm}\mathbb{Z}_{\chi}^{\pm}(Y_c)^{-\gamma}. \tag{98}$$

In Eq. (98), the critical contribution of the scale factor Y_c is explicit. The remaining adjustable crossover parameter $\vartheta_{\mathcal{L}}$ of Eq. (68) is characteristic of the Ising-like extended asymptotic domain $\Delta\tau^* \lesssim \mathcal{L}_{\mathrm{EAD}}^{f}$ where the theoretical crossover functions and experimental data agree. Within the Ising-like preasymptotic domain [see Eq. (57)] where the two-term Wegner-like expansions are expected to be valid, the comparison of the first confluent amplitudes for master and theoretical descriptions, enables one to write $\vartheta_{\mathcal{L}}$ as follows:

$$\vartheta_{\mathcal{L}} \equiv \vartheta \quad (=Y_c\Theta^{\{1f\}}) \tag{99}$$

with

$$t \equiv \left(\frac{\mathcal{Z}_{\chi}^{1,\pm}}{\mathbb{Z}_{\chi}^{1,\pm}}\right)^{1/\Delta}\mathcal{T}^* \quad (=\Theta^{\{1f\}}|\mathcal{T}^*|). \tag{100}$$

As considered from the basic input of the scale dilatation method, Eq. (100) agrees with the scale dilatation of the temperature field

$$\mathcal{T}^* = Y_c|\Delta\tau^*|. \tag{101}$$

Note that the extension $\mathcal{T}^* \lesssim \mathcal{L}_{\mathrm{PAD}}^{\{1f\}}$ of the Ising-like preasymptotic domain of the $\{1f\}$ subclass can then be immediately obtained from Eq. (57), with

$$\mathcal{L}_{\mathrm{PAD}}^{\{1f\}} = \frac{\mathcal{L}_{\mathrm{PAD}}^{\mathrm{Ising}}}{\Theta^{\{1f\}}} \approx 4.43 \times 10^{-4} \tag{102}$$

[see for example the full arrow labeled "PAD" in Fig. 1(c)].

F. Closed master modification of the mean crossover functions and master extension $\mathcal{L}_{\mathrm{EAD}}^{\{1f\}}$ of the extended asymptotic domain

Obviously, the equivalent approach at exact criticality and along the critical isotherm occurs in virtue of the two-scale-factor universality which implies a second unequivocal relation between ψ_ρ and Z_c. However, we can anticipate such a result only from the thermodynamic definitions of the susceptibilities $\chi_{\mathrm{th}}=(\partial m_{\mathrm{th}}/\partial h)_t$ and $\mathcal{X}_{\mathrm{qf}}^*=(\partial\mathcal{M}_{\mathrm{qf}}^*/\partial\mathcal{H}_{\mathrm{qf}}^*)_{\mathcal{T}^*}$, introducing the scale factor $\Psi^{\{1f\}}$ through the following linearized equations:

$$h = \Psi^{\{1f\}}\mathcal{H}_{\mathrm{qf}}^* = \Psi^{\{1f\}}(\Lambda_{qe}^*)^2\mathcal{H}^*, \tag{103}$$

$$m_{\mathrm{th}} = (\mathbb{L}^{\{1f\}})^{-d}(\Psi^{\{1f\}})^{-1}|\mathcal{M}_{\mathrm{qf}}^*| = (\mathbb{L}^{\{1f\}})^{-d}(\Psi^{\{1f\}})^{-1}\Lambda_{qe}^*|\mathcal{M}^*|, \tag{104}$$

where $\Psi^{\{1f\}}$ is a master (i.e., unique) parameter characteristic of the (critical) isothermal line for the $\{1f\}$ subclass ($\Psi^{\{1f\}}$ has the same value whatever the sign of the order parameter). From comparison between either Eqs. (20), (23), (48), and (103) or Eqs. (21), (24), (50), and (104), it is immediate to show that $\chi_{\mathrm{th}}=(\mathbb{L}^{\{1f\}})^{-d}(\Psi^{\{1f\}})^{-1}\mathcal{X}_{\mathrm{qf}}^*$ and, in addition, to obtain the following expected relation:

$$\psi_\rho = (Z_c)^{-1/2}\Psi^{\{1f\}}. \tag{105}$$

The unequivocal link between the scale factors needed either by the theoretical description or by the master description is given by Eqs. (94), (99), and (105). Therefore, the leading theoretical and master amplitudes of the susceptibility and the order parameter are related by the equations

$$\mathcal{Z}_{\chi}^{\pm}\mathbb{Z}_{\chi}^{\pm} = (\mathbb{L}^{\{1f\}})^d(\Psi^{\{1f\}})^2(\Theta^{\{1f\}})^{-\gamma}, \tag{106}$$

$$\frac{\mathcal{Z}_M}{\mathbb{Z}_M} = (\mathbb{L}^{\{1f\}})^d\Psi^{\{1f\}}(\Theta^{\{1f\}})^{\beta}, \tag{107}$$

while the leading theoretical and master amplitudes of the correlation length and the heat capacity are related by the equations

$$\mathcal{Z}_{\xi}^{\pm}\mathbb{Z}_{\xi}^{\pm} = [\mathbb{L}^{\{1f\}}(\Theta^{\{1f\}})^{\nu}]^{-1}, \tag{108}$$

$$\frac{\mathcal{Z}_C^{\pm}}{\alpha\mathbb{Z}_C^{\pm}} = (\mathbb{L}^{\{1f\}})^d(\Theta^{\{1f\}})^{2-\alpha}, \tag{109}$$

Finally, the characteristic set

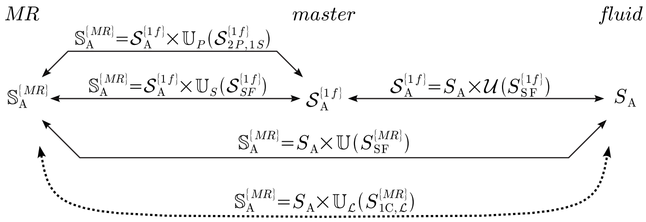

FIG. 2. Schematic links, corresponding to Eqs. (41), (64), (79), (95), and (111), between three-amplitude characterization $\mathbb{S}_A^{\{MR\}}$ of Eq. (58), $\mathcal{S}_A^{\{1f\}}$ of Eq. (40), and $\mathcal{S}_A$ of Eq. (38), for the theoretical, master, and physical singular behaviors, respectively. The only needed fluid-dependent parameters are given by $Q_c^{\min}=\{(\beta_c)^{-1},\alpha_c,Y_c,Z_c\}$ of Eq. (3) and by a supplementary adjustable parameter $\Lambda_{qe}^{*}\neq 1$ [see Eq. (9)] in the case of quantum fluids.

$$\mathcal{S}_{SF}^{\{1f\}}=\left\{\begin{array}{c}\Theta^{\{1f\}}=4.288\times 10^{-3}\\ \mathbb{L}^{\{1f\}}=25.585\\ \Psi^{\{1f\}}=1.75505\times 10^{-4}\end{array}\right\} \qquad (110)$$

is Ising-like equivalent to the one of Eq. (96), while Eqs. (87), (106), and (108) satisfy the following condensed functional form:

$$\mathbb{S}_A^{\{MR\}}=\{\mathcal{S}_A^{\{1f\}}\times\mathbb{U}_S(\mathcal{S}_{SF}^{\{1f\}})\}, \qquad (111)$$

where $\mathbb{U}_S$ are universal scaling functions of $\mathcal{S}_{SF}^{\{1f\}}$. Equation (111) closes the modifications of the theoretical functions of the $\Phi_3(1)$ class in order to provide accurate description of the master singular behavior of the $\{1f\}$ subclass.

Accordingly, each modified function reads as follows:

$$\mathcal{P}_{qf}^{*}(\mathcal{T}^{*})=\mathbb{Z}_P^{\{1f\}}F_P(t) \qquad (112)$$

with $t=\Theta^{\{1f\}}\mathcal{T}^{*}$ and $F_P(t)$ defined in I. All the master prefactors $\mathbb{Z}_P^{\{1f\}}$ can then be calculated using the relations given in column 3 of Table III(a). The master prefactors $\mathbb{Z}_C^{\{1f\}}$ and $\mathbb{Z}_M^{\{1f\}}$ are for the heat capacity case and the order parameter case, respectively. Within the Ising-like preasymptotic domain, Eq. (112) can be approximated by Eq. (30).

Alternatively but equivalently, each physical property can also be fitted by the following modified function:

$$P_{\text{expt}}^{*}(|\Delta\tau^{*}|)=\mathbb{P}_{0,\mathcal{L}}^{*}\mathbb{Z}_P^{\pm}|\Delta\tau^{*}|^{-e_P}\prod_{i=1}^{3}\left(1+X_{i,P}^{\pm}|t|^{D(|t|)}\right)^{Y_{i,P}^{\pm}} \qquad (113)$$

with $|t|=\vartheta|\Delta\tau^{*}|=\Theta^{\{1f\}}Y_c|\Delta\tau^{*}|$ and where the explicit three-term product of the function $\Pi_P(|t|^{D(|t|)})$ [see Eq. (42)], the function $D(|t|)$ [see Eq. (43)], and the universal quantities $\mathbb{Z}_P^{\pm}$, e_P, $X_{i,P}^{\pm}$, $Y_{i,P}^{\pm}$, are given in I. All the physical prefactors $\mathbb{P}_{0,\mathcal{L}}^{*}$ can also be calculated using the equations given in column 3, of Table III(b), where the physical prefactors $\mathbb{C}_{0,\mathcal{L}}$ and $\mathbb{M}_{0,\mathcal{L}}$ are for the heat capacity case and the order parameter case, respectively [see Eq. (113)].

