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Near-perfect absorption by a flat metamaterial plate

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The paper shows that under certain circumstances a monochromatic filament source located above a plane surface coated with a metamaterial does not illuminate the upper half space. New designs of electromagnetic field absorbers and resonators are suggested. They can be constructed with the help of metamaterials.

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I. INTRODUCTION

Interest in field propagation along an imperfect surface has about a century-old history, tracing back to Sommerfeld's solution of the classical problem for a dipole radiating above a plane with finite conductivity. Later, as radio broadcasting evolved, a lot of publications appeared which dealt with electromagnetic field propagation in the presence of an absorbing half space with the aim of minimizing the signal losses.

However, there is another aspect of the same problem, and it is connected with various applications, such as problems of electromagnetic compatibility or of reducing backscattering from elongated cavities. In these cases the attenuation in the process of incidence of electromagnetic wave onto a surface should be made as large as possible to reduce the reflected signal energy. A detailed consideration of this and a number of other similar problems is impossible without perception of the task formulated below.

Let us assume that a harmonically radiating current filament is located at y_0 above and parallel to the plane $y=0$. What impedance of the surface $y=0$ should be chosen in order to make the surface absorb the greatest portion of the energy radiated by the filament source? What portion of the total energy is absorbed in that case?

It may seem that the surface can absorb not more than half of the radiated energy, because the filamentary current in open space radiates equal amounts of energy to the upper ($y>y_0$) and the lower ($y<y_0$) half spaces. Detailed calculations show that a surface characterized by any constant value of impedance can absorb not more than one-half of the energy radiated by the source. However, the situation can become different if the impedance changes along the surface.

This paper will demonstrate the possibility of a complete disappearance of the energy flux in the space $y>y_0$ and of the concentration of energy in the lower half space. We shall demonstrate that under some circumstances a monochromatic filamentary source located above a plane surface coated with a metamaterial will not illuminate the upper half space at all.

II. TOTAL TRANSMISSION OF POINT SOURCE RADIATION INTO A HALF SPACE

A. Ray picture

Let a focusing flat plane (Veselago's lens [1]) with a thickness of $d=y_0/2$ made of a metamaterial with $\varepsilon=-1$, $\mu=-1$ be inserted between the source and a metal plane at the height $h=y_0/4$ (Fig. 1). Then focusing occurs right at the surface of the conducting plane. Once the total phase advance along ray paths is calculated, taking into account the negative phase velocity of the wave traveling through the plane and the field reversal due to the reflection from the conducting plane, one can discover that the reflected and emitted rays are antiphased in the half space of $y>y_0$. This means the incident and secondary fields mutually cancel each other.

One might guess that the analogous effect of inhibited radiation in the upper half space would be expected regardless of whether a metamaterial plate is present or not, and that the immersion of a metamaterial plate between a source and a mirror adds nothing to the well-known picture of the radiation of a source placed above a conducting plane (see, e.g., Refs. [2,3]). Of course, that is far from true, and a few details will help us to clarify this point.

Let a point harmonic source (electrical dipole) be placed parallel to and above a conducting plane. Its directional patterns, i.e., the angular dependencies of the far field calculated at different distances between the source and the plate, have a common feature, namely, the presence of specific oscillations caused by the interference between the fields of the source and its mirror image (see, e.g., Ref. [2], p. 323, Fig. 7-6). The field maxima are reached in the directions where the source field (1) and the image field (2) sum up in cophase, and minima are registered in the directions where these fields appear to be antiphased; see Figs. 2(a) and 2(b), respectively.

Thus, when a source is located above a conducting plane, it is impossible to completely suppress the field radiated in

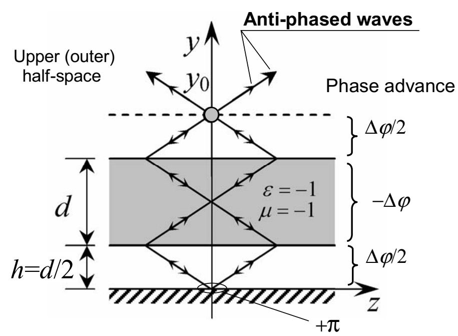

FIG. 1. Radiation into the outer half space can be totally canceled by locating a metamaterial plate between the electrical filamentary source and a conducting plane.

