Multifractality and scale invariance in human heartbeat dynamics

Emily S. C. Ching
Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong

Yue-Kin Tsang
Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California 92093-0213, USA
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Human heart rate is known to display complex fluctuations. Evidence of multifractality in heart rate fluctuations in healthy state has been reported [Ivanov *et al.*, Nature (London) **399**, 461 (1999)]. This multifractal character could be manifested as the dependence of the probability density functions (PDFs) of the interbeat interval increments, which are the differences in two interbeat intervals that are separated by n beats, on n. On the other hand, "scale invariance in the PDFs of detrended healthy human heart rate increments" was recently reported [Kiyono *et al.*, Phys. Rev. Lett. **93**, 178103 (2004)]. In this paper, we clarify that the scale invariance reported is actually exhibited by the PDFs of the increments of the "detrended" integrated healthy interbeat interval and should, therefore, be more accurately referred as the scale invariance or n independence of the PDFs of the sum of n detrended interbeat intervals. Indeed, we demonstrate explicitly that the PDFs of detrended healthy interbeat interval increments are scale or n dependent in accord with its multifractal character. Our work also establishes that this n independence of the PDFs of the sum of n detrended interbeat intervals is a general feature of human heartbeat dynamics, shared by heart rate fluctuations in both healthy and pathological states.

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I. INTRODUCTION

The heart interbeat interval in human is known to display complex fluctuations, referred to as heart rate variability (HRV). In the past decade, many analyses [1–10] have been carried out to characterize the statistical features of human HRV, with an aim to gain understanding of human heartbeat dynamics. In these studies, possible different statistical features of HRV in different physiological states have been reported. In particular, an intriguing finding is the multifractality in healthy HRV and the loss of this multifractal character in pathological HRV in patients with congestive heart failure [3]. Such multifractal complexity in healthy HRV was further shown to be related to the intrinsic properties of the control mechanisms in human heartbeat dynamics and is not simply due to changes in external stimulation and the degree of physical activity [4].

In another complicated phenomenon of fluid turbulence, physical measurements are also known to be multifractal [11]. In fluid turbulence, it is common to study structure functions, which are the statistical moments of the increments of the signals at different scales, and their scaling behavior. Multifractality manifests itself as a nonlinear dependence of the scaling exponents on the order of the structure functions. This nonlinear dependence is equivalent to the scale dependence of the probability density functions (PDFs) of the increments of the signals at different scales. These ideas of structure functions in fluid turbulence were employed to analyze healthy HRV and similar multifractality, i.e., a scale or n dependence of the PDFs of the interbeat interval increments, which are the differences of two interbeat intervals separated by n beats, was indeed found [12]. This analogy of human HRV to fluid turbulence was further exploited and a hierarchical structure found in fluid turbulence [13] was shown to exist also in human HRV, with different parameters for heart rate fluctuations in healthy and pathological states [14]. The different values of the parameters can thus be used to quantify the multifractal character of healthy HRV and its loss in pathological HRV more easily [14]. On the other hand, in a recent analysis [15] "scale invariance in the PDFs of detrended healthy human heart rate increments" was reported, and interpreted as an indication that healthy heartbeat dynamics are in a critical state. At first sight, this finding of scale invariant PDFs appears to be in contradiction to the multifractal character of healthy HRV as discussed above, and, therefore, needs clarification.

In this paper, we clarify that the analysis of Ref. [15] consists of first integrating the interbeat interval data then taking the increments after the removal of local "trend" so that the scale invariance reported is actually exhibited by the PDFs of the increments of detrended integrated interbeat interval. We further show that these increments of detrended integrated interbeat interval can be more accurately understood as the sum of detrended interbeat intervals. Our work establishes that this scale invariance of the PDFs of the sum of detrended interbeat intervals is a general feature of human heartbeat dynamics, shared by heart rate fluctuations in both healthy and pathological states. Thus, such scale invariance cannot be an indication of healthy human heartbeat dynamics being in a critical state. We also understand that the essential effect of the detrending procedure is to eliminate the local average from the heart interbeat data.

