Promotion of cooperation induced by appropriate payoff aspirations in a small-world networked game

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Based on learning theory, we adopt a stochastic learning updating rule to investigate the evolution of cooperation in the Prisoner's Dilemma game on Newman-Watts small-world networks with different payoff aspiration levels. Interestingly, simulation results show that the mechanism of intermediate aspiration promoting cooperation resembles a resonancelike behavior, and there exists a ping-pong vibration of cooperation for large payoff aspiration. To explain the nontrivial dependence of the cooperation level on the aspiration level, we investigate the fractions of links, provide analytical results of the cooperation level, and find that the simulation results are in close agreement with analytical ones. Our work may be helpful in understanding the cooperative behavior induced by the aspiration level in society.

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Cooperation can be found widespread in a realistic world; understanding the emergence and persistence of cooperation among selfish individuals is one of the fundamental and central problems. Evolutionary game theory has provided a powerful framework to solve this problem [1,2]. As one typical game, the Prisoner's Dilemma game (PDG) has become a worldwide known paradigm for studying the evolution of cooperation. In the original PDG, two players simultaneously decide whether to cooperate or defect to obtain some payoffs. They will receive R if both cooperate, and P if both defect. While a player cooperates and the other defects, they get S and T, respectively. The rank of the four payoff values is $T>R>P>S$, and it is easy to recognize that it is best to defect for rational individuals irrespective of what the opponent does in a single round of the PDG. Since the unstable cooperative behavior is opposite to the observations in a realistic world, suitable extensions based on the original PDG are proposed in order for the emergence of cooperation. The tit-for-tat strategy, for example, can remarkably enhance cooperative behavior in repeated games and become known worldwide [3,4]. Another well-known rule outperforming tit-for-tat is win-stay-lose-shift (WSLS) [5]. More interestingly, the work of Nowak and May showed that the PDG on a simple spatial structure can lead to the emergence and persistence of cooperation [6]. Subsequently, much attention has been given to evolutionary games on different graphs since graph theory provides a natural and meaningful framework to describe the population structure on which the evolution of cooperation is studied.

It is well accepted that the updating rule plays an important role in the evolution of cooperation, and players can update their strategies by adopting different rules in the evolutionary games. The deterministic rule proposed by Nowark and May [6], and the stochastic evolutionary rule introduced by Szabó and Töke [7], are the commonly used updating rules. Apart from these, players can adopt a self-questioning mechanism to update strategies [8,9]. Moreover, players can update their strategies depending on the difference between the actual and aspiration payoffs based on learning theory [5,10]. Indeed, learning theory has provided a reasonable framework to study the evolutionary games since players can be viewed as adaptive agents. Therefore it is natural and interesting to consider other updating rules based on learning theory for structured populations. Our motivation is to explore how the learning rule affects the evolution of cooperation on small-world networks.

Presently, we adopt a modified stochastic learning rule to investigate the cooperation level on Newman-Watts (NW) networks [11]. We adopt NW networks as the population structure since real social networks are small world implying the combination of abundant localized interaction and sufficient shortcuts, not similar to regular or random graphs. As the typical model of small-world networks, NW networks are close to real social networks, and can describe the population structure to be more realistic. Moreover, NW networks facilitate extensions of mean-field approximation to study evolutionary dynamics because of the features [12,13]. Here, we introduce payoff aspirations for players, and players update their strategies with a stochastic probability depending on the difference between their collecting payoffs from neighbors and own payoff aspirations; such updating is different from the deterministic switch in WSLS. Interestingly, we find that appropriate intermediate aspirations can lead to the best cooperation level.

Now we would like to describe our evolutionary game model in detail. We consider the evolutionary PDG with players located on the NW networks, where a parameter p controls the fraction of edges randomly added to the one-dimensional lattice for nearest and next-nearest interactions with periodic boundary conditions [11]. The relationship between the average degree $\bar{k}$ and p is $\bar{k}=4(1+p)$. In evolutionary games on graphs, each player who occupies one site of the graph can only follow two simple strategies: $s=C$ or $s=D$, interacts only with its neighbors, and gains the payoffs according to payoff parameters as mentioned above. Following previous works [14,15], we use a simplified version of