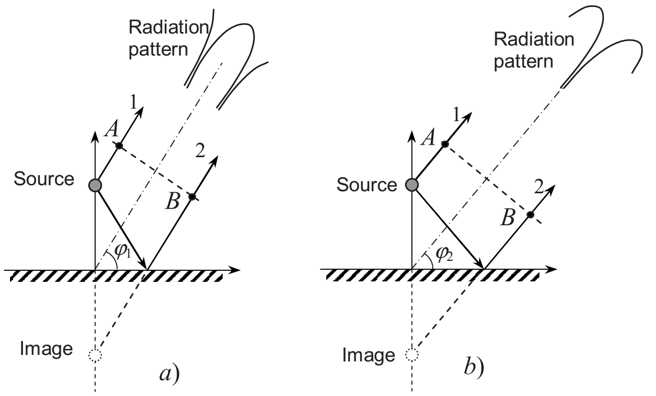

FIG. 2. In the φ_1 direction the phase difference of the fields 1 and 2 at the points A and B on the wave front is equal to $\varphi_B - \varphi_A = 2\pi n$, n being an integer (a), and in the direction of φ_2 the phase difference is equal to $\varphi_B - \varphi_A = 2\pi m + \pi$, m being an integer (b).

the upper half space without using an auxiliary device. By proper choice of the source altitude (in wavelengths) one can inhibit radiation only in several prescribed directions that would be entailed by the field enhancement in other directions. As a whole, all the power radiated by the source is transferred into the upper half space.

As is shown in Fig. 1 and confirmed by rigorous calculations (see Sec. II B below), quite another situation may be arranged by placing a metamaterial plate between the source and a mirror. Provided the metamaterial plate is made of matter with $\varepsilon \approx -1$, $\mu \approx -1$ and a simple geometrical relationship (not dependent upon the wavelength) is maintained, namely, the plate thickness amounts to one-half of the source height, one can secure nearly complete absence of the field in the upper half space, i.e., radiation suppression in all the directions above the source. Physically, this means that almost all the energy radiated by an omnidirectional source will be transferred not in the upper half space but into the metamaterial plate, where energy accumulation and absorption (in the realistic case of the presence of losses) would occur.

Naturally, the perfect field suppression in the upper half space becomes possible only because the metamaterial layer completely compensates the phase advance of the mirror source field. Hence, one can hardly expect to obtain the same result by some other means, i.e., without using a material where a wave propagates with negative phase velocity (a backward wave material).

Of course, varying the distance between the source and the mirror without corresponding correction of the thickness of the metamaterial layer would result in a lack of coincidence of the source and its image. At rather big values of this discrepancy (more than, say, a few tenths of the wavelength) a nonuniform or even oscillating picture of the field interference should arise, which is quite similar to that shown in Refs. [2,3]. This case is not discussed here for brevity and because it is obvious.

B. Full-wave solution

The geometry of the problem is presented in Fig. 3. A

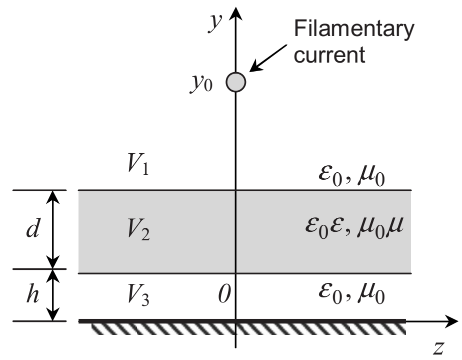

FIG. 3. Geometry of the problem.