This paper is organized as follows. We first review the statistical character of multifractality in healthy HRV in Sec. II. In Sec. III, we study the analysis of Ref. [15], hereafter referred to as the KSASHY analysis, and show that the scale invariance reported for healthy HRV in this analysis should be more accurately understood as the scale or n invariance of the PDFs of the sum of n "detrended interbeat intervals" (to be defined precisely). We further demonstrate that such scale invariance also exists in pathological heart rate fluctuations in patients with congestive heart failure. We then show explicitly that the detrended interbeat interval increments, just as in the untreated interbeat interval increments, exhibit different statistical characteristics in healthy and in pathological physiological states. In particular, for healthy HRV, the PDFs of the detrended interbeat interval increments between n beats are scale or n dependent, in accord with the multifractal character of healthy HRV. On the other hand, for pathological HRV in patients with congestive heart failure, the PDFs of the detrended interbeat interval increments are scale invariant and well approximated by Gaussian. This is consistent with the reported loss of multifractal character in pathological HRV [3]. In Sec. IV, we show that this general scale invariance in human heartbeat dynamics is nontrivial in that multifractal turbulent temperature measurements in thermal convective flow do not exhibit similar statistical feature. In Sec. V, we show that the essential effect of the detrending procedure is to eliminate the local average from the interbeat interval data, thus explaining why the detrended interbeat interval increments are essentially the same as the untreated interbeat interval increments. Finally, we summarize and conclude our paper in Sec. VI.

II. STATISTICAL SIGNATURE OF MULTIFRACTALITY

For completeness, we first review how the multifractality of healthy human HRV can be studied using the ideas of structure functions in fluid turbulence. Consider a dataset of human heart interbeat intervals $b(i)$, where i is the beat number or beat index. The interbeat interval is also known as RR interval as it is the time interval between successive "R" peaks, corresponding to the heart beats, in the electrocardiogram (ECG) time signal. The value of $b(i)$ varies from beat to beat and this variation is the human HRV. Following the ideas of structure functions in turbulent fluid flows, one defines [12,14] the interbeat interval increments between n beats as

$$\Delta_n b(i) = b(i+n) - b(i), \tag{1}$$

which are the differences between two interbeat intervals separated by n beats. The pth order structure functions S_p's are the pth order statistical moments of the interbeat interval increments

$$S_p(n) = \langle |\Delta_n b(i)|^p \rangle. \tag{2}$$

Analogous to velocity or temperature structure functions in turbulent fluid flows, $S_p(n)$ for human HRV exhibits power-law dependence on n:

$$S_p(n) \sim n^{\zeta_p} \tag{3}$$

for some intermediate values of n [12]. We write Eq. (3) as

$$[S_p(n)]^{1/p} \sim n^{\zeta_p/p}. \tag{4}$$

If ζ_p/p is a constant, or in other words, if ζ_p is proportional to p, then the heart rate fluctuations would be monofractal or simply fractal. Otherwise, if ζ_p is not proportional to p, then the heart rate fluctuations would be multifractal. Moreover, the standardized PDFs (with mean subtracted then normalized by the standard deviation) of Δb_n are scale invariant, i.e., independent of n, if and only if ζ_p is proportional to p.

For healthy HRV, it was found [12] that ζ_p varies with p in a nonlinear fashion. This nonlinear dependence of ζ_p on p or, equivalently, the scale dependence of the standardized PDFs of $\Delta_n b$ on n is thus a characteristic signature of the multifractal character of healthy human HRV.

III. SCALE INVARIANCE IN THE KSASHY ANALYSIS

In contrast to physical measurements in turbulent fluid flows, interbeat interval data are often nonstationary. This nonstationarity is one possible reason for the relatively poor quality of scaling in HRV as compared to that in turbulent fluid flows. To eliminate the nonstationarity, a "detrended fluctuation analysis" has been introduced [16], which was further developed by Refs. [15,17] to study "detrended heart rate increments." The resulting KSASHY analysis consists of the following steps. First, the interbeat interval data $b(j)$ are "integrated" to give $B(m)$:

$$B(m) = \sum_{j=1}^{m} b(j). \tag{5}$$

Second, the integrated data of $B(m)$ are divided into segments of size $2n$, and the datapoints in each segment are fitted by the best qth-order polynomial. This polynomial fit represents the "trend" in the corresponding segment. Third, these polynomial fits, denoted by $B_n(m)$, are subtracted from $B(m)$ to get $B^*(m)$, which are then "detrended:"

$$B^*(m) = B(m) - B_n(m). \tag{6}$$

Finally, the standardized PDFs of the increments of B^*

$$\Delta_n B^*(i) = B^*(i+n) - B^*(i) \tag{7}$$

for different values of n are studied. Note that in Refs. [15,17], $\Delta_n B^*(i)$ was denoted as $\Delta_n B(i)$ and the symbol s was used in place of n. The standardized PDFs of $\Delta_n B^*(i)$ for healthy heart interbeat data were found to be independent of n, and this was referred to as "scale invariance in the PDFs of detrended healthy human heart rate increments" in Ref. [15]. However, as discussed, $\Delta_n B^*$ is the increment of detrended integrated interbeat interval, and is thus not obviously related to the increment of heart rate or interbeat interval. Indeed, we shall show below that $\Delta_n B^*$ would be more accurately understood as the sum of n detrended healthy interbeat intervals.