As a summarizing remark related to the schematic Fig. 2, the theoretical amplitude set $\mathbb{S}_A^{\{MR\}}$ of Eq. (58), the master amplitude set $\mathcal{S}_A^{\{1f\}}$ of Eq. (40), and the physical amplitude set S_A of Eq. (38), are unequivocally related only using Y_c and Z_c or ϑ [see Eq. (99)] and ψ_ρ [see Eq. (105)] as entry parameters [assuming that $(\beta_c)^{-1}$, α_c, and Λ_{qe}^{*} are known].

In addition, we can also account for the results of previous analyses of different singular properties for several one-component fluids where each master singular behavior is well fitted by the corresponding crossover functions in the extended asymptotic domain which corresponds to $\ell_{qf}^{*}\gtrsim 3-4$ [see, for example, the dashed arrow labeled "EAD" in Fig. 1(c), for the susceptibility case]. Indeed, the effective extension $\mathcal{L}_{EAD}^{+,\{1f\}}$, where this modified theoretical description seems to be valid, corresponds to a temperaturelike range such as

$$\mathcal{T}^{*}\lesssim\mathcal{L}_{EAD}^{+,\{1f\}}\simeq 0.07-0.1. \qquad (114)$$

Equations (102) and (114) are of crucial importance for scientists interested in analyzing liquid-gas critical point phenomena without adjustable parameters, since they are the master (experimental) answer to the unsolved theoretical question: How large is the range in which the asymptotic universal features are valid in pure fluids? Moreover, when $Q_c^{\min}=\{(\beta_c)^{-1},\alpha_c,Y_c,Z_c\}$ and Λ_{qe}^{*} are known, we note that each modified crossover function of Eq. (112) can act beyond the Ising-like preasymptotic domain, i.e., within the two-decade range $10^{-2}\lesssim\mathcal{T}^{*}\lesssim 1$ corresponding to the blued areas of Fig. 1, to confirm that the critical Ising-like anomalies characterized by a limited numbers of critical parameters would dominate in a large range around the liquid-gas critical point. Such a modified theoretical analysis of the available fluid data at finite temperature distance appears then similar to the one initially proposed to provide the first test of the scaling hypothesis for the one-component fluids by using effective universal equations of state with only two adjustable dimensionless parameters. As a typical example, we analyze the isothermal susceptibility for 12 different fluids in Appendix B, using the well-known linear model of a parametric equation of state (EOS) [17] with $\gamma_{EOS}=1.19$ (and $\beta_{EOS}=0.355$ to close thermodynamics scaling laws). Furthermore, Eqs. (102) and (114) offer explicit Ising-like criteria to control the development of any empirical multiparameter equation of state where such a minimal critical parameter set $Q_c^{\min}$ is customarily used (see, for example, Ref. [48] and references therein).

IV. CONCLUSIONS

We have shown that the needed information to describe the singular behavior of one-components fluids within the Ising-like preasymptotic domain was provided by a minimum set of four scale factors which characterize the thermodynamics inside the volume of the critical interaction cell. We have illustrated the Ising-like scaling nature of the scale dilatation method able to demonstrate the master singular behavior of the one-component fluid subclass. Using the mean crossover function for susceptibility in the homogeneous phase, which complements a previous study of the correlation length in the homogeneous phase, we have demonstrated that the universal features predicted by the massive renormalization scheme are then accounted for by introducing one common crossover parameter and appropriate prefactors, only two among the latter being fluid-characteristic prefactors. By defining three master constants able to relate the theoretical fields and densities to the master ones, the corresponding master modifications of the mean crossover functions were obtained from identification with the asymptotical master singular behavior of the one-component fluid subclass. The four critical coordinates which localize the gas-liquid critical point on the pressure, volume, and temperature phase surface provide then the four scale factors needed to calculate the singular behavior of any correlation function or thermodynamical property, in a well-controlled effective extension of the asymptotic critical domain for any one-component fluid belonging to this subclass, in agreement with the idea first introduced by one of us. In the case where quantum effects can be non-negligible, a single supplementary adjustable parameter seems needed to correctly account for them.

ACKNOWLEDGMENT

The authors are indebted to C. Bervillier for valuable discussion and constructive comments during this work.

APPENDIX A: SCALING EQUIVALENCE FOR A ONE-PARAMETER CROSSOVER MODELING WITHIN THE PREASYMPTOTIC DOMAIN

The use of Eq. (61) in the hierarchical Eqs. (61)–(63) needs that the characteristic scale factor ϑ is the first mandatory parameter to be determined, whatever the renormalization scheme (at $h=0$). For scaling understanding, Eq. (61) must be expressed in the universal form of Eq. (75), i.e., such that

$$a_\chi^+ \vartheta^{-\Delta} = \mathbb{Z}_\chi^{1,+}. \tag{A1}$$

Such a theoretical scaling form of Eq. (A1) [or Eq. (75)] is then provided from any phenomenological model which use a single crossover (temperaturelike) parameter $\Delta\tau_{\chi,M}^*$ related to the (system-dependent) Ginzburg number G (the subscript M refers to the selected model). Although the crossover phenomenon can be general upon approach to the Ising-like critical point, such a modeling, in which G is a tuneable parameter, is essential to check carefully its description with the aim of discussing the shape and the extension of the crossover curves (leading, for example, to distinguish a wide variety of Ising-like experimental systems, including simple fluids, binary liquids, micellar solutions, polymer mixtures, etc.). However, for the one-component fluid case, our interest can be restricted to the crossover temperature scale estimated by three crossover modeling selected in Table IV, i.e., (i) the massive renormalization (MR) scheme [8,10] and (ii) the minimal-subtraction renormalization (MSR) scheme [19,25], both modeling without tuneable G, and (iii) the parametric model of the equation of state (CPM) [31], with tuneable G. The universal form of the first confluent amplitude for the susceptibility case, is then given by the equation

TABLE IV. Estimated universal values of the confluent exponent (column 2) and confluent crossover parameter (column 3) of the scaling forms of Eq. (A2) for the first confluent correction term in the susceptibility case. Column 1, label M of the different crossover models (see references given in the last column). Column 5, order of magnitude for the ratio of the crossover parameters obtained using MSR or CPM fitting, from reference to the MR fitting (see text).

M	Δ	$g_{\chi,M}^{1,+}$	$(g_{\chi,M}^{1,+})^{-1/\Delta}$	$\left(\frac{g_{\chi,M}^{1,+}}{\mathbb{Z}_\chi^{1,+}}\right)^{1/\Delta}$ $(=\vartheta\Delta\tau_{\chi,M}^*)$	Ref.
MSR	0.504	0.525	3.591	3.9×10^{-2}	[23]
CPM	0.51	0.590	2.814	4.9×10^{-3}	[31]
		$\mathbb{Z}_\chi^{1,+}$	$(\mathbb{Z}_\chi^{1,+})^{-1/\Delta}$		
MR	0.50189	8.56347	0.013859	1	[8]

$$a_\chi^+(\Delta\tau_{\chi,M}^*)^\Delta = g_{\chi,M}^{1,+}, \tag{A2}$$

where $g_{\chi,M}^{1,+}$ is a universal constant given in Table IV. The differences in the estimates of $g_{\chi,M}^{1,+}$ come from differences in several theoretical aspects: the extension of the renormalization procedures, the nature of the asymptotic limit of $\Delta\tau^*/G$, the nature of the nonuniversal corrections, the precision of numerical calculations, etc. Therefore, we cannot expect a practical understanding from each value given in Table IV. However, in spite of these numerical differences, the scaling form of Eqs. (75), (A2), and (A3) provides analytic equivalence between the three models, since each model exactly accounts for the same Ising-like critical crossover using a single crossover parameter, especially for the temperature dependence of the effective exponent [26]. The crossover temperature scale $\Delta\tau_{\chi,M}^*$ takes a small finite value and can then be comparable to ϑ, via the "sensor" $\Delta\tau_\Delta^*=t_0^*/\vartheta$ [see Eq. (39) in I] of the mean crossover functions (see also Ref. [46]). As illustrated by the point-to-point transformations in Figs. 3(a) and 3(b), and numerical values given in column 5, Table IV, $\Delta\tau_{\chi,M}^*$ is then scaled by ϑ through the universal scaling equation

$$\vartheta\Delta\tau_{\chi,M}^* = \left(\frac{g_{\chi,M}^{1,+}}{\mathbb{Z}_\chi^{1,+}}\right)^{1/\Delta} = \text{universal constant}, \tag{A3}$$

where $\Delta\tau_{\chi,\mathrm{MSR}}^*=b_+^*(\mu^2/a)(1-u/u^*)^{1/\Delta}$ for the minimal-subtraction renormalization scheme, and $\Delta\tau_{\chi,\mathrm{CPM}}^*=[c_t^*/(\bar{u}\Lambda)^2](1-\bar{u})^{1/\Delta}$ for the crossover parametric model, are the so-called effective Ginzburg numbers (see Refs. [25,31] for the notations and definitions of the above quantities).

