Facilitated movement of inertial Brownian motors driven by a load under an asymmetric potential

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Based on recent work [L. Machura, M. Kostur, P. Talkner, J. Luczka, and P. Hanggi, Phys. Rev. Lett. **98**, 040601 (2007)], we extend the study of inertial Brownian motors to the case of an asymmetric potential. It is found that some transport phenomena appear in the presence of an asymmetric potential. Within tailored parameter regimes, there exists two optimal values of the load at which the mean velocity takes its maximum, which means that a load can facilitate the transport in the two parameter regimes. In addition, the phenomenon of multiple current reversals can be observed when the load is increased.

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Recently, Brownian motors have attracted considerable attention simulated by research on molecular motors. These systems can be modeled, for instance, by considering Brownian particles in a periodic asymmetric potential and acted upon by an external time-dependent force of zero average [1–3]. Typical examples are rocking ratchets [4], flashing ratchets [5], diffusion ratchets [6], correlation ratchets [7], white-shot-noise ratchets [8], and entropic ratchets [9,10].

Most studies have referred to consideration of the overdamped case in which the inertial term due to the finite mass of the particle is neglected. However, there are some pervious works on inertial Brownian motors and some transport phenomena were observed [11]. In particular, Machura and co-workers [12] studied the transport of inertial Brownian particles moving in a symmetric periodic potential under the influence of both a time periodic and a constant, biasing driving force. They found that thermal equilibrium fluctuations can induce the phenomenon of absolute negative mobility. Absolute negative mobility is the rather surprising opposite behavior in the form of a permanent motion against a (not too large) static force of whatever direction. Haljas and co-workers [13] had found that an interplay of three-level colored and thermal noises can also generate the phenomenon of absolute negative mobility. In this paper, we extend the previous work [12] to the case of an asymmetric potential. We emphasize on finding whether a load can facilitate the transport of inertial Brownian particles.

Let us consider the one-dimensional inertial Brownian particles driven by a time-dependent external force and a load, under the influence of an asymmetric periodic potential. The particle is driven by an unbiased time-periodic monochromatic force of strength A and an angular frequency Ω. The equation of motion reads as [12,14]

$$m\ddot{x} + \gamma\dot{x} = -V'(x) + A\cos(\Omega t) + F + \sqrt{2\gamma k_B T}\xi(t), \quad (1)$$

where the dot and prime denote derivatives with respect to t and x, respectively. The parameter γ denotes the friction coefficient, T is the temperature, and k_B is the Boltzmann constant. $V(x)$ is the external asymmetric periodic potential and has the period L and a barrier height ΔV. The quantity F denotes the external, constant force. The thermal fluctuations due to the coupling of the particle with the environment are modeled by a zero-mean, Gaussian white noise $\xi(t)$ with autocorrelation function $\langle\xi(t)\xi(s)\rangle = \delta(t-s)$ satisfying Einstein's fluctuation-dissipation relation.

Upon introducing characteristic length scale and time scale [12,14], Eq. (1) can be rewritten in dimensionless form

$$\ddot{\hat{x}} + \hat{\gamma}\dot{\hat{x}} = -\hat{V}'(\hat{x}) + a\cos(\omega\hat{t}) + f_0 + \sqrt{2\hat{\gamma}D}\hat{\xi}(\hat{t}), \quad (2)$$

with

$$\hat{x} = \frac{x}{L}, \quad \hat{t} = \frac{t}{\tau_0}, \quad \tau_0^2 = \frac{mL^2}{\Delta V}, \quad \hat{\gamma} = \frac{\gamma\tau_0}{m}, \quad \hat{V}(\hat{x}) = \frac{V(x)}{\Delta V},$$

$$a = \frac{AL}{\Delta V}, \quad \omega = \Omega\tau_0, \quad D = \frac{k_B T}{\Delta V}, \quad f_0 = \frac{FL}{\Delta V}, \quad (3)$$

and the zero-mean white noise $\hat{\xi}(\hat{t})$ has autocorrelation function $\langle\hat{\xi}(\hat{t})\hat{\xi}(\hat{s})\rangle = \delta(\hat{t}-\hat{s})$.

In the following, mostly for the sake of simplicity, we shall only use dimensionless variables and shall omit the "caret" notation in all quantities. In addition, our emphasis is on finding whether a load can facilitate the transport, so we take a new quantity $f=-f_0$. For the asymmetric ratchet potential $V(x)$, we choose

$$V(x) = \sin(2\pi x) + \frac{\Delta}{4}\sin(4\pi x), \quad (4)$$

where Δ is the asymmetry parameter of the potential. We restrict the discussion here to a set of optimal driving parameters [12], reading $a=4.2$, $\omega=4.9$, and $\gamma=0.9$.