To proceed, we define the detrended interbeat interval, denoted by $b^*(i)$. Its definition naturally follows from the detrending procedure described above as

$$B^*(m) = \sum_{j=1}^{m} b^*(j). \tag{8}$$

Thus

$$\Delta_n B^*(i) = \sum_{j=i+1}^{i+n} b^*(j) \tag{9}$$

is more accurately understood as the sum of n detrended interbeat intervals. On the other hand, one would naturally define the detrended interbeat interval increment between n beats to be

$$\Delta_n b^*(i) = b^*(i+n) - b^*(i). \tag{10}$$

It is then clear that $\Delta_n B^*(i)$ generally has no obvious relation to $\Delta_n b^*(i)$. In particular, the observation of scale-invariant or n-independent standardized PDFs of $\Delta_n B^*$ does not necessarily imply that the standardized PDFs of $\Delta_n b^*$ are also n independent. Indeed, one expects the contrary, namely, the standardized PDFs of $\Delta_n b^*$ should depend on n as healthy human HRV is multifractal.

To clarify this issue, we study the scaling behavior of the statistical moments of $\Delta_n B^*$ and $\Delta_n b^*$. We analyze healthy interbeat data that are taken from a database of 18 sets of daytime normal sinus rhythm data downloaded from public domain [18]. We follow the KSASHY analysis described above to get $\Delta_n B^*(i)$. We find that a polynomial of degree 3 ($q=3$) is sufficient to fit the "trend" as found in Ref. [15]. To get the detrended interbeat interval increment $b^*(i)$, we use Eq. (8) to get

$$b^*(i) = B^*(i) - B^*(i-1) \tag{11}$$

for both $B^*(i-1)$ and $B^*(i)$ belonging to the same segment and skip that datapoint when $B^*(i-1)$ and $B^*(i)$ fall into different (consecutive) segments. Next, we evaluate the statistical moments

$$\hat{S}_p(n) \equiv \langle |\Delta_n B^*(i)|^p \rangle, \tag{12}$$

$$S_p^*(n) \equiv \langle |\Delta_n b^*(i)|^p \rangle. \tag{13}$$

As seen from Fig. 1, $\hat{S}_p(n)$ exhibits power-law or scaling behavior with n with exponents $\hat{\zeta}_p$:

$$\hat{S}_p(n) \sim n^{\hat{\zeta}_p} \tag{14}$$

for n between 16 to 1024 and p between 0.2 to 3. On the other hand, $S_p^*(n)$ exhibits better scaling behavior with n with exponents ζ_p^*:

$$S_p^*(n) \sim n^{\zeta_p^*} \tag{15}$$

for n between 32 to 1024 and p between 0.2 to 3 (see Fig. 2).

As the standardized PDFs of $\Delta_n B^*$ were found to be n independent [15], the scaling exponents of the statistical moments of $\Delta_n B^*$, $\hat{\zeta}_p$, should be proportional to p. On the other hand, because of the multifractal character of healthy HRV [3], one expects that the scaling exponents of the statistical moments of $\Delta_n b^*$, ζ_p^*, would behave similar to the scaling exponents of their untreated counterparts ζ_p and have a nonlinear dependence on p. In Fig. 3, we plot the relative scaling exponents $\hat{\zeta}_p/\hat{\zeta}_2$ and ζ_p^*/ζ_2^* as a function of p. Indeed $\hat{\zeta}_p$ is proportional to p, confirming that the standardized PDFs of $\Delta_n B^*$ are indeed scale invariant as reported in Ref. [15]. But

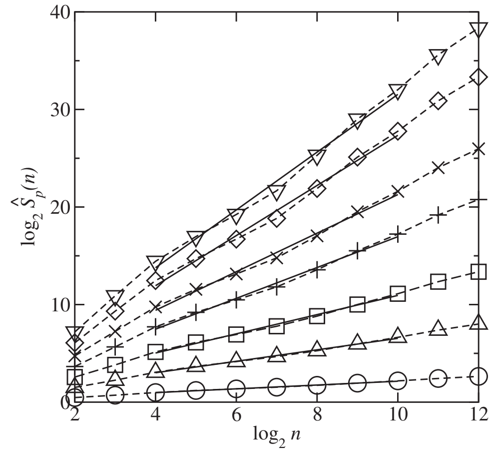

FIG. 1. The statistical moments $\hat{S}_p(n)$ of the sum of detrended heartbeat intervals [see Eq. (12) for definition] for healthy heartbeat data for $p=0.2$ (circles), $p=0.6$ (triangles), $p=1.0$ (squares), $p=1.6$ (pluses), $p=2.0$ (crosses), $p=2.6$ (diamonds), and $p=3.0$ (inverted triangles). The curves have been shifted vertically for clarity.

we have clarified that this scale invariance is exhibited by the standardized PDFs of the sum of n detrended interbeat intervals. Also, as expected, ζ_p^* is not proportional to p but changes with p in a nonlinear manner, in accord with the multifractal character of healthy human HRV. This nonlinear dependence of ζ_p^* on p thus clarifies that for healthy human HRV that is multifractal, the standardized PDFs of detrended interbeat interval increments between n beats, similar to those of the untreated interbeat interval increments, do depend on n. Indeed, the standardized PDFs change from flatter than Gaussian for small n to Gaussian for larger n.