Correlatively but uniquely, when Eqs. (79) or (A2) are valid (i.e., when the Ising-like critical crossover is characterized by a single parameter), we must extend the scaling analysis to the leading amplitudes, expressing again Eqs. (62) and (63) in the universal form of Eq. (64), i.e., such as

$$\xi_0^+(g_0\vartheta^\nu) = \xi^+\mathbb{L}^{\{1f\}}\vartheta^\nu = (\mathbb{Z}_\xi^+)^{-1}, \tag{A4}$$

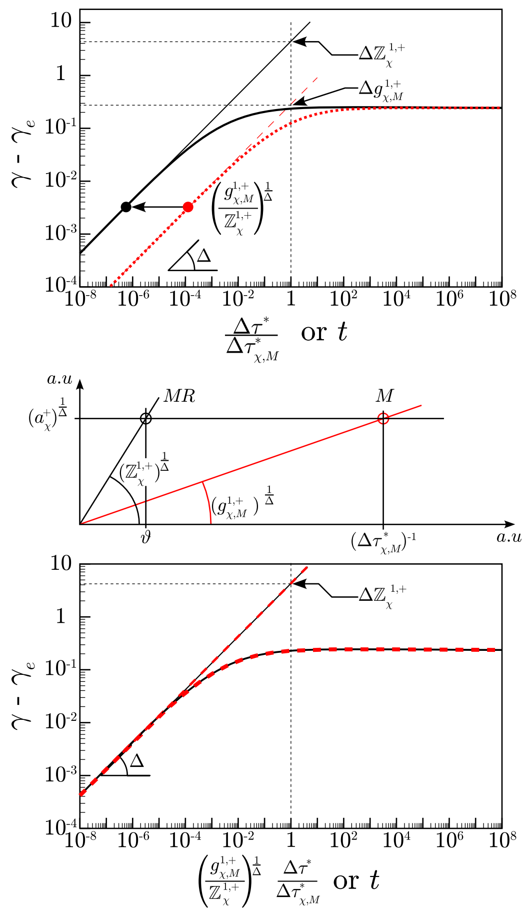

FIG. 3. (Color online) Schematic illustrations [for $\gamma-\gamma_e$ and $(a_\chi^+)^{1/\Delta}$] of the Ising-like preasymptotic equivalence between the dimensionless crossover temperature scale $\Delta\tau^*_{\chi,M}$ needed by the one-parameter crossover model M (dashed red line), and the scale factor ϑ needed by the massive renormalization (MR) scheme (full black line) [see Eqs. (75), (A2), and (A3), and Table IV]; the small difference in the respective Δ values is neglected.

$$\Gamma^+(\mathbb{L}^{\{1f\}})^{-d}(\psi_\rho)^{-2}\vartheta^\gamma = (\mathbb{Z}_\chi^+)^{-1}. \quad \text{(A5)}$$

Obviously, as for the confluent amplitude, we can close the asymptotic identification between the three (MR, MSR, CPM) models, introducing two supplementary universal numbers which relate unequivocally the scale factors $\mathbb{L}^{\{1f\}}$ and ψ_ρ of the massive renormalization scheme to the equivalent two free parameters of another crossover approach (see also Refs. [36,44] and the Appendix B below).

APPENDIX B: EFFECTIVE CROSSOVER FUNCTION BEYOND THE ISING-LIKE PREASYMPTOTIC DOMAIN

1. Effective exponent and effective amplitude

According to the above asymptotic analysis of the equivalence between crossover modeling, the scale transformations of the variables which produce the universal collapse of the Ising-like crossover curves can be illustrated by using not only effective exponents [26] but also effective amplitudes (see also Ref. [46]). Indeed, from $\chi_{\text{th}}(t)$ of Eq. (45), the local value of the effective (theoretical) exponent $\gamma_{e,\text{th}}(t)$ is defined by the equation

$$\gamma_{e,\text{th}}(t) = -\frac{\partial \ln[\chi_{\text{th}}(t)]}{\partial \ln t}. \quad \text{(B1)}$$

The local value of its attached effective (theoretical) amplitude $\mathbb{Z}^+_{e,\chi}(t)$ is defined by the equation

$$\mathbb{Z}^+_{\chi,e}(t) = \frac{\chi_{\text{th}}(t)}{t^{-\gamma_{e,\text{th}}}}. \quad \text{(B2)}$$

Therefore, $\gamma_{e,\text{th}}(t)$ and $\mathbb{Z}^+_{\chi,e}(t)$ have equivalent universal features as $\chi_{\text{th}}(t)$. By eliminating t (then simultaneously eliminating the scale factor $\vartheta_{\mathcal{L}}$ since $t=\vartheta_{\mathcal{L}}\Delta\tau^*$), the classical-to-critical crossover is characterized by a single (i.e., universal) function $\mathbb{Z}^+_{\chi,e}(\gamma_{e,\text{th}})$ over the complete range $\gamma_{\text{MF}} \leq \gamma_{e,\text{th}}(t) \leq \gamma$. This result is here represented by the top (black dot-dashed) curve in Fig. 4. Its limiting Ising-like critical point takes universal coordinates $\{\gamma;(\mathbb{Z}^+_\chi)^{-1}\}$ (see the top cross in Fig. 4.

In a similar way, from the physical function $\kappa^*_{T,\text{expt}}(\Delta\tau^*)$ of Eq. (67) which fits the experimental results using $\vartheta_{\mathcal{L}}$ [see Eq. (68)] and $\mathbb{X}^*_{0,\mathcal{L}}$ [see Eq. (98)], the local (physical) exponent is defined by

$$\gamma_{e,\text{expt}}(\Delta\tau^*) = -\frac{\partial \ln[\kappa^*_{T,\text{expt}}(\Delta\tau^*)]}{\partial \ln(\Delta\tau^*)} \quad \text{(B3)}$$

and its related local (physical) amplitude by

$$\Gamma^+_e(\Delta\tau^*) = \frac{\kappa^*_{T,\text{expt}}(\Delta\tau^*)}{(\Delta\tau^*)^{-\gamma_{e,\text{expt}}}}. \quad \text{(B4)}$$

Eliminating $\Delta\tau^*$ from Eqs. (B3) and (B4), the corresponding physical function $\Gamma^+_e(\gamma_{e,\text{expt}})$ is represented in Fig. 4 by the bottom (red dashed) curve, selecting xenon as a typical example [46]. Its related Ising-like critical point takes the physical coordinates $\{\gamma;\Gamma^+\}$, as represented by the bottom cross in Fig. 4 [with $\Gamma^+(\text{Xe})=0.057\,824$). For quantitative comparison in this "physical" part of Fig. 4, we have also represented the experimental lower (black dashed) curve for Γ^+_e values obtained from the Güttinger and Cannell fit of their susceptibility measurements [47] (bold part of the curve), and from several pVT measurements reported in Table V (full points labeled 1 to 4, open circle labeled P).

Finally, considering the master singular behavior $\mathcal{X}^*_{\text{qf}}(\mathcal{T}^*)$ of Eq. (85)) using $\Theta^{\{1f\}}$ [see Eq. (68)] and $\mathbb{Z}^{\{1f\}}_\chi$ [see Eq. (98)], we can define the local (master) exponent by

$$\gamma_{e,1f}(\mathcal{T}^*) = -\frac{\partial \ln[\mathcal{X}^*_{\text{qf}}(\mathcal{T}^*)]}{\partial \ln(\mathcal{T}^*)} \quad \text{(B5)}$$

and its related local (master) amplitude by

$$\mathcal{Z}^+_{\chi,e}(\mathcal{T}^*) = \frac{\mathcal{X}^*_{\text{qf}}(\mathcal{T}^*)}{(\mathcal{T}^*)^{-\gamma_{e,1f}}}. \quad \text{(B6)}$$

After $\mathcal{T}^*$ elimination between Eqs. (B5) and (B6), the master function $\mathcal{Z}^+_{\chi,e}(\gamma_{e,1f})$ can also be represented by the unique median (blue double-dot-dashed) curve in Fig. 4. Its Ising-