rigorous solution was obtained for arbitrary values of ε, μ, h, d, and y_0 in each of three regions V_1, V_2, and V_3 for the case of excitation by a filament of electric current $\vec{J} = \vec{i}_x I_0 \delta(y - y_0)\delta(z)$. The vector potential $\vec{A} = \vec{i}_x A$ (the electric field amplitude is proportional to this potential) is represented in the form of a Fourier integral, in particular, with $y > y_0$ (i.e., in the region V_1), $A = A^i + A^s$,

$$A^i = \frac{I_0}{4\pi}\int_{-\infty}^{\infty} \frac{1}{q_0} e^{-q_0|y-y_0|} e^{-i\xi z} d\xi,$$

$$A^s = \frac{I_0}{4\pi}\int_{-\infty}^{\infty} f(\xi)\frac{1}{q_0} e^{-q_0[y-(h+d)]} e^{-i\xi z} d\xi,$$

$$f = e^{q_0[(h+d)-y_0]}\frac{e^{-qd}(\mu q_0 + q)S^+ - e^{qd}(\mu q_0 - q)S^-}{e^{-qd}(\mu q_0 - q)S^+ - e^{qd}(\mu q_0 + q)S^-},$$

$$S^{\pm} = \mu q_0 (e^{-2hq_0} + 1) \pm q(e^{-2hq_0} - 1),$$

$$q_0 = \begin{cases} i(k_0^2 - \xi^2)^{1/2}, & |\xi| \le \mathrm{Re}(k_0), \\ (\xi^2 - k_0^2)^{1/2}, & |\xi| > \mathrm{Re}(k_0), \end{cases}$$

$$q = \begin{cases} i(k^2 - \xi^2)^{1/2}, & |\xi| \le \mathrm{Re}(k), \\ (\xi^2 - k^2)^{1/2}, & |\xi| > \mathrm{Re}(k), \end{cases}$$

where A^i corresponds to the field of the source located in free space, $k_0 = 2\pi/\lambda$ and $k = k_0\sqrt{\varepsilon}\sqrt{\mu}$ are the wave numbers of the external space and of the plate material, respectively, λ is the wavelength, y_0 is the point of filament location, d is the plate thickness (see Fig. 3), the time dependence is selected in the form $\exp(i\omega t)$, and the principal values of the radicals are used. In the special case of $\varepsilon = \mu = 1$, the expression for A^s takes the form of the mirror source field,

$$A^s = -\frac{I_0}{4\pi}\int_{-\infty}^{\infty} \frac{1}{q_0} e^{-q_0(y+y_0)} e^{-i\xi z} d\xi,$$

while in the other case, when $\varepsilon = \mu = -1$, $y_0 = 2d$, $h = d/2$, A^s and A^i mutually cancel each other at $y > y_0$:

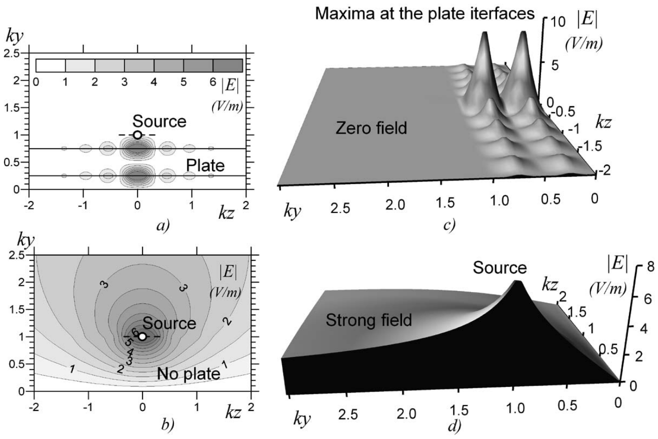

FIG. 4. Field in the presence (a), (c) and at the absence (b), (d) of a metamaterial plate.

$$f(\xi) = - e^{q_0(5d/2-y_0)},$$

$$A^s = - \frac{I_0}{4\pi}\int_{-\infty}^{\infty} \frac{1}{q_0} e^{-q_0(y-2d)} e^{-i\xi z} d\xi = - A^i,$$

so the rigorous solution of the corresponding boundary problem results in the same conclusion regarding potentially total field suppression in the upper half space, similar to the one given above in Sec. II.