We have also compared the scaling exponents of the detrended interbeat interval increments ζ_p^* with those of the untreated interbeat interval increments ζ_p and as shown in Fig. 3, the two sets of exponents agree well with one another. This indicates that the detrending procedure does not change much the scaling exponents of the interbeat interval increments. We shall return to understand this in Sec. V.

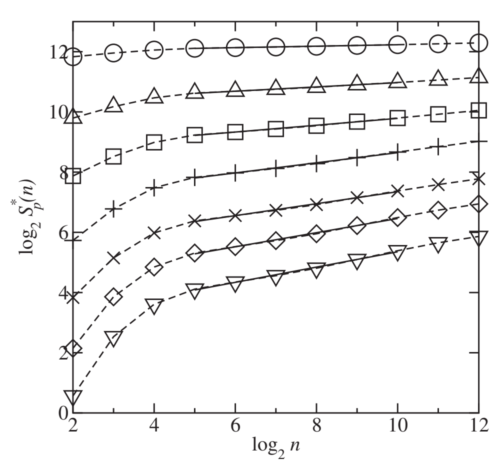

FIG. 2. The statistical moments $S_p^*(n)$ of detrended heart rate intervals [see Eq. (13) for definition] for healthy heartbeat data for p ranges from 0.2 to 3.0. Same symbols as in Fig. 1. The curves have been shifted vertically for clarity.

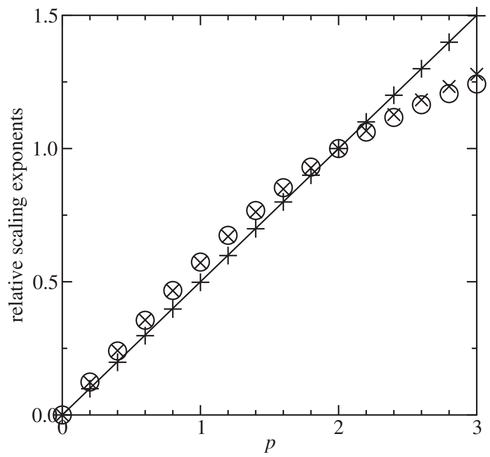

FIG. 3. The relative scaling exponents $\hat{\zeta}_p/\hat{\zeta}_2$ (pluses), ζ_p^*/ζ_2^* (crosses), and ζ_p/ζ_2 (circles) as a function of p for healthy heartbeat data. It can be seen that $\hat{\zeta}_p/\hat{\zeta}_2$ is close to $p/2$ which is shown as the solid line.

As discussed in Sec. I, it was suggested [15] that this scale invariance of the standardized PDFs of $\Delta_n B^*$, the sum of n detrended interbeat intervals, is an indication of healthy human heartbeat dynamics being in a critical state. To check this suggestion, it would be useful to perform the same analysis to human HRV in a pathological state. We repeat the KSASHY analysis using 45 sets of daytime interbeat data from congestive heart failure patients, also downloaded from the same public domain [18]. The results for $\hat{\zeta}_p$ and ζ_p^* in this case are shown in Fig. 4. Note that ζ_p^* is now approximately proportional to p, confirming that the multifractality is lost in pathological HRV [3]. In this pathological state, we have found that the scale-invariant standardized PDFs of the detrended interbeat interval increments are well approximated by Gaussian.

On the other hand, it can be seen that $\hat{\zeta}_p$ is again proportional to p, demonstrating that the scale invariance of the standardized PDFs of the sum of detrended interbeat intervals is not restricted to healthy HRV but also exhibited by pathological HRV in congestive heart failure patients. Moreover, the scale-invariant or n-independent standardized PDFs are approximately exponential for both healthy and pathological HRV as shown in Figs. 5 and 6. Since this scale invariance is found generally in heart rate fluctuations in both healthy and pathological state, it could not be an indication that healthy heartbeat dynamics are in a critical state. Common feature for both healthy and diseased human HRV was also reported before [22]; it would be interesting to explore whether this earlier feature and the present one are related.