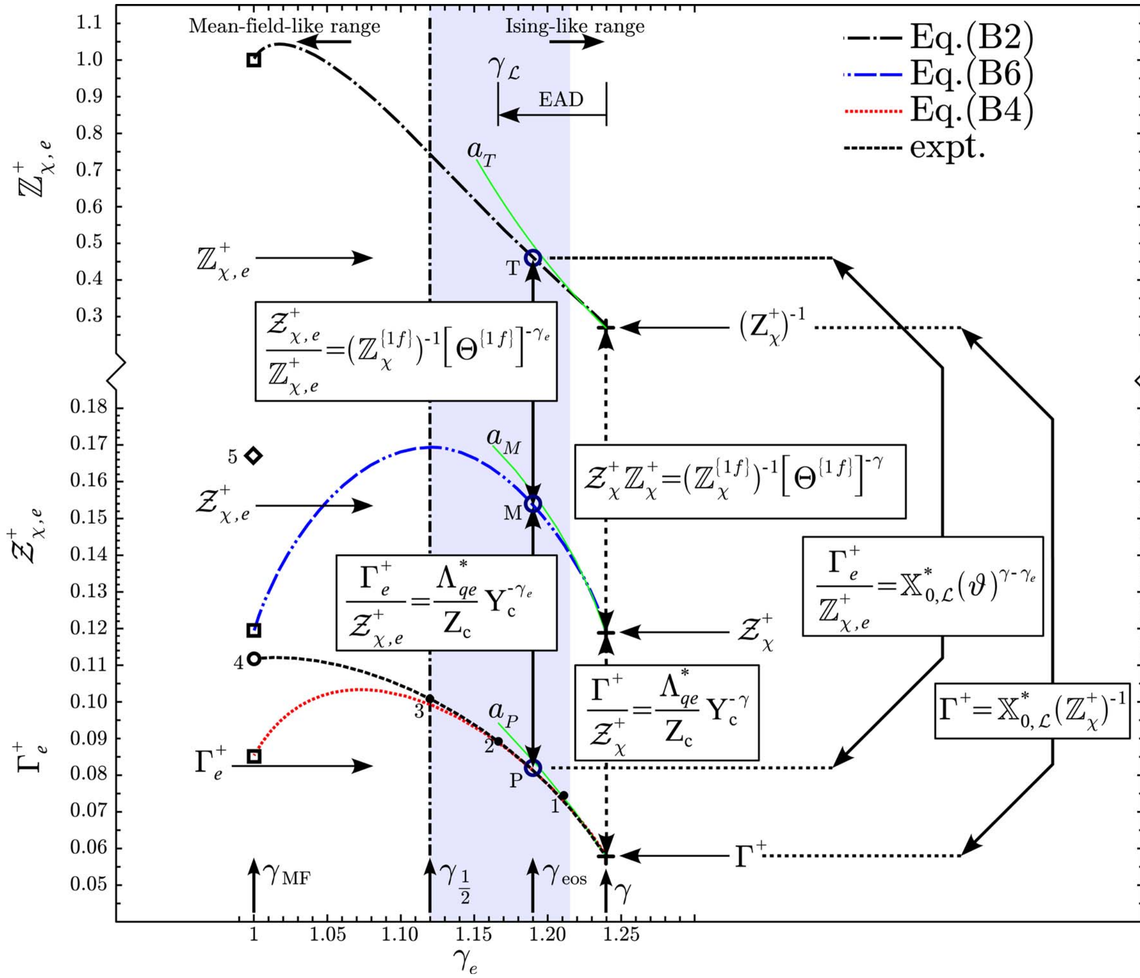

FIG. 4. (Color online) Theoretical estimations of the effective mean [$\mathbb{Z}^+_{\chi,e}$, upper black dot-dashed curve, Eq. (B2)], master [$\mathcal{Z}^+_{\chi,e}$, median blue double-dot-dashed curve, Eq. (B6)], and physical (xenon) [Γ^+_e, lower red dashed curve, Eq. (B4)] amplitudes as a function of the effective exponent γ_e for the susceptibility case along the critical isochore in the homogeneous domain. Double (dotted at $\gamma_e=\gamma$, full at $\gamma_e=\gamma_{EOS}$) arrays, point-to-point (pluses at $\gamma_e=\gamma$, open circles at $\gamma_e=\gamma_{EOS}$, open squares at $\gamma_e=\gamma_{MF}$) transformations between effective functions using Y_c and Z_c, or, alternatively but equivalently, ϑ and $\mathbb{X}^*_{0,\mathcal{L}}$ (each relation associated with the transformation at γ and γ_e constant value is illustrated in an attached rectangular box); lower black dashed bold curve (labeled expt), Γ^+_e from Güttinger and Cannell's fit for xenon susceptibility [47] (see also text and Table V); M coordinates $\gamma_{EOS}=1.19$ and $\mathcal{Z}^+_{\chi,e}=0.153\,74$, tangent line at the point M to the theoretical curve of Eq. (85) in Fig. 1; other quantities, points, and symbols, see text.

TABLE V. Column 1, index of the points in Figs. 4 and 5, columns 2 and 3, effective power law description of pVT measurements in xenon (see Ref. [46] for detail and data sources); columns 4–6, calculated values of the (geometrical) mean temperature $\langle\Delta\tau^*_{pVT}\rangle = \sqrt{\Delta\tau^*_{min}\Delta\tau^*_{max}}$ of pVT measurements (column 4), theoretical local temperature $\Delta\tau^*_{th}(\gamma_{e,th})$ satisfying the condition $\gamma_{e,th}(t)=\gamma_{e,pVT}(\Delta\tau^*)$ (column 5), and theoretical local amplitude $\Gamma^+_{e,th}$ for $\gamma_{e,th}(t)=\gamma_{e,pVT}(\Delta\tau^*)$ (column 6).

	$\gamma_{e,pVT}$	$\Gamma^+_{e,pVT}$	$\langle\Delta\tau^*_{pVT}\rangle$ $\langle\Delta\tau^*_{pVT}\rangle=\sqrt{\Delta\tau^*_{min}\Delta\tau^*_{max}}$	$\Delta\tau^*_{th}(\gamma_{e,th})$ $\gamma_{e,th}(t)=\gamma_{e,pVT}(\Delta\tau^*)$	$\Gamma^+_{e,th}$
1	1.211 ± 0.01	0.0743 ± 0.015	2.07×10^{-3}	2.95×10^{-3}	0.07263
2	1.16665	0.089	2.24×10^{-2}	3.338×10^{-2}	0.08859
3	$1.1198(=\gamma_{1/2})$	0.101	1.21×10^{-1}	1.928×10^{-1}	0.09960
4	$1(=\gamma_{MF})$	0.11	$7.1\ 10^{-1}$	∞	0.08507
5(vdW)	$1(=\gamma_{vdW})$	$\frac{1}{6}(=\Gamma^+_{vdW})$		∞	0.08507
P(EOS)	$1.19(=\gamma_{EOS})$	$0.0793(=\Gamma^+_{EOS})$	1.13×10^{-2}	1.135×10^{-2}	0.08084

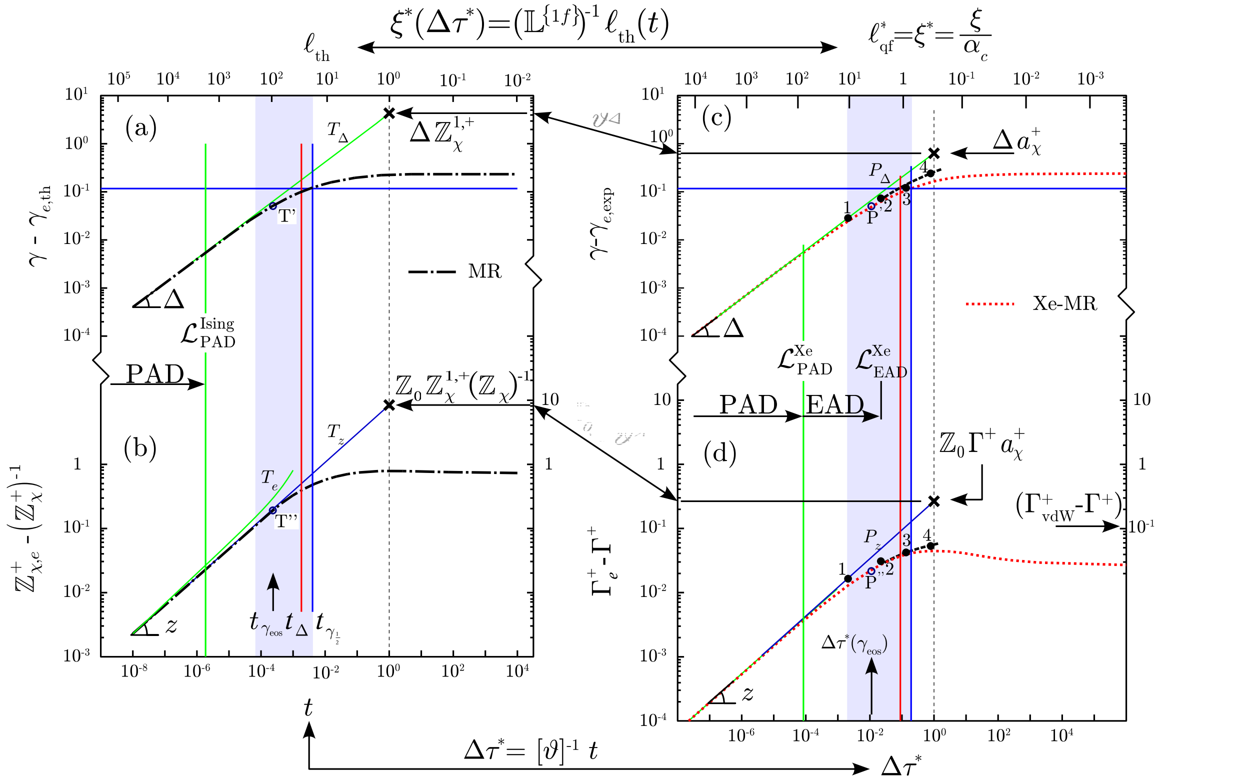

FIG. 5. (Color online) (a) Dot-dashed (black) curve (labeled MR), $\gamma-\gamma_{e,\mathrm{th}}$ as a function of t, calculated from the theoretical crossover function of Eq. (45) for susceptibility (log-log scale); (green) line T_Δ, limiting singular behavior [see Eq. (B18)] within the Ising-like preasymptotic domain of extension $t^* < \mathcal{L}_{\mathrm{PAD}}^{\mathrm{Ising}}$ [vertical green line, see Eq. (57)]; curve T_Δ of slope Δ crossing the vertical line $t=1(\times)$, value of the first confluent amplitude of Eq. (56); vertical (blue and pink) lines, $t_{\gamma_{1/2}}$ and t_Δ coordinates for $\gamma_{e,\mathrm{th}}=\gamma_{1/2}$ and $D(t_\Delta)=\Delta_{1/2}$, respectively. (b) Dot-dashed (black) curve (labeled MR), same as for (a) for $(\mathbb{Z}_{\chi,e}^{+})^{-1}-(\mathbb{Z}_{\chi}^{+})^{-1}$ as a function of t, calculated from Eq. (45); (green and blue) lines T_e and T_z, logarithmic singularity and power law approximation [see Eqs. (B14) and (B15), respectively, and text for detail]. (c), (d) Dotted (red) curves (labeled Xe-MR), corresponding xenon quantities $\gamma-\gamma_e$ and $\Gamma_e^{+}-\Gamma^{+}$ as a function of $\Delta\tau^*=[\vartheta(\mathrm{Xe})]^{-1}t$; points P and 1–4, pVT results (see Ref. [46] for detail) given in Table V (see also Fig. 4); (green and blue) lines P_Δ and P_z, xenon counterparts of the theoretical (green and blue) lines T_Δ and T_z. $\times$ at $t=\Delta\tau^*=1$, point-to-point transformations between first confluent amplitudes $\mathbb{Z}_{\chi}^{1,+}$ and a_{χ}^{+} (upper arrow) and between leading amplitudes $(\mathbb{Z}_{\chi}^{+})^{-1}$ and Γ^{+} (lower arrow).

like critical point takes the master coordinates $\{\gamma;\mathcal{Z}_{\chi}^{+}\}$, corresponding to the median cross in Fig. 4.

