Crossover in the power-law behavior of confined energy in a composite granular chain

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We present a numerical study of the impact energy decay in a composite granular chain containing two heavy and one light sections. We observe a marked crossover in the power-law behavior of the impact-energy decay. The average reflection frequency first increases with a decreasing acceleration, and arrives at its maximum at "crossing" time then decays almost exponentially. The analysis demonstrates that this phenomenon is related to the structural transition from compression to dilation state in both heavy-particle sections. The further calculations suggest the dependence relation of the power-law exponent (γ_{cb}) in compression state on the mass ratio (m_2/m_1) and the Hertz law exponent (n) of the composite granular chain $\gamma_{cb}\sim\left(\frac{m_2}{m_1}\right)^{1/(n+1)}$.

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Granular materials have recently received increasing interest because of their ubiquity around us and many known applications, but it is difficult to understand their intrinsic dynamic properties due to the strong nonlinearity of forces between particles and the complex distributions. One-dimensional chains of elastic spheres are the simplest granular systems and a very active area of research [1–15] . As reported in the pioneering work of Nesterenko [1] the propagation of an elastic impulse in a granular chain possesses soliton-like features. Experimental [16] studies have confirmed the existence of such solitary waves. Despite large progress has been made on this subject, the physics of granular matter remains interesting and new behaviors or effects are to be found and investigated.

Granular materials have long been used to protect things from impact, such as packing sandbags around bunkers. Researchers are trying to develop more sophisticated granular materials that provide better protection from blasts. Recently, Hong [17] constructed a "granular container" using a series of sections with particles and by theoretical analysis he predicted that this granular container can trap energy in a particular region and release the trapped energy little by little in the form of separate solitary waves over time. Nesterenko *et al.* [18] demonstrated experimentally the efficiency of solitonlike and shocklike pulse trapping and disintegration in a composite granular protector and proved that its efficiency depends on the particle's arrangements. Hong [17] studied the decay law of the impulse energy in granular protectors and found a very interesting universal power-law behavior in time for leakage of the impulse energy remained inside various granular containers $E_R=At^{-\gamma}$, where the exponent γ is a universal dimensionless constant and the constant A depends on the structure of the granular container, such as the length of the container and the arrangement of the granules. This power-law behavior originates from the decrease in the speed of a solitary wave after the reflection accompanying transmission at the interface from light into heavy granules. However, little is known of the exponent. If it is available that the theoretical dependence relation of the exponent (γ) on the mechanical properties such as the elastic constant (k), the mass (m), and the Hertz law exponent (n), this should not only be of great benefit to understand the dynamics of composite granular chain and but also be crucial to design the shock protector.

In this paper we revisit the leakage of impulse energy trapped inside a composite granular chain. We observe a marked crossover in this power-law behavior of the decay of the trapped energy and find that the crossover links to the structural transition from compression to dilation state in the heavy-particle sections. We then extend our study by exploring the influencing factors of the exponent in compression state and obtain their dependence relation.

In the present work we modeled the symmetric granular chain as a collection of spherical particles which contains two sets of particles and is divided into two heavy and one light sections as shown in Fig. 1. The particles on both heavy regions (denoted by dark balls) have masses m_1 and radii R_1, the particles of the light section (denoted by gray balls) have masses m_2 and radii R_2. There exist two interfaces in the granular chain. Both ends of the chain are free to move. Initially, every particle is placed barely in touch with one another. The particles repel one another only when they are in contact. The contact force between neighboring particles follows Hertz's law [19]: $F(\delta_{i,i+1})=k_{i,i+1}(\delta_{i,i+1})^{3/2}$ if $\delta_{i,i+1}\geqslant 0$ and $F(\delta_{i,i+1})=0$ if $\delta_{i,i+1}<0$. The overlap between two adjacent particles, $\delta_{i,i+1}$, is written as $\delta_{i,i+1}=R_i+R_{i+1}-(x_{i+1}-x_i)$, x_i is the position of grain i. The elastic constant is denoted by k. Under no precompression and in the frictionless medium, the equation of motion of particle i is written as

$$\frac{d^2x_i}{dt^2}=\frac{1}{m_i}[k_{i-1,i}(\delta_{i-1,i})^{3/2}-k_{i,i+1}(\delta_{i,i+1})^{3/2}]. \quad (1)$$