This is illustrated by Fig. 4, which shows the absolute values of the total field in the vicinity of the source (in the plane perpendicular to the filament of electrical current). Contour plots are given in Figs. 4(a) and 4(b) and corresponding three-dimensiongal images of the field distribution are shown in Figs. 4(c) and 4(d).

Figures 4(a) and 4(c) depict the results obtained at $ky_0 = 1$, $d=2h=y_0/2$, $\varepsilon=\mu=-1-i0.001$. Figures 4(b) and 4(d) refer to the case of plate absence, when $\varepsilon=\mu=1$. Note that, in the presence of a metamaterial plate, the field in the region $y > y_0$ is almost equal to zero, in contrast to the second case, when the field of the filamentary source does not attenuate.

Note that, in the case of an easy-to-manufacture metamaterial plate with rather high losses, say, $\varepsilon=\mu=-1-i0.1$, the conclusions remain the same. The major portion of the energy, as much as 99.4%, is transferred into the lower half space and dissipates there. This is illustrated by Fig. 5, where

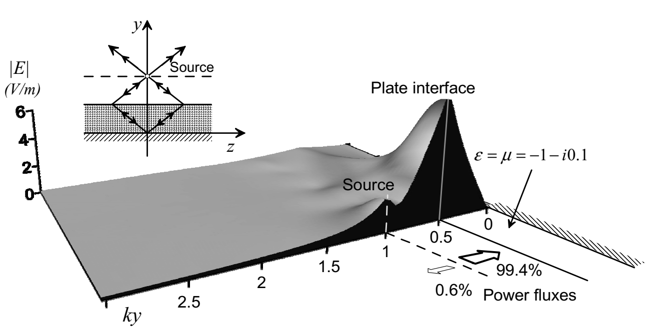

FIG. 5. Field power transfer in the presence of a realistic absorptive metamaterial plate.

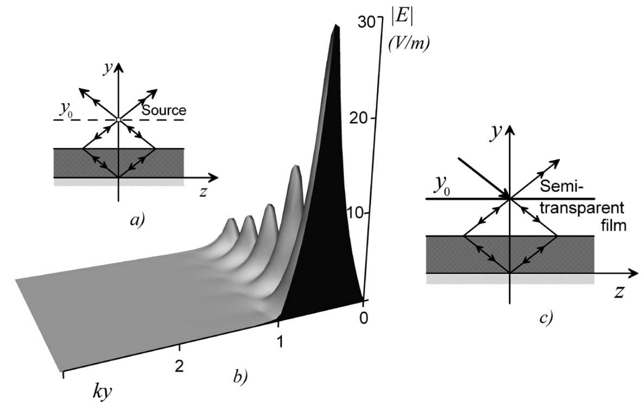

FIG. 6. Open resonator (a), (b) and absorbing coating (c) with a metamaterial plate.

the field distribution and portions of the power fluxes propagating into each half space are shown. Of course, microwave experiments on the discussed phenomena with a linear antenna operating as a source should indicate a strong reflected wave in the antenna feeder. This wave can be easily suppressed by a number of techniques commonly used in microwave engineering.

C. Open resonator

The regions in Figs. 4(a) and 4(c) with high field concentration due to accumulating reactive energy attract great attention. They occupy rather limited space (here, as small as half a wavelength) and arise next to the metamaterial plate faces when field compensation in the upper half space occurs. These maxima reach especially high values when the plate is arranged side by side with the conducting surface, $h=0$. Figure 6(a) shows the ray picture and Fig. 6(b) the field distribution. Thus, the structures shown in Figs. 1 and Fig. 6(a) may serve as prototypes for designing open resonators without the usual restrictions on the thickness of the system in terms of wavelength. Note that previously a different idea for a "thin" metamaterial-based resonator of "closed" type was suggested [4] (the metamaterial sheet was sandwiched between a pair of conducting plates). Another design for an open resonator is also known [5], it is based on the negative refraction property of a photonic crystal or metamaterial prism.