Our main interest can then be focused on the point-to-point transformation at constant γ_e between these three curves, using only two fluid-dependent parameters, either $\vartheta_{\mathcal{L}}$ and $\mathbb{X}_{0,\mathcal{L}}^{*}$ for the physical quantities, or $\Theta^{\{1f\}}$ and $\mathbb{Z}_{\chi}^{\{1f\}}$ for the master quantities. We recall that, when $\vartheta_{\mathcal{L}}$ $(\Theta^{\{1f\}})$ and $\mathbb{Z}_{\xi}^{\{1f\}} \equiv \mathbb{L}^{\{1f\}}$ are known, $\mathbb{X}_{0,\mathcal{L}}^{*}$ gives unequivocal determination of ψ_ρ $(\Psi^{\{1f\}})$. Now, introducing also Y_c and Z_c, the complete set of the relations between the—theoretical, master, and physical—amplitudes are summarized in Fig. 4. Consequently, this figure closes the master description of $\mathcal{X}_{\mathrm{qf}}^{*}(\gamma_{e,1f})$ establishing unequivocal links between the three parameter sets $\{\vartheta_{\mathcal{L}};\mathbb{X}_{0,\mathcal{L}}^{*}\}$, $\{\Theta^{\{1f\}};\mathbb{Z}_{\chi}^{\{1f\}}\}$, and $\{Y_c;Z_c\}$. Figure 4 contains explicit equations of the schematic links given in Fig. 2 for the isothermal susceptibility case (with the implicit master condition $\mathbb{Z}_{\xi}^{\{1f\}} \equiv \mathbb{L}^{\{1f\}} = g_0\alpha_c$ fixing g_0).

Hereafter we discuss the experimental results obtained at large distance from the critical point, i.e., *beyond* the Ising-like preasymtotic domain where practical estimations of γ_e are significantly different from γ (an analysis of the Ising-like preasymptotic domain very close to the Ising-like limit $\gamma_e \to \gamma$ will be in consideration in Ref. [46]; see also the last section of this Appendix B). In particular, we focus our present attention on the range $1.215 \gtrsim \gamma_e \gtrsim \gamma_{1/2} \approx 1.12$ corresponding to the blued area in Fig. 4 [obviously equivalent to the blued area in Fig. 1(c)].

We start with the xenon (Xe) case selected as a standard one-component fluid. We can then estimate the (theoretical, master, and physical) crossover functions for the correlation length and the isothermal compressibility of xenon, using $\{Y_c=4.93846; Z_c=0.286021\}$ and $\{\vartheta_{\mathcal{L}}=0.021\,175; \mathbb{X}_{0,\mathcal{L}}^{*}=0.214\,503\}$ (or $\{\vartheta=0.021\,175; \psi_\rho=3.28165\times10^{-4}; g_0=29.0245\ \mathrm{nm}^{-1}\}$), with $\alpha_c=0.881\,498$ nm and $\mathbb{L}_{0,\mathcal{L}}^{*}=0.444\,008$ (for details, see Ref. [46]). As a basic application, we can define the correspondence between theoretical and physical temperature ranges and between theoretical and physical correlation length ranges for description of either $\gamma-\gamma_{e,\mathrm{th}}$ and $\mathbb{Z}_{\chi,e}^{+}-(\mathbb{Z}_{\chi}^{+})^{-1}$ as a function of t and as a function of ℓ_{th}, or $\gamma-\gamma_{\ell,\mathrm{expt}}$ and $\Gamma_e^{+}-\Gamma^{+}$ as a function of $\Delta\tau^*$ and as a function of ξ^*. Each result is illustrated by a (black dot-dashed or red dotted) curve in each part (a)–(d) of Fig. 5. Now, the blued areas in Fig. 5 correspond either to the theoretical ranges $10^{-4} \lesssim t \lesssim 2\times10^{-3}$ (bottom axis) and $180 \gtrsim \ell_{\mathrm{th}} \gtrsim 18$ (top axis) in (a) and (b), or the physical (xenon) ranges $5\times10^{-3} \lesssim \Delta\tau^* \lesssim 10^{-1}$ (bottom axis) and $10.5 \gtrsim \xi^* \gtrsim 0.73$ (top axis) in (c) and (d).

The following compares these theoretical predictions to the $\gamma_{e,pVT}$ and $\Gamma^+_{e,pVT}$ values obtained from *pVT* measurements [49–53] (see also details in Ref. [46]). We recall that the *pVT* measurements were performed at *finite* distance to the critical point, such that the $\kappa^*_{T,pVT}$ data obtained from *pVT* data can be fitted by an effective power law

$$\kappa^*_{T,pVT} = \Gamma^+_{e,pVT}(\Delta\tau^*)^{-\gamma_{e,pVT}} \tag{B7}$$

valid only in a restricted temperature range defined by $\Delta\tau^*_{\min} \le \Delta\tau^* \le \Delta\tau^*_{\max}$. The measured (exponent and amplitude) parameters $\{\gamma_{e,pVT};\Gamma^+_{e,pVT}\}$ are then associated with the temperature range $\{\Delta\tau^*_{\min};\Delta\tau^*_{\max}\}$ of central value $\langle\Delta\tau^*_{e,pVT}\rangle = \sqrt{\Delta\tau^*_{\min}\Delta\tau^*_{\max}}$ (in logarithmic scale) located beyond the Ising-like preasymptotic domain. Therefore, we can represent these results by points of respective coordinates $\{\gamma_{e,pVT};\Gamma^+_{e,pVT}\}$, $\{\gamma_{e,pVT};\langle\Delta\tau^*_{pVT}\rangle\}$, and $\{\Gamma^+_{e,pVT};\langle\Delta\tau^*_{pVT}\rangle\}$ in each appropriate binary diagram.

The four points (labeled 1–4) illustrated in Figs. 4, 5(c), and 5(d) correspond to the xenon results reported in rows labeled 1–4, respectively, of Table V. The points labeled 1 and 2 follow the general trend of the theoretical curves. This result confirms that, in spite of a large correlated error bar in the adjustable exponent and amplitude parameters, the variations of their respective central values agree with a two-parameter description within the Ising-like side of the crossover domain where $\gamma > \gamma_{e,pVT} > 1.17$. However, the point labeled 3, and more significantly the point labeled 4, show that the *pVT* experimental results are not in agreement with the mean-field behavior predicted by the crossover function within the mean-field-like side where $\gamma_{1/2} \gtrsim \gamma_{e,pVT} > \gamma_{\mathrm{MF}}$. The failure of the classical corresponding state theory is also illustrated by the point labeled 5 in Fig. 4, which corresponds to the result obtained from the van der Waals equation of state [see the line labeled 5(vdW) in Table V].

To translate the $\mathcal{L}^{+,\{1f\}}_{\mathrm{EAD}}$ master value [Eq. (114)] into a $\gamma_{\mathcal{L}}$ master value which delimits the effective range of the extended asymptotic domain in Fig. 4, one needs to consider the upper horizontal axis of Figs. 5(c) and 5(d), which measures the master correlation length $\xi^* = \xi/\alpha_c(\mathrm{Xe})$ [i.e., the dimensionless ratio which compares the size of the critical fluctuation to the actual range of the microscopic interaction, with $\Lambda^*_{qe}(\mathrm{Xe}) = 1$ in xenon case]. As a matter of fact, the value $\mathcal{L}^{\mathrm{Xe}}_{\mathrm{EAD}} \approx 2\times10^{-2}$ corresponds to the value $\mathcal{L}^{+,\{1f\}}_{\mathrm{EAD}}/Y_c(\mathrm{Xe})$ where $\xi^* \approx 3$. Therefore, the associated *local* value is $\gamma_{\mathcal{L}} \approx 1.16–1.17$. This value descriminates the "non-Ising-like" range $\gamma_e < \gamma_{\mathcal{L}}$ [including the value $\gamma_{1/2} = (\gamma + \gamma_{\mathrm{MF}})/2 \approx 1.12$] where the effective classical-to-critical crossover for xenon is no longer accounted for by the theoretical crossover function, as shown in Fig. 4, where an increasing difference is observed between the curves labeled Xe-MR and the dotted curve labeled Xe-expt when $\gamma_e \to \gamma_{\mathrm{MF}} = 1$.