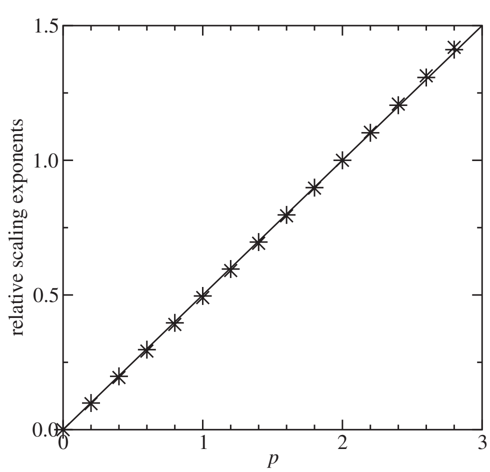

FIG. 4. The relative scaling exponents $\hat{\zeta}_p/\hat{\zeta}_2$ (pluses) and ζ_p^*/ζ_2^* (crosses) as a function of p for pathological heartbeat data from congestive heart failure patient. Both of them are close to the solid line of $p/2$.

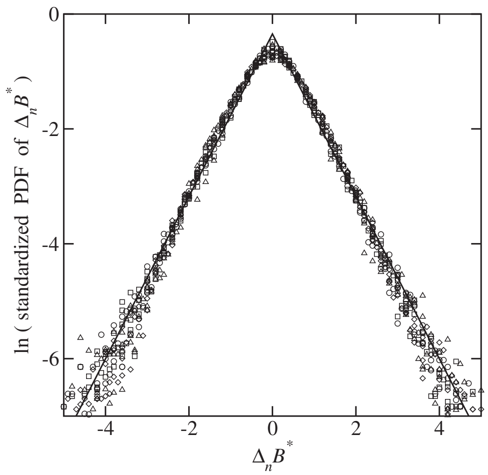

FIG. 5. Standardized PDFs for $\Delta_n B^*$ for healthy heartbeat data with $n=4$ (circles), $n=16$ (squares), $n=64$ (diamonds), and $n=256$ (triangles). Data from four different healthy subjects are shown and seen to coincide with one another. These n-independent PDFs are seen to be well approximated by the standard exponential distribution (solid line).

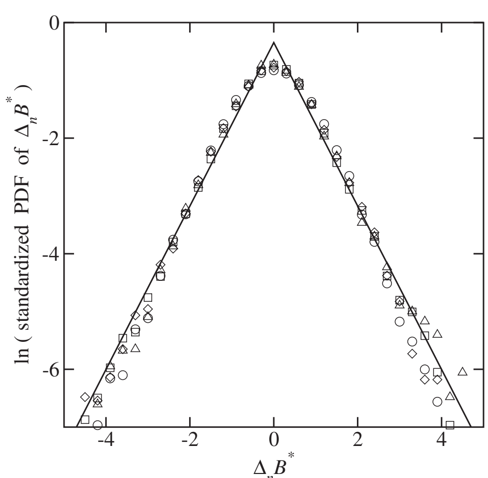

FIG. 6. Standardized PDFs for $\Delta_n B^*$ for heartbeat data from a congestive heart failure patient with $n=4$ (circles), $n=16$ (squares), $n=64$ (diamonds), and $n=256$ (triangles). Again, the scale invariant PDFs are well approximated by the standard exponential distribution (solid line).

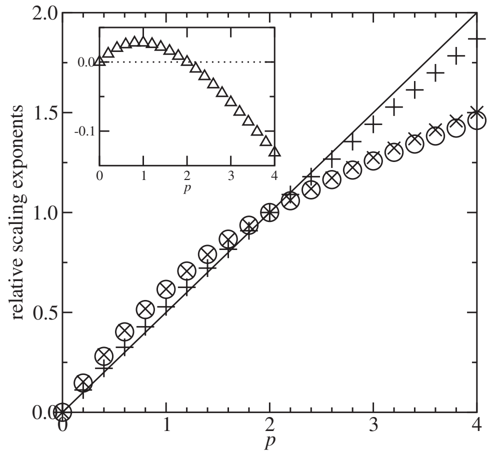

FIG. 7. The three relative scaling exponents $\hat{\xi}_p/\hat{\xi}_2$ (pluses), ξ_p^*/ξ_2^* (crosses), and ξ_p/ξ_2 (circles) for temperature measurements in turbulent convective flows. All the three relative exponents deviate from $p/2$ (the solid line). The deviation $\hat{\xi}_p/\hat{\xi}_2 - p/2$ is plotted versus p in the inset to show clearly that $\hat{\xi}_p/\hat{\xi}_2$ is not proportional to p.

