

Growing trees in internet news groups and forums

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We present an empirical study of the networks created by users within internet news groups and forums and show that they organize themselves into scale-free trees. The structure of these trees depends on the topic under discussion; specialist topics have trees with a short shallow structure whereas more universal topics are discussed widely and have a deeper tree structure. For news groups we find that the distribution of the time intervals between when a message is posted and when it receives a response exhibits a composite power-law behavior. From our statistics we can see if the news group or forum is free or is overseen by a moderator. The correlation function of activity, the number of messages posted in a given time, shows long-range correlations connected with the users' daily routines. The distribution of distances between each message and its root is exponential for most news groups and power law for the forums. For both formats we find that the relation between the supremacy (the total number of nodes that are under the node i , including node i) and the degree is linear $s(k) \sim k$, in contrast to the analytical relation for the Barabási-Albert network.

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I. INTRODUCTION

One of the most important features of the internet is the opportunity it offers people to exchange opinions with one another. Now anyone can participate in a discussion or debate online and the global reach of the internet allows a single person's opinion to be shared with people from all over the world. Thus each of us can now be a source of information, not only for our relatives and friends, but for the whole world. We can offer our opinion to a very wide range of people and receive feedback on this opinion. Thus internet discussions are potentially important in helping to shape people's opinions and behavior and in the spreading of ideas and information. In this way the internet is a medium which is very different than traditional media such as newspapers, radio, and television. The use of the internet has led to an explosion of interest within other academic disciplines in phenomena such as social contagion, viral marketing, and stealth marketing. Despite this importance, scientific research into internet discussions has been rather limited.

There have only been a few scientific papers examining internet discussion networks. Makowiec and Bykowska [1] considered the three most popular web log or blog web pages in Poland. They provided an analysis of the network structure of blogs and gave a sociological explanation of the results. In related work, Zhongbao and Changshui [2] examined the network properties of bulletin board systems (BBSs), which are similar to the news groups examined in this paper. They [2] studied a network in which edges were between users and were able to identify distinct communities within the network of users. BBS and user networks were also studied by Goh *et al.* [3] who found intercommunities and intracommunities with different topological properties. The intercommunity was a homogenous system, in which members all knew each other, while intercommunities were characterized by a power-law degree distribution. Capocci *et al.* [4] investigated the largest internet encyclopedia, Wikipedia. A bow-tie-like, scale-free structure with almost neutral

mixing was found. Only small and medium nodes exhibited linear preferential attachment. Valverde and Solé [5] focused on technology development communities, such as open source communities, by looking at e-mail exchanges. Nonlocal growth rules based on a betweenness centrality model were examined and compared with the empirical data. The temporal properties of e-mail exchange groups were studied by Barabási [6].

Internet forums and news groups are similar to BBS networks, but in contrast to previous work [1–3,5], here we place an edge between messages and focus on the network of ideas or opinions posted by users, rather than networks between the users themselves. In this way we obtain treelike networks with a central topic, the root node, and the surrounding threads.

In the last few years there has been much work characterizing the topology of real networks [7–13]. This work has shown that our world is more complex than we had originally imagined and has led to the development of the idea of a complex network. The most significant result arising from these studies is that a power-law degree distribution appears to be very common in real complex networks.

In this paper we examine empirically a variety of basic structural and temporal properties of the internet discussion networks that are created by internet users. The paper is organized as follows. In the next section we introduce the different types of internet discussions and describe the scope of our empirical study. In Sec. III we describe our results, both topological and temporal, before summarizing our findings in the final section.

II. TYPES OF INTERNET DISCUSSIONS

Almost all internet discussions take place through the medium of forums. Most internet information portals, on subjects such as politics, accidents, sports, etc., include forums as part of their website. New topics are introduced to these forums on a daily basis. Some portals give people fixed fo-

runs to discuss common topics such as love, work, and sports. Users cannot put unmoderated messages into these forums; most forums have a person or computer program—a “moderator”—that acts as a referee for the comments posted rejecting those that are deemed unsuitable.