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payoffs, and make $T=b$, $R=1$, and $P=S=0$, where b represents the advantage of defectors over cooperators, being typically constrained to the interval $1<b<2$. On the other hand, we define a parameter A as the average aspiration level of the players, and each player calculates its aspiration payoff based on the parameter A, which is typically constrained to the interval $0\leq A\leq b$ [16]. For one certain individual x, the aspiration payoff P_{xa} is defined as $P_{xa}=k_xA$, where k_x is the connectivity of player x. During the evolutionary process, player x will compare the collecting payoff P_x from neighbors with the aspiration level P_{xa}, and change its current strategy to its opposite strategy with a probability depending on the difference (P_x-P_{xa}) as

$$W_x = \frac{1}{1+\exp[(P_x-P_{xa})/K]}, \tag{1}$$

where K characterizes the noise effects in the strategy adoption process. The aspiration level P_{xa} provides the benchmark which is used to evaluate whether player x satisfies its current strategy. This evolutionary rule is stochastic in comparison with the deterministic switch in WSLS. The rationale is that players can make use of their own payoff information efficiently, and evaluate their satisfaction levels toward their current strategies more accurately. This probability characterizes the exact extent of changing their current strategies. Herein, we simply set $K=0.1$ and $p=0.5$ $(\bar{k}=6)$ respectively in this Brief Report, and concentrate on how the average payoff aspiration affects the evolution of cooperation on this social network.

In what follows, we will show the simulation results carried out on NW networks with $N=1000$ individuals. Initially, the two strategies of C and D are randomly distributed among the players with the equal probability 0.5. The key quantity for characterizing the cooperative behavior of the system is the density of cooperators ρ_c, which is defined as the fraction of cooperators in the whole population. In all simulations, ρ_c is obtained by averaging over the last 2000 generations of the entire 20 000 generations [17]. Each data point results from 10 different network realizations with 10 runs for each realization. The above model is simulated with synchronous update [18].

Figure 1 shows the cooperation level ρ_c as a function of the temptation to defect b for different values of A. One can see that ρ_c monotonically decreases as b increases no matter what the value of A is. It also shows that changing the value of A can influence the cooperation level for fixed b, and there may exist appropriate payoff aspirations promoting cooperation. To quantify the ability of aspiration level A to favor and maintain cooperation for various b more precisely, we study ρ_c as a function of A and b together, as shown in Fig. 2. One can find that there exists an intermediate aspiration level, resulting in the maximum value of ρ_c for different values of b. This phenomenon reveals that there exists somewhat resonant behavior reflected by the optimal cooperation level at the intermediate aspirations. The similar behavior can be found in other recent works, induced by different mechanisms [12,19–22].Additionally, the maximum cooperation level at optimal intermediate aspiration level decreases with

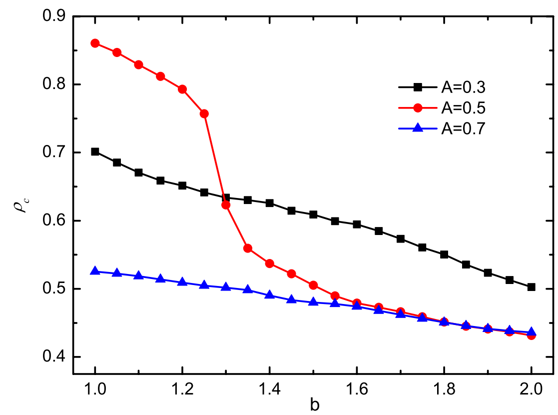

FIG. 1. (Color online) Cooperation level vs the temptation to defect b for different values of A.

increasing the temptation to defect b. It demonstrates that this positive effect on cooperation at the intermediate aspiration level can be restricted by the favored defection action [12,21].

In order to help us understand the resonance-like behavior induced by the average aspiration level A, it is necessary and interesting to investigate the fractions of links (C-C, C-D, and D-D links; see Fig. 3 and Fig. 4) so that we can scrutinize the microscopic evolution of cooperation. Accordingly, we can explain the nontrivial dependence of the cooperation level on A. Actually, in combination with the symmetry condition $P_{CD}=P_{DC}$ and the constraint $P_{CC}+P_{CD}+P_{DC}+P_{DD}=1$, the density of cooperators ρ_c can be obtained by $\rho_c=P_{CC}+P_{CD}$ [18,22–24]. Herein, we want to stress that this relation is valid only for the translationally invariant (regular) structures. However, we can find that there are little differences between the values of ρ_c and $P_{CC}+P_{CD}$ on NW networks, as shown in Fig 3. It shows that the fractions of links cannot exactly estimate the cooperation level, but can correctly depict the trends, that is, the changes of cooperation for the average aspiration level A on NW networks. Whereafter, we explain the nontrivial dependence of the coopera-