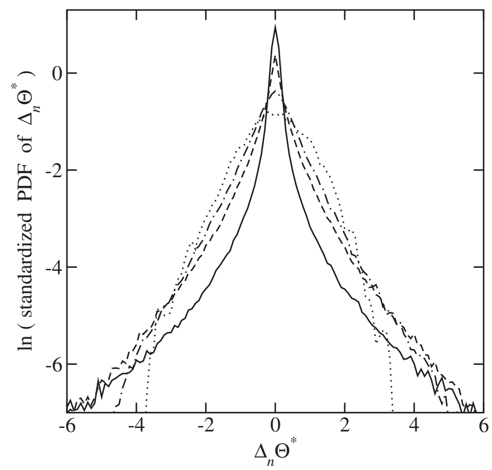

FIG. 8. The standardized PDFs of $\Delta_n\Theta^*$ for temperature measurements taken in turbulent thermal convective flows with $n=4$ (solid), $n=32$ (dashed), $n=256$ (dot-dashed), and $n=4096$ (dotted). The dependence of the standardized PDFs on n is clearly seen.

IV. KSASHY ANALYSIS FOR TURBULENT TEMPERATURE MEASUREMENTS

It is natural to ask whether this general scale invariance found in human heartbeat dynamics is trivial, i.e., whether it exists for any fluctuating data. In this section, we shall see that such scale invariance is absent in temperature data in turbulent flows so the answer to the above question is no.

Specifically, we apply the KSASHY analysis to temperature measurements taken in turbulent thermal convective flows [19]. In place of $b(i)$, we now have $\theta(t_i)$, the temperature measurement taken at time t_i. In the experiment, the measurements were sampled at a constant frequency of 320 Hz such that $t_i = i\,\delta t$ with $\delta t = 1/320$ s. The standardized PDFs of the temperature increments $\Delta_n\theta(t_i) = \theta(t_{i+n}) - \theta(t_i)$ have been studied and found to change with n [20] thus the temperature data in turbulent thermal convection are multifractal. Also, the temperature structure functions $R_p(n) \equiv \langle |\Delta_n\theta(t_i)|^p \rangle$ have been studied and found to have good relative scaling [21]

$$R_p(n) \sim [R_2(n)]^{\xi_p/\xi_2}. \tag{16}$$

We calculate $\Theta(t_m) = \Sigma_{j=1}^m \theta(t_j)$ and repeat the KSASHY analysis, as discussed in Sec. III with $B(m)$ replaced by $\Theta(t_m)$, to obtain $\Delta_n\Theta^*(t_i)$ and $\Delta_n\theta^*(t_i)$. We then calculate the corresponding statistical moments $\hat{R}_p(n) \equiv \langle |\Delta_n\Theta^*(t_i)|^p \rangle$ and $R_p^*(n) \equiv \langle |\Delta_n\theta^*(t_i)|^p \rangle$ and their respective relative exponents $\hat{\xi}_p/\hat{\xi}_2$ and ξ_p^*/ξ_2^*, defined by

$$\hat{R}_p(n) \sim [\hat{R}_2(n)]^{\hat{\xi}_p/\hat{\xi}_2}, \tag{17}$$

$$R_p^*(n) \sim [R_2^*(n)]^{\xi_p^*/\xi_2^*}, \tag{18}$$

Our results are shown in Fig. 7. Again we find that ξ_p^*/ξ_2^* deviates from $p/2$, as expected from the multifractal character of the turbulent temperature measurements. However, interestingly $\hat{\xi}_p/\hat{\xi}_2$ deviates from $p/2$ too, showing that the standardized PDFs of $\Delta_n\Theta^*$ are scale dependent and changing with n. To show this deviation more clearly, we plot $\hat{\xi}_p/\hat{\xi}_2 - p/2$ versus p in the inset of Fig. 7.

Indeed, the standardized PDFs of $\Delta_n\Theta^*$ changes from stretched-exponential to exponential to Gaussian as n increases from 4 to 4096, as shown explicitly in Fig. 8. This change of the standardized PDFs of $\Delta_n\Theta^*$, the sum of detrended temperature measurements taken over n sampling intervals, with n is similar to the change of the standardized PDFs of the temperature increments $\Delta_n\theta$ with n as reported in Ref. [20].

As can be seen in Fig. 7, ξ_p^*/ξ_2^* are close to ξ_p/ξ_2, indicating again that the detrending procedure does not affect the scaling exponents of the temperature increments. Next, we shall understand this.

V. THE ESSENTIAL EFFECT OF THE DETRENDING PROCEDURE

In this section, we shall explore and understand what the detrending procedure does to the data. As discussed in Sec. III, the "trend" is estimated by a polynomial fit in each segment of the integrated interbeat interval data $B(m)$, and we have used a polynomial of degree 3. We check that our results do not change much when a polynomial of a lower degree is used instead. In particular, we obtain similar results by using a linear fit of the different segments of $B(m)$. Actually, in the original detrended fluctuation analysis, a linear least-squares-fit was adopted [16]. In the following, we shall derive explicit results for "detrended" B^* when the "trend" is estimated by a linear fit.