Another type of internet discussion are news groups. These are run on servers that normally contain an enormous number of fixed topics. The access to a news group depends on its construction and usually to become a member of a news group one needs a computer program—a client of the news group. Nowadays, all e-mail programs such as Microsoft Outlook or Mozilla Firefox have such a client. Recently due to the growing interest in internet discussions, a lot of news groups can be accessed through the web page, for instance, news groups on the Google web page. In the former case, which is discussed in our paper, the connection to the server is controlled by the administrator. Some servers are designed for everybody and others themed for a group of people such as students at a university, employees of a company, etc. The administrator of a server can block access of the people that break its rules.

A third popular medium of internet discussion is a blog. There are a number of websites where people can establish their own blog, which usually takes the form of a diary of their day-to-day life. Other people can discuss the blogs and express their opinions about them to other readers or the owners of the blogs themselves. The bloggers are usually able to place links to other blogs, which are either on a topic related to their own or of general interest to them, on their website. These links create a network of blog owners [1].

Typical construction of internet discussions. For a news group and an internet forum the topic of the discussion is a root node. The threads that initiate new discussions are connected directly to the root node. When people contribute to a forum they can either write a commentary on a previous opinion or start a new thread. Every message is indexed by the name of the author, its place in the hierarchy, and its time of posting. In this paper we treat each message as a node. We create a link between a message and a responding or answering message. Figure 1 shows the typical structure of a small internet discussion.

We have investigated the network structure and temporal properties of 3 forums and 15 news groups, whose data was collected from two sources: The internet forum on the website www.onet.pl and the news groups on the server news.student.pw.edu.pl. In the case of news groups, the people who can contribute to a discussion is limited by the fact that only computers inside the university’s network are allowed to login. Because of this only students and academic staff have access to these discussions, and there are around 30 000 of them each year. We did not measure the number of active users of these news groups, but we suppose that there are less than 5000.

The internet forum on www.onet.pl is part of the largest Polish news portal, which is used by around 50% of all Polish internet users. Almost all internet discussions that we have collected were created at different times. However, for internet forums the period of collected data is between 2001–2005 and for news groups the period is 2002–2005.

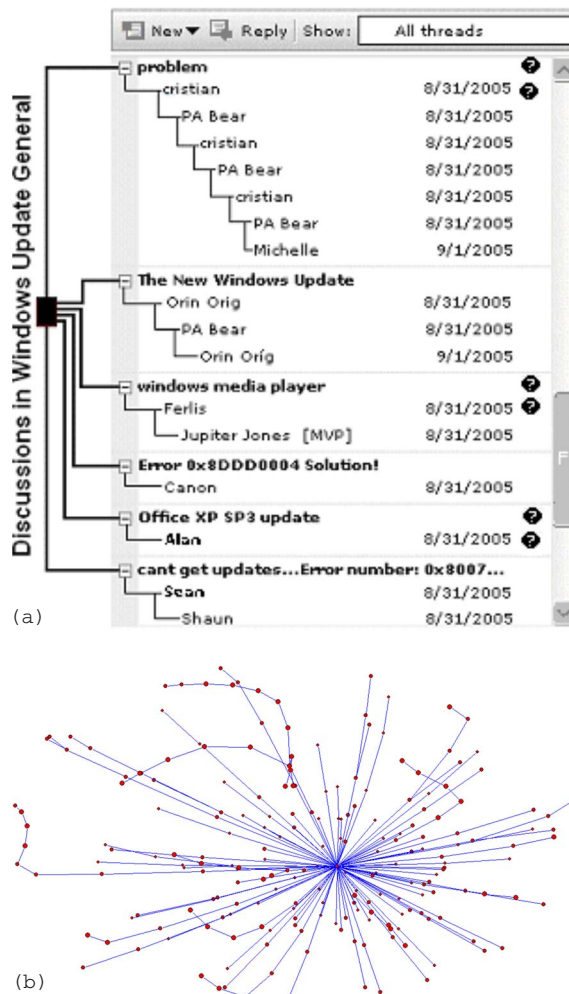


FIG. 1. (Color online) (a) The typical structure of an internet discussion. The black lines show links between messages and the responses to them. (b) The treelike structure of the small news group “Physics,” $N=220$ nodes.