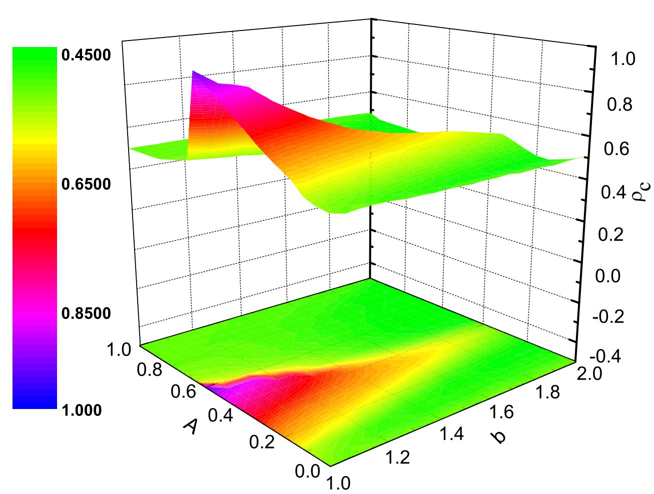

FIG. 2. (Color online) Frequency of cooperators as a function of the average aspiration level A and temptation to defection b.

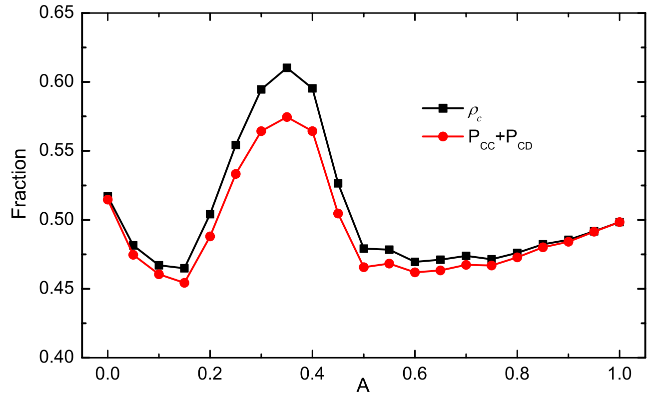

FIG. 3. (Color online) Cooperation level and $P_{CC}+P_{CD}$ as a function of the average aspiration level A for $b=1.6$.

tion level on A by combining the results of the fractions of links. In our study, in the case of small A, the payoffs of almost all players are above the aspiration payoffs, hence both C players and D players update their strategies with a very small probability; therefore all fractions of links keep unchanged and the cooperation level keeps at its initial level with increasing evolutionary generations. In this situation the system is in the "frozen" state [see Fig. 4(a)] and hence the cooperation level is about 0.5, whereas in the case of very large A, almost all players update their strategies simultaneously with a very high probability. Very interestingly, in this situation C players and D players mutually switch to each other, P_{CC} and P_{DD} switch to each other synchronously, and P_{CD} keeps unchanged with the increment of generations [see Fig. 4(b)]. This phenomenon can be regarded as the "ping-pong effect" [9], and the cooperation level vibrates around 0.5. Moreover, we do not show the results when A is above 1 in Fig. 3, because we find that when A is above 1 for $b=1.6$, the ping-pong effect still exists [25], and the cooperation level of the whole system is fixed as 0.5 for synchronous update [see Fig. 5(b)]. In the case of intermediate A, there exists a situation where the payoffs of a number of D

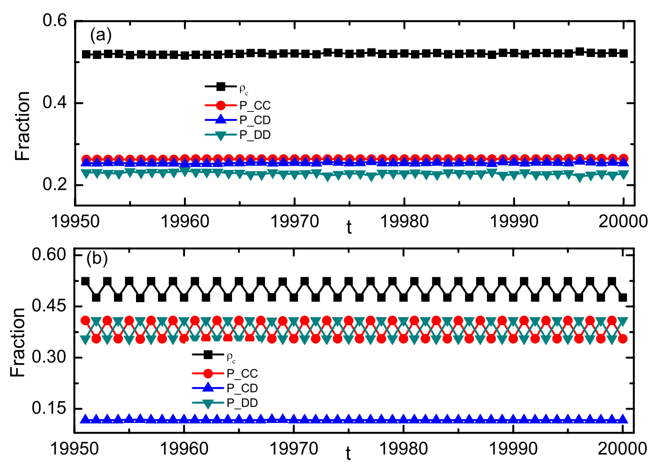

FIG. 4. (Color online) Time evolution of the cooperation level and fractions of links for $b=1.6$ with different average aspiration levels: (a) $A=0.01$; (b) $A=1.2$.