Let us focus on the lth segment of $B(m)$ with $m_1 \leqslant m \leqslant m_2$, where $m_1 = (l-1)(2n)+1$ and $m_2 = l(2n)$ for some l. l runs from 1,2,3,…, for all the segments. Denote the best linear fit to this segment by $a_l m + c_l$ where the fitting constants a_l and c_l depend on l. The fitting constant a_l can be reasonably well approximated by the slope in this segment

$$a_l \approx \frac{B(m_2) - B(m_1)}{2n-1}. \tag{19}$$

Using Eq. (5), we have

$$a_l \approx \frac{\sum_{j=m_1+1}^{m_2} b(j)}{2n-1} \approx \frac{\sum_{j=m_1}^{m2} b(j)}{2n} \equiv \bar{b}_l, \tag{20}$$

where $\bar{b}_l$ is the local average of $b(j)$ in the lth segment. Recall from Sec. III that B^* is B subtracting the best linear fit and use Eq. (5), we have

$$B^*(m) \approx \sum_{j=1}^{m} [b(j) - \bar{b}_l] - c_l \tag{21}$$

and

$$B^*(m+n) \approx \begin{cases} \sum_{j=1}^{m+n} [b(j) - \bar{b}_l] - c_l, & m+n \leqslant m_2, \\ \sum_{j=1}^{m+n} [b(j) - b_{l+1}] - c_{l+1}, & m+n > m_2. \end{cases} \tag{22}$$

Thus using Eq. (7), we get

$$\Delta_n B^*(m) \approx \sum_{j=m+1}^{m+n} [b(j) - \bar{b}_l] \tag{23}$$

for $m+n \leqslant m_2$ and

$$\Delta_n B^*(m) \approx \sum_{j=m+1}^{m_2} [b(j) - \bar{b}_l] + \sum_{j=m_2+1}^{m+n} [b(j) - \bar{b}_{l+1}] \tag{24}$$

for $m+n > m_2$. To obtain Eq. (24), we make use of the approximation that the two linear fits of the lth and $(l+1)$th segments intersect at $m=m_2$:

$$b_l m_2 + c_l \approx b_{l+1} m_2 + c_{l+1} \Rightarrow c_{l+1} - c_l \approx (\bar{b}_l - \bar{b}_{l+1}) m_2. \tag{25}$$

Comparing Eqs. (23) and (24) with (9), we see immediately that the detrended interbeat interval b^* is given approximately by

$$b^*(j) \approx b(j) - \bar{b}_l. \tag{26}$$

Hence the essential effect of the removal of the linear trend from the integrated interbeat interval data $B(m)$ is to subtract the local average from the interbeat interval time series $b(j)$. For a general time series, the detrending procedure of first integrating the data then removing the linear trend is essentially the same as subtracting the local average from the original time series.

To verify this directly, we redo the analysis for the heart interbeat data with the local average subtracted and compare the results obtained with those from the KSASHY analysis. We define

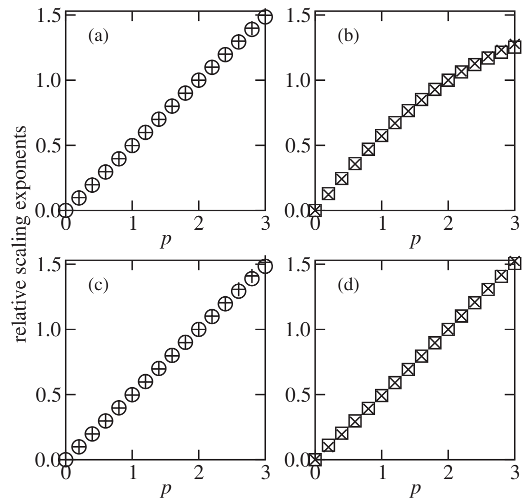

FIG. 9. Comparison of the relative scaling exponents $\hat{\zeta}_p/\hat{\zeta}_2$ (pluses) and ζ_p^*/ζ_2^* (crosses) with $\hat{\alpha}_p/\hat{\alpha}_2$ (circles) and α_p/α_2 (squares) obtained, respectively, from the KSASHY analysis and from the analysis eliminating the local mean. The comparison for healthy heartbeat data is shown in (a) and (b) while that for pathological heartbeat data from congestive heart failure patients is shown in (c) and (d). Good agreement between $\hat{\zeta}_p$ and $\hat{\alpha}_p$ and between ζ_p^* and α_p is seen.