The governing parameters in Eq. (1) is the ratio of k and m, that is, the decrease of elastic constant is equivalent to the increase of mass. Therefore, as introduced by Sinkovits and Sen [3] and adapted in the work of Hong [17], we fixed the elastic constant (k=5657) and changed the mass to discriminate different particles. The diameter is 100. An impulse defined by an initial velocity at time t=0 was initiated at the

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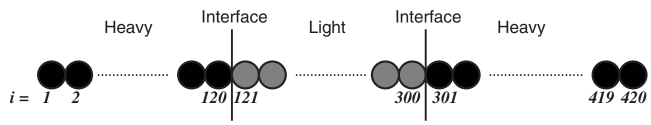

FIG. 1. Schematic setup of the symmetric granular chain containing two sets of particles.

first ($i=1$) particle. The fifth-order Gear predictor-corrector method [20] was used to integrate the particle equation of motion and the time step was 1.25×10^{-5}.

In Fig. 2, we present the log-log plot of the remained energy (E_R) inside the light-section particles versus the elapsed time (t). We make special note of the fact that the dependences of the natural logarithm of E_R on the natural logarithm of t are linear over both, what we now call, the compression and the dilation branches. The "crossing" time $t_c(\approx 120)$ corresponds to the transition point from compression to dilation branch. This crossover has not been observed before. The linearity clearly shows that, as reported by Hong [17], the remained energy inside the light-section particles decays with the time in a behavior $E_R(t)=At^{-\gamma}$. The crossover yields two power-law regions, the slow releasing of E_R in the compression and the fast releasing in the dilation. That is, the exponent γ in the compression branch is less than that in the dilation branch $\gamma_{cb}<\gamma_{db}$. The same crossover behaviors in the time dependence of E_R have also been observed in other granular chains with different details such as contact force or grains number in the light section. Therefore, the presently observed crossover behavior in the power-law dependence of E_R on time is universal in these granular containers.

As is well known recently, when the solitary wave passes from heavier particles into lighter ones, it breaks into a train of weaker, slower pulses [2,14,17]. On the other hand, when a pulse moves from lighter particles to heavier ones, part of it is reflected back through the interface. So the light-section chains are filled with weak, slow pulses bouncing back and forth between the two interfaces and release the trapped energy of the impact in both directions very slowly. It is both heavy-light and light-heavy interfaces that lead to energy trapping in the middle light section. Therefore, the decay of remaining energy inside the light section is dependent on the number of reflections at both interfaces. The number of the reflections per unit time decreases with the decrease in the speed of the solitary wave and decreases with the increase of the length of the light section. Here, it can be ignored the light-section length dependence of the reflections number. Because the solitary-wave speed does not keep a constant but decreases after reflection, the number of reflections per unit time decreases with the time. On the basis of the power-law behavior of E_R, Hong [17] inferred that the averaged number of reflections per unit time at interfaces $[N_R(t)]$ is inversely proportion to the time $N_R(t)\propto\frac{1}{t}$. That is, the result of $E_R(t)=At^{-\gamma}$ originates from the relation of $N_R(t)=\gamma/t$. Our calculations show that the $N_R(t)$ exhibits a much complex behavior due to the appearance of second multipulse structures at the interfaces. But the average reflection frequency [that corresponds to $\frac{1}{t}\int_{t_0}^{t}N_R(t)dt$] shown by the dotted line in

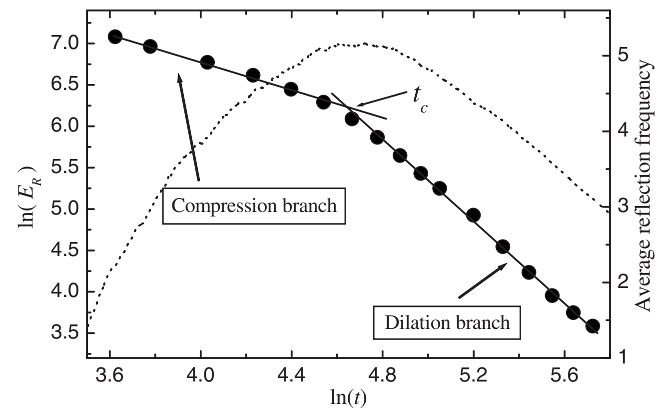

FIG. 2. The time dependence of the impact energy remained inside the light-section particles (solid circles) and the average reflection frequency at both interfaces (dotted line). Two solid lines are guides to the eyes and correspond to the compression and dilation branches, respectively, they intersect at time t_c.