Accounting for an extended (Ising-like) asymptotic domain defined by $\gamma_e < \gamma_{\mathcal{L}}$, we are also able to reconsider the results previously obtained using a universal scaled form of the equation of state with universal values of the exponents significantly different from the Ising ones. As a typical example, the xenon results obtained from the restricted linear model of a parametric equation of state with $\gamma_{\mathrm{EOS}} = 1.19$ [see the line labeled *P*(EOS) in Table V] are in excellent agreement with the theoretical crossover function, as illustrated by the point labeled *P* in Fig. 4. Moreover, using as a $\Delta\tau^*$ coordinate the theoretical value $\Delta\tau^*_{\mathrm{th}}(\gamma_{\mathrm{EOS}}) = 1.135\times10^{-2}$ [Eq. (B3)], we can show that these results are also well accounted for in the theoretical temperature dependence [see the corresponding points labeled P' and P'' in Figs. 5(c) and 5(d), respectively]. Such results confirm that *two* xenon parameters involved in an universal form of the equation of state can be used as Ising-like characteristic factors to be related to the two scale factors Y_c and Z_c, as illustrated in the next section for the case of the linear parametric equation of state.

Master crossover provided by a restricted linear model of a parametric equation of state

In the 1970s, the first analyses of the two-scale factor universality for one-component fluids used effective scaled forms of the equation of state to fit the *pVT* data measured at finite distance to the critical point (for detail, see Refs. [17,18,54–60]). Such a thermodynamic approach to universality was based on a limited number of characteristic parameters for each pure fluid, using effective universal values for the critical exponents. We limit the present purpose to the well-known restricted linear model of the parametric equation of state [56], with application to several different fluids [57–60]. The two main interests for such a choice are the following.

(i) The effective thermodynamic exponents have been precisely fixed at (non-Ising) values of $\gamma_{\mathrm{EOS}} = 1.190$, $\beta_{\mathrm{EOS}} = 0.355$, $\alpha_{\mathrm{EOS}} = 0.100$ (the subscript EOS recalls the origin of these effective values). As shown in Fig. 4, the value $\gamma_{\mathrm{EOS}} = 1.190$ is precisely within the selected γ_e range beyond the Ising-like preasymptotic domain, but well inside the extended asymptotic domain $\gamma_e < \gamma_{\mathcal{L}}$, which corresponds to $\Delta\tau^* \lesssim \mathcal{L}^{\mathrm{Xe}}_{\mathrm{EAD}}$.

(ii) The effective values of the thermodynamic amplitude Γ^+_{EOS} [see below Eq. (B8)] of the isothermal compressibility were then obtained using only two adjustable (fluid-dependent) parameters (namely, *k* and *a*), which are the two characteristic parameters involved in the scaled equation of state. As shown in Fig. 4, "equivalent" values of $\Gamma^+_e(\gamma_{\mathrm{EOS}})$ at $\gamma_{\mathrm{EOS}} = 1.190$ can be simultaneously obtained by a scale transformation between the point *P* (on the physical curve) and the point *M* (on the master curve), which also involves only two characteristic parameters (namely, Y_c and Z_c) (admitting that the parameter Λ^*_{qe} which accounts for quantum effects is known).

Therefore, both in quantity (two), and in nature (Ising-like), the fluid-dependent parameters *k* and *a* appear equivalent to Y_c and Z_c, except from the noticeable distinction of their respective determinations, outside the Ising-like preasymptotic domain for the $\{k;a\}$ pair, asymptotically close to the critical point for the $\{Y_c;Z_c\}$ pair.

Now we compare the corresponding values of Γ^+_{EOS} and $\Gamma^+_e(\gamma_{\mathrm{EOS}})$ for 12 selected fluids. From the linear model of the

TABLE VI. Two-parameter universality of the effective amplitude of the isothermal compressibility estimated from the linear model of a parametric equation of state and the master modification of the theoretical function.

Fluid	x_0 [58]	a [58]	k Eq. (B10)	Γ^+_{EOS} Eq. (B9)	Y_c	Z_c	$\Gamma^+_e(\gamma_{\mathrm{EOS}})$ Eq. (B11)	$\Delta\tau^*(\gamma_{\mathrm{EOS}})$ Eq. (B13)	$r\%(\Gamma^+_e)$
$^3He^{(*)}$	0.489	4.63	0.9235	0.1995	2.3984	0.30129	$0.20003^{(*)}$	2.326×10^{-2}	−0.283
Ar	0.183	16.5	1.309	0.07934	4.3288	0.2896	0.09284	1.289×10^{-2}	−14.5
Kr	0.183	16.5	1.309	0.07934	4.9437	0.2913	0.07887	1.128×10^{-2}	0.6
Xe	0.183	16.5	1.309	0.07934	4.9385	0.28602	0.08084	1.135×10^{-2}	−1.86
O_2	0.183	15.6	1.309	0.08392	4.9864	0.28797	0.07890	1.119×10^{-2}	6.36
N_2	0.164	18.2	1.361	0.07478	5.3701	0.28887	0.07201	1.039×10^{-2}	3.84
CH_4	0.164	17.0	1.361	0.08006	4.9838	0.28678	0.07928	1.119×10^{-2}	0.99
C_2H_4	0.166	17.5	1.355	0.07744	5.3487	0.2813	0.07431	1.043×10^{-2}	4.22
CO_2	0.141	21.8	1.436	0.06587	6.0104	0.27438	0.06631	0.928×10^{-2}	0.653
NH_3	0.109	21.4	1.361	0.07353	6.3019	0.24294	0.07079	0.885×10^{-2}	3.88
H_2O	0.100	22.3	1.622	0.07275	6.8552	0.22912	0.0679	0.814×10^{-2}	7.14
D_2O	0.100	22.3	1.622	0.07275	7.0728	0.22783	0.0658	0.799×10^{-2}	10.6

parametric equation of state, Eq. (B7) can be rewritten as

$$\kappa^*_T = \Gamma^+_{\mathrm{EOS}}(\Delta\tau^*)^{-\gamma_{\mathrm{EOS}}}, \tag{B8}$$

where Γ^+_{EOS} is related to the characteristic parameters k and a as follows:

$$\Gamma^+_{\mathrm{EOS}} = \frac{k}{a}. \tag{B9}$$

Considering then a restricted form of the linear model such as analyzed in Ref. [58], k can be estimated from the relation

$$k = \left(\frac{x_0}{b^2_{\mathrm{SLH}}-1}\right)^{-\beta_{\mathrm{EOS}}} \tag{B10}$$

where $b^2_{\mathrm{SLH}}=1.3908$ is a universal quantity [56] while x_0 is a fluid-dependent parameter related to the value of the effective amplitude of the coexistence curve (associated with the value $\beta_{\mathrm{EOS}}=0.355$ of the effective exponent). The values of x_0 and a can be found in Ref. [58]. They are reported with the corresponding k values in Table VI (columns 2–4, respectively) for the selected 12 fluids (column 1). The related values of Γ^+_{EOS} obtained by using Eq. (B9) are given in Table VI (column 5).

The unequivocal scale transformation between the points P and M is given by the relation (see Fig. 4)

$$\Gamma^+_e(\gamma_{\mathrm{EOS}}) = \mathcal{Z}^+_{\chi,e}(\gamma_{\mathrm{EOS}})\frac{\Lambda^*_{qe}(Y_c)^{-\gamma_{\mathrm{EOS}}}}{Z_c}, \tag{B11}$$

where $\mathcal{Z}^+_{\chi,e}(\gamma_{\mathrm{EOS}})$ is the effective master amplitude for the $\mathcal{T}^*(\gamma_{e,1f})$ value satisfying the condition $\gamma_{e,1f}=\gamma_{\mathrm{EOS}}=1.19$. For practical use of Eq. (B11), the crucial advantage is given by the unequivocal scale transformation between the points T (on the theoretical curve) and M (on the master curve) illustrated in Fig. 4. That provides immediately $\mathcal{Z}^+_{\chi,e}(\gamma_{\mathrm{EOS}}) =\mathbb{Z}^+_{\chi,e}(\gamma_{\mathrm{EOS}})[\mathbb{Z}^{\{1f\}}_\chi(\Theta^{\{1f\}})^{\gamma_{\mathrm{EOS}}}]^{-1}$ and $\mathcal{T}^*(\gamma_{\mathrm{EOS}})=t(\gamma_{\mathrm{EOS}}) \times(\Theta^{\{1f\}})^{-1}$. Using Eqs. (B1) and (B2), the theoretical function $\chi_{\mathrm{th}}(t)$ leads to the values $t(\gamma_{\mathrm{EOS}})=2.392\times10^{-4}$ and $\mathbb{Z}^+_{\chi,e}(\gamma_{\mathrm{EOS}})=0.456\,414$ related to coordinates of the points labeled T' and T'' in Figs. 5(a) and 5(b), respectively. Using the numerical values of $\mathbb{Z}^{\{1f\}}_\chi$ and $\Theta^{\{1f\}}$ given in Table III, we obtain $\mathcal{T}^*(\gamma_{\mathrm{EOS}})=5.579\times10^{-2}$ and $\mathcal{Z}^+_{\chi,e}(\gamma_{\mathrm{EOS}})=0.153\,74$. Subsidiarily, in Fig. 1(c), we note that the master curve $\chi^*_{\mathrm{qf}}(\mathcal{T}^*)$ of Eq. (85) has a tangent curve of slope $-\gamma_{\mathrm{EOS}}$ at the point M of $\mathcal{T}^*(\gamma_{\mathrm{EOS}})$ coordinate which corresponds to the effective power law

$$\mathcal{X}^+_{\mathrm{EOS}}(\mathcal{T}^*) = 0.153\,74(\mathcal{T}^*)^{-1.19}. \tag{B12}$$

The values of Y_c and Z_c for the selected fluids are reported in Table VI (columns 6 and 7, respectively). The estimated values of $\Gamma^+_e(\gamma_{\mathrm{EOS}})$ using Eq. (B11) are given in Table VI (column 8). Each physical curve $\kappa^*_T(\Delta\tau^*)$ of Eq. (67) has a tangent curve of slope $-\gamma_{\mathrm{EOS}}$ at the point of the $\Delta\tau^*$ coordinate:

$$\Delta\tau^*(\gamma_{\mathrm{EOS}}) = \frac{\mathcal{T}^*(\gamma_{\mathrm{EOS}})}{Y_c} \tag{B13}$$

(see column 9 of Table VI), which corresponds to the effective power law $\Gamma^+_e(\Delta\tau^*)=\Gamma^+_e(\gamma_{\mathrm{EOS}})(\Delta\tau^*)^{-\gamma_{\mathrm{EOS}}}$.