$$\tilde{b}(j) = b(j) - \bar{b}_l, \tag{27}$$

$$\tilde{B}(m) = \sum_{i=1}^{m} \tilde{b}(i), \tag{28}$$

and study the scaling behavior of the statistical moments of

$$\Delta_n \tilde{b}(j) = \tilde{b}(j+n) - \tilde{b}(j), \tag{29}$$

$$\Delta_n \tilde{B}(j) = \tilde{B}(j+n) - \tilde{B}(j) = \sum_{i=j+1}^{j+n} \tilde{b}(i) \tag{30}$$

with n. The corresponding exponents are denoted by α_p and $\hat{\alpha}_p$, which are defined by

$$\langle |\Delta_n \tilde{b}(i)|^p \rangle \sim n^{\alpha_p}, \tag{31}$$

$$\langle |\Delta_n \tilde{B}(i)|^p \rangle \sim n^{\hat{\alpha}_p}. \tag{32}$$

We compare α_p and $\hat{\alpha}_p$ with ζ_p^* and $\hat{\zeta}_p$ respectively. As seen from Fig. 9, the two sets of exponents are in good agreement for both healthy and pathological HRV, confirming that the essential effect of the detrending procedure is to eliminate the local average from the heart interbeat data. As a result, the detrended interbeat interval increment $b^*(j+n) - b^*(j) \approx \tilde{b}(j+n) - \tilde{b}(j)$ will be close to the untreated interbeat interval increment $b(j+n) - b(j)$, that is, the interbeat interval increments are not affected much by the detrending procedure. This explains why ζ_p^* are close to ζ_p (see Fig. 3) and similarly why ξ_p^* are close to ξ_p (see Fig. 7). On the other hand, $B^*(j+n) - B^*(j) \approx \tilde{B}(j+n) - \tilde{B}(j) = \Sigma_{i=j+1}^{n} b(i) - \bar{b}_l$ are generally different from $B(j+n)-B(j)=\Sigma_{i=j+1}^{n}b(i)$, and thus the sum of detrended interbeat intervals could have different statistical features from those of the sum of untreated interbeat intervals.

VI. SUMMARY AND CONCLUSIONS

Understanding the nature of the complicated human HRV and thus human heartbeat dynamics has been the subject of many studies. In these studies, possible different statistical features of HRV in different physiological states have been reported. In particular, an interesting and intriguing finding [3] is that in the healthy state, human heart rate fluctuations display multifractality, and that this multifractal character is lost for heart rate fluctuations in pathological state such as congestive heart failure. Based on an analogy with measurements in turbulent fluid flows, which are also known to have multifractal character, such multifractality in healthy HRV can be manifested as a scale dependence or n dependence of the standardized PDFs of the increment of interbeat intervals between n beats. In the KSASHY analysis [15], "scale invariance of the PDFs of detrended healthy human heart rate increments" was reported. However, in this analysis, the interbeat interval data were first integrated, then detrended and the increments taken and studied. We have shown that these increments of the detrended integrated data can be better understood as the sum of detrended interbeat intervals. It is not surprising that the increments of the integrated data would have different statistical features from the increments of the original data. Thus the scale invariance reported in the KSASHY analysis should not be interpreted as the scale invariance of the PDFs of healthy interbeat interval increments. In fact, we have shown explicitly that this is not the case. Rather, the standardized PDFs of detrended healthy interbeat interval increments are scale dependent, as expected from the multifractal character of healthy HRV. Moreover, we have confirmed that the standardized PDFs of detrended interbeat interval increments become scale invariant for pathological HRV, in accord with the loss of the multifractal character of heart rate fluctuations in pathological state.

We have understood the essential effect of the detrending procedure is to eliminate the local average from the heart interbeat data. We have further found that this scale invariance of the PDFs of the sum of n detrended heartbeat intervals, i.e., the sum of n interbeat intervals with the local average subtracted, is displayed also by heart rate fluctuations of congestive heart failure patients. In both the healthy and pathological states, such scale-invariant PDFs are close to an exponential distribution. Since this scale invariance is a general feature, it cannot be an indication of the healthy heartbeat dynamics being critical, in contrast to what was claimed in Ref. [15]. On the other hand, such scale invariance is absent in the multifractal temperature measurements in turbulent thermal convective flows.

In short, human heart rate fluctuations display various intriguing characteristics. The standardized PDFs of the interbeat interval increments between n beats are scale or n dependent for healthy heart rate fluctuations but are scale invariant or n independent for pathological heart rate fluctuations. This multifractal character in healthy HRV and its loss in pathological HRV can be exploited as a diagnostic tool for heart diseases such as congestive heart failure. On the other hand, for both healthy and pathological heart rate fluctuations, the standardized PDFs of the sum of n interbeat intervals, with the local average subtracted, are close to exponential distributions and are thus n independent. The implications of these intriguing features of multifractality and scale invariance for human heartbeat dynamics remain to be fully understood.

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