Fig. 2 first increases with a decreasing acceleration when $t<t_c$; and arrives at its maximum at t_c; then decays almost exponentially when $t>t_c$. This behavior is in good agreement with the crossover in the power-law behavior in the impact-energy decay.

The recent experimental observation in one-dimensional nonlinear composite granular chain has shown that the energy-trapping effect in the granular chain is enhanced by using a magnetically induced precompression [21]. The physical explanation for such enhancement is related to the gaps opening. In addition, gaps opening was also observed in a one-dimensional granular chain in the process of solitary wave collision, which is related to the generation of secondary solitary wave [22,23]. Using the numerical approach, Vergara [24] has found that the scattering process of the solitary wave is elastic at one interface, while at another interface the transmitted solitary wave has stopped its movement during a time that becomes longer with the increase in the ratio between masses at the interfaces. He concluded that this effect could be traced back to the phenomenon of gaps opening. Therefore, an important issue to address is whether our presently obtained result $\gamma_{cb}<\gamma_{db}$ is related to the gaps opening in the granular chain too, even though our simulated system has not be introduced precompression. To answer this question and to explore the dynamic origin of the presently observed crossover behavior, the parameter Δ_{ij} is introduced and quantified as follows:

$$\Delta_{ij}=\frac{x_j-x_i-[R_i+2(R_{i+1}+\cdots+R_{j-1})+R_j]}{R_i+2(R_{i+1}+\cdots+R_{j-1})+R_j}\times 100\,\% , \tag{2}$$

which represents the degree of compression or dilation of the granular chain between particle i and j. The chain between particle i and j is in dilation state if $\Delta_{ij}>0$, which means that a series of gaps open between the particles, and in compression state if $\Delta_{ij}<0$ and no gaps between particles. The initial states of our simulated system corresponds to $\Delta_{ij}\equiv 0$ for any i and j.