The residuals $r\%(\Gamma^+_e)=100((\Gamma^+_{\mathrm{EOS}}/\Gamma^+_e(\gamma_{\mathrm{EOS}}))-1)$ (see column 10, Table VI), generally lower than the typical experimental uncertainty estimated to be 10%, confirm that the universal features observed beyond the Ising-like preasymptotic domain but within the Ising-like extended asymptotic domain, i.e., $\Delta\tau^*\lesssim t_{\mathrm{EAD}}/\vartheta=\mathcal{L}^{+,\{1f\}}_{\mathrm{EAD}}/Y_c=\mathcal{L}^f_{\mathrm{EAD}}$ with $t_{\mathrm{EAD}} =\Theta^{\{1f\}}\mathcal{L}^{+,\{1f\}}_{\mathrm{EAD}}$ and $\mathcal{L}^{+,\{1f\}}_{\mathrm{EAD}}\simeq0.07-0.1$, are well characterized by the two critical scale factors Y_c and Z_c of each fluid f.

Universal approximation of the logarithmic singularity of effective amplitudes

Another practical application of the point-to-point transformations given in Fig. 4 can be obtained by focusing our attention on the logarithmic singularity of any first derivative $(\partial \mathbb{Z}^{+}_{P,e}/\partial e_{P,e})_{e_{P,e}\to e_P}$ close to the Ising-like critical point, for any effective amplitude power law $\mathbb{Z}^{+}_{P,e}(t)=F_P(t)/t^{-e_{P,e}}$ estimated from any crossover function $F_P(t)$ given in Ref. [8] {with $e_{P,e}(t)=-\partial \ln[F_P(t)]/\partial \ln t$} (see Refs. [8,46] for details). For the susceptibility case, the logarithmic singularity of $(\partial \mathbb{Z}^{+}_{\chi,e}/\partial \gamma_{e,\text{th}})_{\gamma_{e,\text{th}}\to\gamma}$ extrapolated beyond the Ising-like preasymptotic domain is illustrated by the curves labeled a_T, a_M, and a_P in Fig. 4. For better evaluation within the preasymptotic domain, the related amplitude singularity in terms of the thermal field dependence is given in Fig. 5(b), for example, by the curve T_e of equation

$$\mathbb{Z}^{+}_{\chi,e}-(\mathbb{Z}^{+}_{\chi})^{-1}=(\mathbb{Z}^{+}_{\chi})^{-1}\mathbb{Z}^{1,+}_{\chi}t^{\Delta \mathbb{Z}^{1,+}_{\chi}t^{\Delta}}[1-\ln(t^{\Delta})]t^{\Delta}. \quad \text{(B14)}$$

We can approximate Eq. (B14) by the following universal power law:

$$\mathbb{Z}^{+}_{\chi,e}-(\mathbb{Z}^{+}_{\chi})^{-1}=\mathbb{Z}_0(\mathbb{Z}^{+}_{\chi})^{-1}\mathbb{Z}^{1,+}_{\chi}t^{z}, \quad \text{(B15)}$$

where $\mathbb{Z}_0=3.7\pm 0.1$ and $z=0.45\pm 0.035$ are independent of the property and the domain. The exponent condition $z<\Delta$, leading to $z/\Delta=1-u<1$, conforms to the logarithmic singularity of the first derivative $(\partial \mathbb{Z}^{+}_{\chi,e}/\partial \gamma_{e,\text{th}})_{\gamma_{e,\text{th}}\to\gamma}$, here approximated by a power law $(\partial \mathbb{Z}^{+}_{\chi,e}/\partial \gamma_{e,\text{th}})_{\gamma_{e,\text{th}}\to\gamma}\propto(\gamma-\gamma_{e,\text{th}})^{-u}$. For practical use, we arbitrarily choose $u=\alpha$, leading us to define $z=\Delta(1-\alpha)$. The validity of this approximation is illustrated by the curve T_z in Fig. 5(b).

Correspondingly, in Fig. 5(d), the physical asymptotic representation of $\Gamma^{+}_{e}-\Gamma^{+}$ is now approximated by the curve P_z of asymptotic equation

$$\Gamma^{+}_{e}-\Gamma^{+}=\mathbb{Z}_0\Gamma^{+}a^{+}_{\chi}(\Delta\tau^*)^{z}=\mathbb{X}_{0,\mathcal{L}}\mathbb{Z}_0\mathbb{Z}^{1,+}_{\chi}(\mathbb{Z}^{+}_{\chi})^{-1}\vartheta^{\Delta}(\Delta\tau^*)^{z}. \quad \text{(B16)}$$

Using Eqs. (B15) and (B16) at $t=\Delta\tau^*=1$, we obtain

$$\Gamma^{+}_{e}(1)-\Gamma^{+}=\mathbb{X}^{*}_{0,\mathcal{L}}\vartheta^{\Delta}[\mathbb{Z}^{+}_{\chi,e}(1)-(\mathbb{Z}^{+}_{\chi})^{-1}]. \quad \text{(B17)}$$

The point-to-point universal transformation which approximates the logarithmic singularity is then illustrated by the two correlated points (symbol ×) at $t=\Delta\tau^*=1$, in Figs. 5(b) and 5(d), respectively. As expected from Fig. 4, this transformation is given by the product $\mathbb{X}^{*}_{0,\mathcal{L}}\vartheta^{\Delta}$.

Obviously, to close the asymptotic behavior within the Ising-like preasymptotic domain we can also consider the respective asymptotic curves labeled T_Δ and P_Δ in Figs. 5(a) and 5(c), of equations

$$\gamma-\gamma_{e,\text{th}}=\Delta\mathbb{Z}^{1,+}_{\chi}t^{\Delta}, \quad \text{(B18)}$$

$$\gamma-\gamma_{e,\text{expt}}=\Delta a^{+}_{\chi}(\Delta\tau)^{\Delta}. \quad \text{(B19)}$$

Here, the point-to-point transformation at $t=\Delta\tau^*=1$ (symbol ×) is given by the scale factor universal power law ϑ^{Δ}.

The above approximation of the logarithmic singularity has practical importance for better analysis of experimental data when the value γ_e is found in the range $\gamma_e=1.21-1.24$, i.e., a value which approaches the theoretical Ising value. As a typical example we use the value $\gamma_{e,pVT}=1.211\pm 0.025$ obtained by Levelt-Sengers *et al.* [58] from their analysis of the *pVT* measurements of Habgood and Schneider [51] in the temperature range $0.2\leq T-T_c\leq 1.8$ K, i.e., $\Delta\tau^*_{\text{min}}=6.9\times 10^{-4}$, $\Delta\tau^*_{\text{max}}=6.2\times 10^{-3}$, and $\langle\Delta\tau^*_{pVT}\rangle=2.07\times 10^{-3}$ (see row 1, column 4, Table IV). This result is then centered near the Ising-like borderline of the blued domain previously analyzed. As evidenced by the matching of the corresponding points labeled 1 with the curves P_Δ and P_z in Figs. 5(c) and 5(d), such a result also appears correctly accounted for using the above approximation. Therefore, using Eqs. (B19) and (B16), we can easily calculate the two values of the *true* confluent and leading amplitudes of the two-term Wegner expansion from the following equations:

$$a^{+}_{\chi}|_{pVT}=\frac{\gamma-\gamma_{e,pVT}}{\Delta\langle\Delta\tau^*_{pVT}\rangle^{\Delta}}=1.266\,66, \quad \text{(B20)}$$

$$\Gamma^{+}|_{pVT}=\frac{\Gamma^{+}_{e,pVT}}{1+\dfrac{\mathbb{Z}_0}{\Delta}(\gamma-\gamma_{e,pVT})\langle\Delta\tau^*_{pVT}\rangle^{z-\Delta}}=0.057\,355. \quad \text{(B21)}$$

These above values are in excellent agreement with the estimated ones $a^{+}_{\chi}=1.23709$ and $\Gamma^{+}=0.057\,824$ [46] from application of the scale dilatation method. In Eq. (B21), we note the practical importance of the prefactor $\mathbb{Z}_0$.