D. Relation to the phenomenon of superresolution

One of the specific features of the Veselago lens is the ability to produce an image with extremely fine details as its resolution is not restricted by the so-called diffraction limit [6]. Later it was shown that the absorption in a metamaterial plays a crucial role in achieving superresolution in practice. The smaller the plate thickness (in wavelengths), the higher will be the upper level of losses needed to secure the desired resolution (see, e.g., Ref. [7]).

Similar conclusions can be drawn regarding the performance of the systems under consideration. But to attain *near-field* compensation in the vicinity of the source (around the point y_0), the mirror image should be developed with superresolution, which is achievable only with extremely low losses in the plate, although, at small ky_0 and kd, one may expect rather good results even using existing metamaterials with noticeable absorption, as was in the case of electrically thin focusing plate [7]. Passing on to the larger values of ky_0, the near field is much more difficult to compensate, and this was proven by calculations. However, rather modest requirements need to be placed on the quality of metamaterial if one intends to compensate only propagating modes of the far field in the upper half space.

E. Electromagnetic wave absorber with special angular properties

Finally, note that the discussed phenomena may be efficiently used to create absorbers of the electromagnetic energy of a plane wave. Their special properties may be achieved, in particular, in an arrangement where the wave path, when it crosses the metamaterial structure, results of in phase advance compensation in a wide angular range. An example of a radio absorber design usable under the incidence of a perpendicularly polarized (TM) plane wave is shown in Fig. 6(c). Provided the electromagnetic response of the semitransparent film, in particular, its transmission and reflection coefficients, is properly chosen, the wave reflected from the film cancels the wave penetrating into and returning back from the region $y < y_0$. This latter wave gets a negative phase correction when propagating in the metamaterial plate and an additional reversal because of the reflection from the conducting plane. It is interesting that the total phase advance of that wave is equal to π at any incidence angle. Therefore, it is possible to achieve a very broad angular range in which such an absorber should operate efficiently, in contrast to classical designs, like the Salisbury screen [8]. In fact, only deviations of the semitransparent film properties impose certain limits on the angular performance. Finally, as there are no fundamental physical restrictions on the thickness of the described absorber, it can be made electrically thin (at least, in principle), like the previously suggested system of complementary metamaterials [9].

III. CONCLUSION

Thus, the results obtained show that the energy radiated by an omnidirectional (point) source can be completely absorbed by a flat surface with special equivalent impedance. In order to suppress both propagating and evanescent components, such a surface should be engineered with the use of metamaterials. Photonic crystals or stacked frequency-selective surfaces are also good candidates for that purpose.

Some opportunities originating from the described phenomenon are presented as well. In addition to the evident option of applying a metamaterial layer to solve electromagnetic compatibility problems, we suggest a new design for an open resonator excited by a point source. An approach to designing radio-absorbing interference coatings is also introduced, which enables one to enhance their properties and to obtain some specific features, e.g., wide angular operational range at small electrical thickness. The latter becomes possible because the required phase relationships for mutual compensation of waves reflected from the media interfaces are achieved by the application of a backward wave medium rather than by increasing the thickness of the coating layers. This is because, when using a metamaterial, negative phase shifts are easily realized, in contrast to the conventional method of securing the effective negative phase advance by implementing thick layers of ordinary material with the thickness exceeding half a wavelength. Finally, one of the possible ways to achieve a weak angular dependency of the wave reflection from a metamaterial-based absorber is schematically shown. Further details of such radio-absorbing coatings design will be considered in a future work.

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