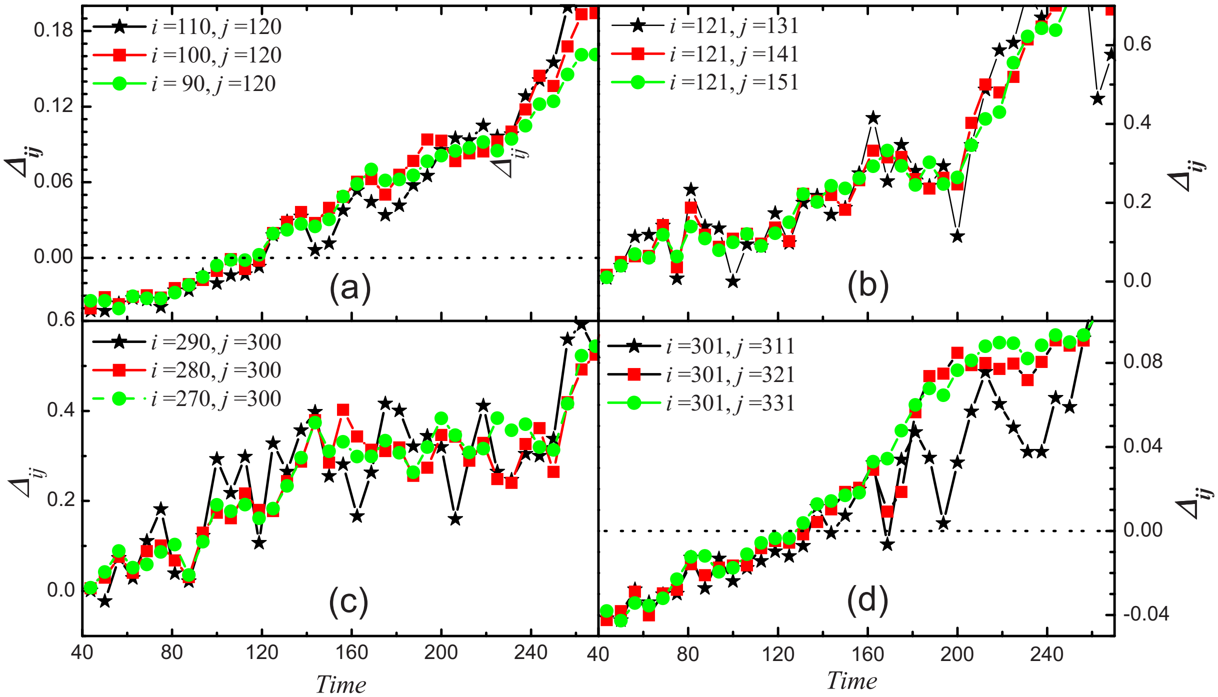

FIG. 3. (Color online) The compression/dilation parameter Δ_{ij} as a function of the time for those heavy (a) and light (b) particles nearby the heavy-light interface and light (c) and heavy (d) particles nearby the light-heavy interface.

Figures 3(a)–3(d) show Δ_{ij} as a function of time for heavy (a) and light (b) particles nearby the heavy-light interface and light (c) and heavy (d) particles nearby the light-heavy interface, respectively. For those particles in the light section as shown in (b) and (c), whether they are close to the heavy-light interface or close to the light-heavy interface, Δ_{ij} keeps a positive value and increases with the time, indicating that during the decay of the trapped energy the particles of the light section are maintained in the dilation state and the dilation is enhanced with the time. In sharp contrast, for those particles in both heavy sections and close to both interfaces, Δ_{ij} increases with the time, at early times it is negative and at late times it increases to be positive, the time when the sign of Δ_{ij} changes from negative to positive is in good agreement with the crossing time t_c mentioned above. In other words, inside both heavy sections, $\Delta_{ij}<0$ before t_c and $\Delta_{ij}>0$ after t_c, which is why we call two linear regions of Fig. 2 with different slopes the compression and dilation branches. Therefore, the crossover behavior in the dependences of E_R on t should resulted from the structural transition in both heavy sections from compression to dilation state. In early times $(t<t_c)$ the particles keep contact and the compression between neighboring particles weakens with the time but almost no gaps appear, whereas in late times $(t<t_c)$ a series of gaps open between them. t_c corresponds to the transition time from no gaps to gaps opening for particles in both heavy sections. Gaps opening in both heavy sections leads to the decrease in the reflected energy at both interfaces but the increase in the transmitted energy, and thus enhances the releasing of the trapped energy in the light section. In the meantime, the structural transition gives rise to a great difference in the reflection frequency at the interfaces between compression and dilation state.

We then extend our calculations about the dependence of γ_{cb} and γ_{db} on the mass ratio of heavy to light particles (m_1/m_2) and the Hertz law exponent (n). In the dilation regime, with the change in the mass ratio of heavy to light particles γ_{db} almost keeps a constant, so in Fig. 4 we only display γ_{cb} as a function of the mass ratio for two cases with various Hertz-law exponents. Note that we fixed the elastic constant and changed the mass to discriminate different particles, and for convenience m_2 is set to be 1. From this figure we can clearly see the influence of m_1 and n on γ_{cb}. With the

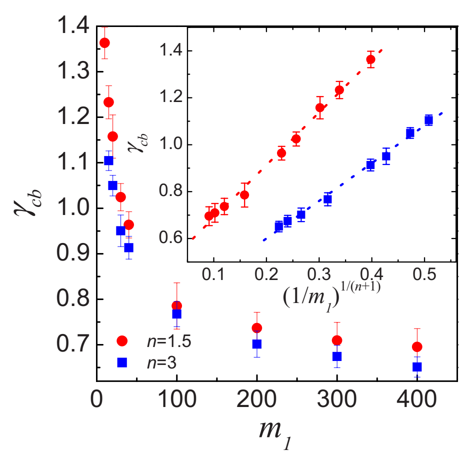

FIG. 4. (Color online) The power law exponent (γ_{cb}) in the compression state as a function of the mass (m_1) of heavy particles for two cases with different Hertz law exponent (n). The inset shows the exponent γ_{cb} plotted against $(1/m_1)^{1/(n+1)}$ illustrating the dependence relation. The dotted lines correspond to the linear fits.

increase of m_1 (i.e., the increase in the mass ratio of heavy to light particles) γ_{cb} decreases steeply and with the increase n of γ_{cb} decreases. When γ_{cb} is replotted against $\left(\frac{1}{m_1}\right)^{1/(n+1)}$ as in the inset of Fig. 4, we find the linear fits are quite satisfactory for $n=1.5$ and $n=3$. So an approximate expression of the exponent γ_{cb} could be obtained $\gamma_{cb} \sim \left(\frac{m_2}{m_1}\right)^{1/(n+1)}$. This could be interpreted in terms of the dimensionless time and velocity $\zeta = \dot{x}/v_0$ and $\tau = t(k/m)^{1/(n+1)} v_0^{(n-1)/(n+1)}$, which transform the equation of motion into a dimensionless form [2].