In conclusion, using Figs. 4 and 5, we have explicitly demonstrated that the two dimensionless scale factors Y_c and Z_c [or alternatively but equivalently $\vartheta_{\mathcal{L}}$ ($\equiv\vartheta$) and $\mathbb{X}^{*}_{0,\mathcal{L}}$], which characterize each one-component fluid f belonging to the $\{1f\}$ subclass, can be used to calculate the isothermal compressibility over the Ising-like extended asymptotic domain $\Delta\tau^*\lesssim\mathcal{L}^{f}_{\text{EAD}}$.

[1] See, for example, J. Zinn-Justin, *Euclidean Field Theory and Critical Phenomena*, 4th ed. (Clarendon, Oxford, 2002).

[2] See, for example, A. Pelissetto and E. Vicari, Phys. Rep. **368**, 549 (2002), and references therein.

[3] K. G. Wilson and M. E. Fisher, Phys. Rev. Lett. **28**, 240 (1972).

[4] K. G. Wilson and J. Kogut, Phys. Rep., Phys. Lett. **12**, 75 (1974).

[5] F. J. Wegner, Phys. Rev. B **5**, 4529 (1972).

[6] A. Pelissetto, P. Rossi, and E. Vicari, Phys. Rev. E **58**, 7146 (1998).

[7] For a review, see, for example, M. A. Anisimov and J. V. Sengers, in *Equations of State for Fluids and Fluid Mixtures*, edited by J. V. Sengers, R. F. Kayser, C. J. Peters, and H. J. White, Jr. (Elsevier, Amsterdam, 2000), part I, pp. 381–434, and references therein.

[8] Y. Garrabos and C. Bervillier, Phys. Rev. E **74**, 021113 (2006).
[9] R. Guida and J. Zinn-Justin, J. Phys. A **31**, 8103 (1998).
[10] C. Bagnuls and C. Bervillier, Phys. Rev. E **65**, 066132 (2002).
[11] V. Privman, P. C. Hohenberg, and A. Aharony, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. B. Lebowitz (Academic Press, New York, 1991), Vol. 14.
[12] Y. Garrabos, Ph.D. thesis, University of Paris VI, 1982.
[13] Y. Garrabos, J. Phys. (France) **46**, 281 (1985).
[14] Y. Garrabos, J. Phys. (France) **47**, 197 (1986).
[15] C. Bagnuls and C. Bervillier, J. Phys. (France) Lett. **45**, L95 (1984); Phys. Rev. B **32**, 7209 (1985); C. Bagnuls, C. Bervillier, D. I. Meiron, and B. G. Nickel, *ibid.* **35**, 3585 (1987). **65**, 149901(E) (2002).
[16] C. Bagnuls, C. Bervillier, and Y. Garrabos, J. Phys. (France) Lett. **45**, L127 (1984).
[17] J. M. H. Levelt Sengers and J. V. Sengers, in *Progress in Liquid Physics*, edited by C. A. Croxton (Wiley, New York, 1978), p. 103.
[18] J. M. H. Levelt-Sengers and J. V. Sengers, in *Perspectives in Statistical Physics*, edited by H. J. Raveché (North-Holland, Amsterdam, 1981), pp. 239–271.
[19] V. Dohm, Z. Phys. B: Condens. Matter **60**, 61 (1985); R. Schloms and V. Dohm, Europhys. Lett. **3**, 413 (1987); Nucl. Phys. B **328**, 639 (1989); Phys. Rev. B **42**, 6142 (1990); H. J. Krause, R. Schloms, and V. Dohm, Z. Phys. B: Condens. Matter **79**, 287 (1990); see also Refs. [24,25] below.
[20] E. Luijten and K. Binder, Europhys. Lett. **47**, 311 (1999).
[21] E. Luijten and H. Meyer, Phys. Rev. E **62**, 3257 (2000).
[22] M. H. Müser and E. Luijten, J. Chem. Phys. **116**, 1621 (2002).
[23] I. Hahn, F. Zhong, M. Barmatz, R. Haussmann, and J. Rudnick, Phys. Rev. E **63**, 055104(R) (2001).
[24] F. Zhong and M. Barmatz, Phys. Rev. E **70**, 066105 (2004).
[25] F. Zhong, M. Barmatz, and I. Hahn, Phys. Rev. E **67**, 021106 (2003).
[26] J. S. Kouvel and M. E. Fisher, Phys. Rev. **136**, A1626 (1964).
[27] Z. Y. Chen, P. C. Albright, and J. V. Sengers, Phys. Rev. A **41**, 3161 (1990).
[28] Z. Y. Chen, A. Abbaci, S. Tang, and J. V. Sengers, Phys. Rev. A **42**, 4470 (1990).
[29] M. A. Anisimov, S. B. Kiselev, J. V. Sengers, and S. Tang, Physica A **188**, 487 (1992).
[30] M. A. Anisimov, A. A. Povodyrev, V. D. Kulikov, and J. V. Sengers, Phys. Rev. Lett. **75**, 3146 (1995).
[31] V. A. Agayan, M. A. Anisimov, and J. V. Sengers, Phys. Rev. E **64**, 026125 (2001).
[32] J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids* (Wiley, New York, 1954).
[33] C. Bagnuls and C. Bervillier, Phys. Rev. Lett. **76**, 4094 (1996).
[34] M. Campostrini, A. Pelissetto, P. Rossi, and E. Vicari, Phys. Rev. E **65**, 066127 (2002).
[35] E. Luijten, H. W. J. Blöte, and K. Binder, Phys. Rev. Lett. **79**, 561 (1997); Phys. Rev. E **54**, 4626 (1996); **56**, 6540 (1997).
[36] Y. Garrabos, Phys. Rev. E **73**, 056110 (2006).
[37] Y. Garrabos, F. Palencia, C. Lecoutre, C. J. Erkey, and B. Le Neindre, Phys. Rev. E **73**, 026125 (2006).
[38] Y. Garrabos, F. Palencia, C. Lecoutre, D. Broseta, B. Le Neindre, and C. Erkey, Phys. Rev. E **75**, 061112 (2007).
[39] Y. Garrabos, F. Palencia, C. Lecoutre, D. Broseta, and B. Le Neindre, Phys. Rev. E **76**, 061109 (2007).
[40] L. Riedel, Chem.-Ing.-Tech. **26**, 83 (1954).
[41] See, for example, R. Reid, J. M. Prausnitz, and B. E. Poling, *The Properties of Gases and Liquids*, 4th ed. (McGraw-Hill, New York, 1988); J. F. Ely and I. M. F. Marrucho, in *Equations of State for Fluids and Fluid Mixtures*, edited by J. V. Sengers, R. F. Kayser, C. J. Peters, and H. J. White, Jr. (Elsevier, Amsterdam, 2000), part I, pp. 289–320; H. W. Xiang, *Corresponding-States Principle and Practice* (Elsevier, Amsterdam, 2005).
[42] R. B. Griffiths and J. C. Wheeler, Phys. Rev. A **2**, 1047 (1970).
[43] Z takes an extremum (minimum) value along the vapor-liquid equilibrium line and continuously decreases as density increases from gas to liquid in coexistence.
[44] Y. Garrabos, B. Le Neindre, R. Wunenburger, C. Lecoutre-Chabot, and D. Beysens, Int. J. Thermophys. **23**, 997 (2002).
[45] This method was only suggested and not used in practice in the concluding discussion given in Ref. [16], due to the large fitting uncertainty in the $\vartheta_{\mathcal{L}}$ determination, added to the large theoretical uncertainties in the Ising-like asymptotic forms of the crossover functions estimated from the dated development of the massive renormalization scheme.
[46] Y. Garrabos, C. Lecoutre, and F. Palencia, http://hal.archives-ouvertes.fr/hal-00165475
[47] H. Güttinger and D. S. Cannell, Phys. Rev. A **24**, 3188 (1981).
[48] See, for example, S. B. Kiselev and J. F. Ely, J. Chem. Phys. **119**, 8645 (2003).
[49] J. A. Beattie, R. J. Barriault, and J. S. Brierley, J. Chem. Phys. **19**, 1219 (1951); **19**, 1222 (1951).
[50] M. A. Weinberger and W. G. Schneider, Can. J. Chem. **30**, 422 (1952); **30**, 847 (1952).
[51] H. W. Habgood and W. G. Schneider, Can. J. Chem. **32**, 98 (1954); **32**, 164 (1954).
[52] A. Michels, T. Wassenaar, and P. Louwerse, Physica (Amsterdam) **20**, 99 (1954).
[53] V. A. Rabinovich, L. A. Tokina, and V. M. Berezin, Teplofiz. Vys. Temp. **11**, 64 (1973); V. A. Abovskii and V. A. Rabinovich, Teploenergetika (Moscow, Russ. Fed.) **3**, 44 (1971); **6**, 45 (1973).
[54] M. S. Green, M. Vicentini-Missoni, and J. M. H. Levelt Sengers, Phys. Rev. Lett. **18**, 1113 (1967).
[55] M. Vicentini-Missoni, J. M. H. Levelt Sengers, and M. S. Green, J. Res. Natl. Bur. Stand., Sect. A **73**, 563 (1969).
[56] P. Shofield, J. D. Lister, and J. F. Ho, Phys. Rev. Lett. **23**, 1098 (1969).
[57] J. M. H. Levelt Sengers, Physica (Amsterdam) **73**, 73 (1974).
[58] J. M. H. Levelt Sengers and J. V. Sengers, Phys. Rev. A **12**, 2622 (1975).
[59] J. M. H. Levelt Sengers, W. L. Greer, and J. V. Sengers, J. Phys. Chem. Ref. Data **5**, 1 (1976).
[60] See Ref. [17], p. 144, Table 4.3.4.