III. EMPIRICAL RESULTS

We study empirically a number of properties of real internet discussions. Our networks are trees and consist of messages, not users, so we are unable to study properties such as the clustering coefficient or to define communities. Similarly it would be fruitless to study node mixing or the betweenness centrality, which were studied in Refs. [1–3,5]. Thus, the structural properties we examine are the degree distribution, the average and maximal distance, the distribution of distances between messages and their root nodes, and the average supremacy [19] of each node as a function of degree. The temporal properties we examine are the distribution of time between a message being posted and there being a response to it, the activity time series, and its correlation function. With the temporal properties we distinguish between network time—time in which one message is posted in one time step and message i is added at time i —and real time—the actual time that messages were posted in our experimental data. Where appropriate, we present results for both an internet forum and a news group for the largest and most representative examples.

TABLE I. We measured 19 internet discussions, 4 from the internet forum www.onet.pl and 15 news groups from the server news.student.pw.edu.pl. The columns contain the name of the discussion, the number of nodes N , the exponent γ of the power law degree distribution, and the maximum distance R_{\max} from the root node. The next column contains the number of threads n_1 over all messages N and the last the average distance from the root node $\langle r \rangle$.

No.	Topic of discussion	N	γ	r_{\max}	n_1/N	$\langle r \rangle$
Onet forums						
1	Poland in the EU	43 027	3.53	13	0.118	4.127
2	Opinions of Poles	36 479	3.28	13	0.103	4.062
3	Situation in Middle East	47 075	3.37	13	0.048	5.701
News groups						
1	Trade	44 266	5.23	24	0.517	1.905
2	Politics	11 706	5.52	46	0.078	7.041
3	Humor	52 525	3.90	76	0.204	3.534
4	Off-topics	21 940	4.71	51	0.188	4.153
5	Linux	11 049	4.87	25	0.208	3.234
6	Pillory	40 495	4.70	62	0.132	5.299
7	Games	34 080	5.37	30	0.293	2.811
8	Games.CS	18 976	4.46	25	0.162	3.698
9	Programming	14 560	5.50	25	0.261	2.948
10	Music	12 461	5.49	20	0.359	2.481
11	Campus.Riviera	15 431	5.08	33	0.326	2.821
12	Campus.Ustronie	31 170	5.10	26	0.317	2.897
13	Electronics	28 199	5.75	18	0.364	2.329
14	Windows	13 684	5.84	32	0.210	3.575
15	Film	32 923	5.16	20	0.306	2.783

A. Degree distribution

All the networks we examined were found to have power-law degree distributions

$$p(k) \sim k^{-\gamma}. \quad (1)$$

Table I lists the topics of these discussions, their size N , the exponent γ of their power-law degree distribution, the maximal distance R_{\max} from the root node, the ratio of the number of threads n_1 over the total number of messages N , and the average distance of all nodes in the network from the root node $\langle r \rangle$.

The internet forums generally have a lower exponent γ than the news groups. In particular, the exponents for forums are in the range $3.28 < \gamma < 4.24$ and for news groups $3.90 < \gamma < 5.84$.

Figure 2 shows a typical degree distribution for the forums and the news groups. The networks have few nodes with high degrees, even for the larger networks, with only seven networks having a maximum degree $k_{\max} > 30$. For the news groups the largest degree is around 20.