In summary, a numerical study has been performed on the impact energy decay in a symmetric granular chain with two interfaces and two sets of particles. We have observed an interesting crossover in the universal power-law behavior of the trapped energy versus time. That is, in early times the log-log curve of the remained energy inside the light-section particles versus the elapsed time changes linearly, at a certain time it has a noticeable bend, then it varies linearly again and becomes steeper. These two regions are called as the compression branch and the dilation branch, respectively, because in the compression branch the heavy particles close to both interfaces keep in contact whereas in the dilation branch there exist a series of gaps between them. There exists a great difference in the average reflection frequency between compression and dilation state. The extended calculations yield an approximate expression of the exponent $\gamma_{cb} \sim \left(\frac{m_2}{m_1}\right)^{1/(n+1)}$. The presently observed crossover in the power-law behavior and the approximate dependence relation of the power-law exponent are important and deserve to be further clarified in the sense that they do form the part of the complete dynamics of the solitary waves in the composite granular chain and one hopes to find novel applications of granular systems against the mechanical impacts.

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[1] V. F. Nesternko, J. Appl. Mech. Tech. Phys. **5**, 733 (1984).
[2] V. F. Nesterenko, *Dynamics of Heterogeneous Materials* (Springer, New York, 2001), Chap. 1.
[3] R. S. Sinkovits and S. Sen, Phys. Rev. Lett. **74**, 2686 (1995).
[4] J. Ji and J. Hong, Phys. Lett. A **260**, 60 (1999).
[5] S. Sen and M. Manciu, Physica A **268**, 644 (1999).
[6] S. Sen, M. Manciu, and F. S. Manciu, Appl. Phys. Lett. **75**, 1479 (1999).
[7] S. Sen, M. Manciu, and F. S. Manciu, Physica A **299**, 551 (2001).
[8] J. Hong and A. Xu, Phys. Rev. E **63**, 061310 (2001).
[9] J. Hong and A. Xu, Appl. Phys. Lett. **81**, 25 (2002).
[10] J. Lee, S. P. Park, and I. Yu, Phys. Rev. E **67**, 066607 (2003).
[11] A. Rosas and K. Lindenberg, Phys. Rev. E **69**, 037601 (2004).
[12] C. Daraio, V. F. Nesterenko, E. B. Herbold, and S. Jin, Phys. Rev. E **72**, 016603 (2005).
[13] S. Job, F. Melo, A. Sokolow, and S. Sen, Phys. Rev. Lett. **94**, 178002 (2005).
[14] Lautaro Vergara, Phys. Rev. Lett. **95**, 108002 (2005).
[15] F. Melo, S. Job, F. Santibanez, and F. Tapia, Phys. Rev. E **73**, 041305 (2006).
[16] C. Coste, E. Falcon, and S. Fauve, Phys. Rev. E **56**, 6104 (1997).
[17] J. Hong, Phys. Rev. Lett. **94**, 108001 (2005).
[18] C. Daraio, V. F. Nesterenko, E. B. Herbold, and S. Jin, Phys. Rev. Lett. **96**, 058002 (2006).
[19] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, Oxford, 1986).
[20] M. P. Allen and D. J. Tildesley, *Computer Simulation of Liquids* (Oxford University Press, Oxford, 1987).
[21] V. F. Nesterenko, C. Daraio, E. B. Herbold, and S. Jin, Phys. Rev. Lett. **95**, 158702 (2005).
[22] M. Manciu, S. Sen, and A. J. Hurd, Phys. Rev. E **63**, 016614 (2000).
[23] F. S. Manciu and S. Sen, Phys. Rev. E **66**, 016616 (2002).
[24] L. Vergara, Phys. Rev. E **73**, 066623 (2006).