Our emphasis is on finding the asymptotic mean velocity which is defined as the average of the velocity over the time and thermal fluctuations. The Fokker-Planck equation corresponding to Eq. (2) cannot be analytically solved, therefore, we carried out extensive numerical simulations [14]. We

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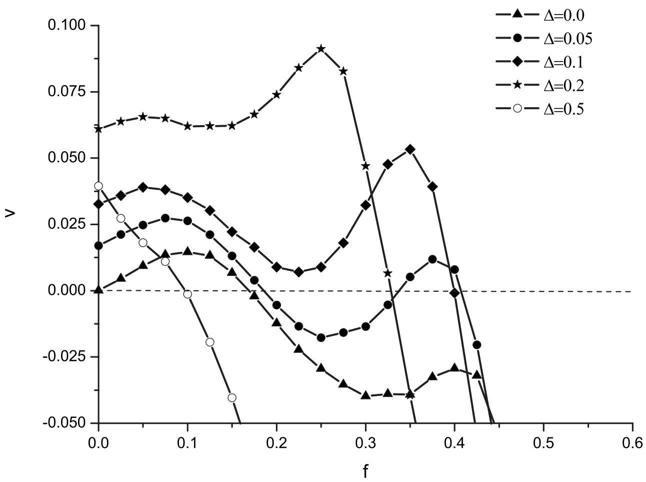

FIG. 1. The asymptotic mean velocity v vs the external force f for different values of the asymmetry parameter Δ at $a=4.2$, $\omega=4.9$, $\gamma=0.9$, and $D=0.001$.

have numerically integrated Eq. (2) by the stochastic Runge-Kutta method of the second order with time step $\Delta t=0.001$. The initial condition of $x(t)$ is taken from a uniform distribution over the dimensionless periodic $L=1$ of the ratchet potential and the initial condition of $v(t)$ is chosen at random from a symmetric, uniform distribution over the interval $[-0.2, 0.2]$. The data obtained were averaged over 500 different trajectories and each trajectory evolved over 5×10^4 periods. The numerical results were shown in Figs. 1 and 2.

Figure 1 shows the asymptotic mean velocity v as a function of the external force f for different values of the asymmetry parameter Δ. When the potential is symmetric ($\Delta=0$), there are two peaks in the curve, one above the zero velocity line and the other below the zero velocity line. The peak above the zero velocity line (the left-hand peak) is already investigated by Machura and co-workers [12], which shows the phenomenon of absolute negative mobility. The peak below the zero velocity line (the right-hand peak) is induced by the high-frequency driving (the time-dependent periodic external force) ratchet effect. When the asymmetry parameter is increased, the position of the right-hand peak moves to a small value of the load f and its height increases. The two peaks may be above the zero velocity line ($\Delta=0.05, 0.1, 0.2$), which shows that a load can facilitate the transport in the two different regimes. However, for a large value of the asymmetry parameter ($\Delta=0.5$), the two peaks disappear and the velocity decreases monotonically with increasing the load f. It is obvious that in this case both the absolute negative mobility effect and the high-frequency driving ratchet effect disappear and the transport is dominated by the noise driving ratchet effect. In addition, we also find that multiple current reversals occur as the load is increased. The Brownian particle changes its direction three times for $\Delta=0.05$. It must be pointed out that multiple current reversals can be detected in overdamped Brownian systems driven by the colored three-level Markovian noises [15].

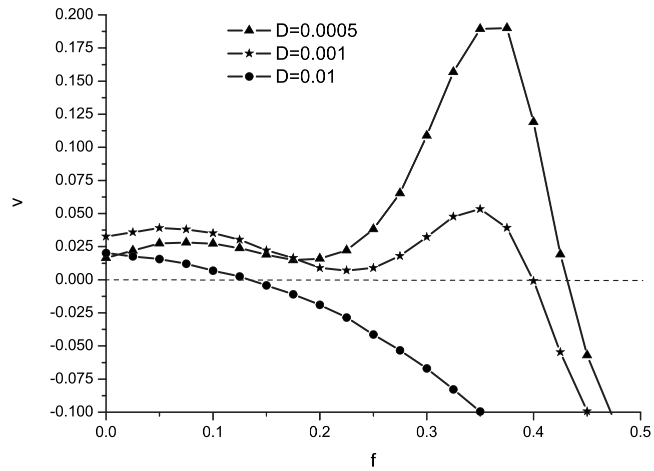

FIG. 2. The asymptotic mean velocity v vs the external force f for different values of D at $a=4.2$, $\omega=4.9$, $\gamma=0.9$, and $\Delta=0.1$.

In Fig. 2, we depict the asymptotic mean velocity as a function of the external force f for different values of the noise intensity D at $\Delta=0.5$. As the noise intensity is increased, the high-frequency driving ratchet effect decreases and the noise driving ratchet effect increases. Thus, the height of the right-hand peak decreases. For very large values of D, both the absolute negative mobility effect and the high-frequency driving ratchet effect disappear, and the two peaks disappear.

In conclusion, in this paper we study the transport of inertial Brownian particles moving in an asymmetric potential. Within tailored parameter regimes given in Ref. [12], we extend their work to the case of an asymmetric potential. It is found that there are two optimal values of the load, at which the mean velocity takes its maximum, which indicates that the load facilitates the transport in the two different regimes. The first optimal value (the left-hand peak) is from absolute negative mobility [12] and the second one is induced by the high-frequency driving ratchet effect. When the noise intensity D or the asymmetry parameter Δ is very large, the two optimal values disappear. In addition, when the load is increased, multiple current reversals may occur within tailored parameter regimes. For example, for $\Delta=0.05$, the particle can change its direction three times as the load is increased.

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