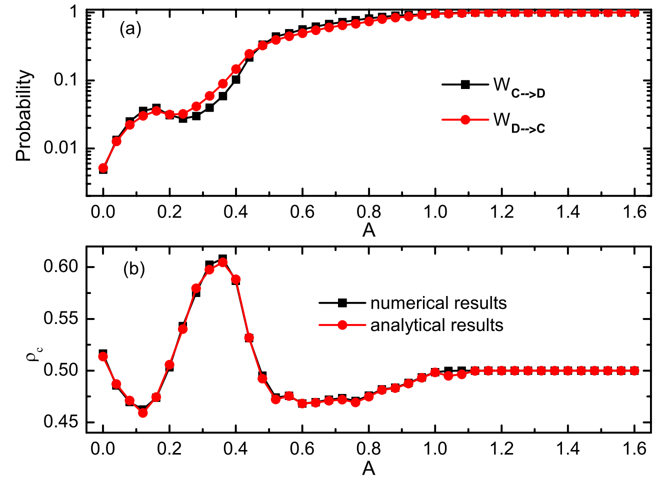

FIG. 5. (Color online) (a) Transition probability and (b) the cooperation level as a function of the average aspiration level A for $b=1.6$ on NW networks. In (b) squares are simulation results and circles are corresponding theoretical analysis.

players are below aspiration levels. At this point, the pattern where C players are surrounded by C players is ultimately stable and favored [18,23,26], because C players can collect larger payoffs in this pattern and hold their current strategies easily. Accordingly high cooperation emerges at an intermediate aspiration level. Combining these different cases of A, there should exist an appropriate intermediate aspiration level A which facilitates the formation of cooperator clusters and induces the maximum value of ρ_c.

On the other hand, to validate the above simulation results, we would like to investigate the transition probability of C players changing into D players $W_{C\to D}$, and D players changing into C players $W_{D\to C}$. Under this updating rule, for each player the probability of changing its current strategy is independent of others' payoffs, therefore the transition probability $W_{C\to D}$ and $W_{D\to C}$ can be respectively defined as

$$W_{C\to D}=\frac{1}{N_C}\sum\nolimits_{s_i=C} W_i,$$

$$W_{D\to C}=\frac{1}{N_D}\sum\nolimits_{s_i=D} W_i, \tag{2}$$

where N_C and N_D ($N_C+N_D=N$) are the numbers of C players and D players in the population, respectively. Since NW networks prevent the emergence of large degree nodes and can help the extension of the mean-field techniques for small-world structures, the motion of ρ_c can be approximately depicted as [24]

$$\dot{\rho}_c=(1-\rho_c)\,W_{D\to C}-\rho_c\,W_{C\to D}. \tag{3}$$

When the system has reached the steady state, $\dot{\rho}_c=0$ and the cooperation level in the steady state can be given as

$$\rho_c=\frac{W_{D\to C}}{W_{D\to C}+W_{C\to D}}=\frac{1}{1+\dfrac{W_{C\to D}}{W_{D\to C}}}.$$

In the following, we provide corresponding results based on the above analysis. Figure 5(a) shows the comparison between $W_{D\to C}$ and $W_{C\to D}$. In combination with Eq. (2), we can also understand the changes of ρ_c for A. Especially, when A is around 0.35, we can find that $W_{D\to C}/W_{C\to D}$ is greater than those for other values of A, therefore the cooperation level obtains its optimal value around $A=0.35$ for $b=1.6$. Furthermore, based on the above analysis, we plot the analytical results as well as the numerical simulations in Fig. 5(b). Since $W_{D\to C}$ and $W_{C\to D}$ cannot be reproduced by theoretical analysis, the transition probability used in Eq. (2) for computing the cooperation level ρ_c is obtained by simulations, as shown in Fig. 5(a). From the comparison between simulation results and theoretical analysis, we can find that analytical results are well consistent with numerical simulations on NW small-world networks [12]. Moreover, we find that the results remain qualitatively unaffected for different values of p and population size N. Thus far, we have illustrated the validity and efficiency of our proposed mode in studying the evolutionary games. Comparing to WSLS, this mechanism makes full use of individual own payoff information so that players can update their states more accurately. The results under this updating rule may reflect the phenomenon in society that, neither too small aspiration level nor too large aspiration level could improve the decision maker's long-run performance remarkably, whereas moderate aspiration level can favor the cooperative behavior.

In summary, we have studied the evolutionary PDG on NW small-world networks for different average aspiration levels under the stochastic updating rule, and found that there exists an appropriate intermediate aspiration level leading to the maximum value of cooperation. We have also found that the system is in the frozen state for small payoff aspiration, whereas the interesting ping-pong effect emerges for large payoff aspiration. To explain the nontrivial role of the average aspiration level in cooperation, we have further investigated the fractions of links, provided theoretical analysis of the cooperation level, and found that the analytical results are well consistent with the numerical simulations of the cooperation level. Our work may be helpful in exploring the role of different payoff aspirations in cooperation and reflecting the realistic phenomenon in social systems.

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