In all networks we examined the number of nodes with degree 1 was similar to the number of nodes with degree 2, that is $p(1) \approx p(2)$. This seems to be because people like to argue and preferentially create chain structures in threads and also because people sometimes respond to their own mes-

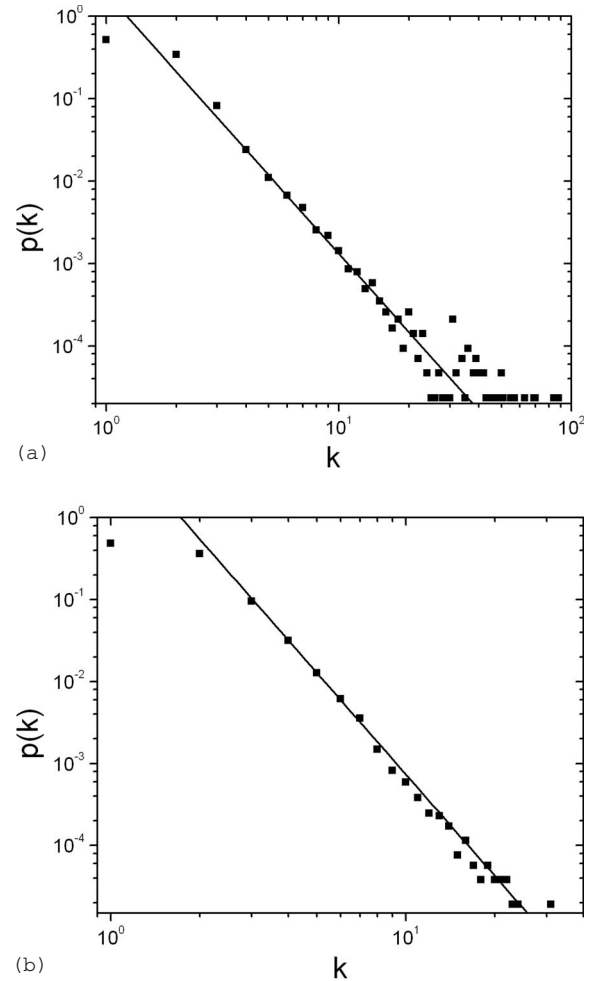


FIG. 2. The degree distribution for the internet forum “Poland in European Union (EU)” (a) and the news group “Humor” (b). The exponents γ are $\gamma=3.53$ for forum “Poland in the EU” (a) and $\gamma=3.90$ for news group “Humor” (b).

sages. This behavior creates more nodes with degrees $k=2$, $k=3$, etc., and shifts the degree distribution toward higher values of k .

B. Time-interval distribution $T(\tau)$

Internet users visit news portals to update themselves on the recent news, and some of them will discuss this news in a forum. In most cases they will only discuss the very latest news and only very interesting topics will be discussed by users over a long period of time. The same rule applies for messages, only interesting or very controversial opinions are discussed for a long time period. This is why messages age very quickly and are soon forgotten. The influence of aging is the reason for the large exponent γ in these networks and for the lack of nodes with large degrees.

There have been a number of attempts to model the effect of aging, see, for instance, Refs. [15–18]. The fundamental quantity in these models is $\pi(k, t, \tau)$, the rate of attaching a new node to a node of degree k and age τ at time t . All these models assume that $\pi(k, t, \tau)$ is a separable function of the

degree and the age of the node. In particular, Dorogovtsev and Mendes [16,17] modeled this aging by assuming that incoming nodes are linked to a node with degree k and age τ with rate $\pi(k,t,\tau)=A(\tau)k$, where $A(\tau)$ is some aging function given by

$$A(\tau) \sim \tau^{-\beta}. \quad (2)$$

They found that the degree distribution of this network remained power law $p(k) \sim k^{-\gamma}$ in the large time limit but with an exponent γ that strongly depends on the exponent β in the aging function [16].

Unfortunately, $A(\tau)$ is not easily measured empirically, as attempts to verify that some real networks were grown by preferential attachment without aging clearly illustrate [20]. Instead, we have measured a related quantity [21], the time interval distribution. This is the distribution of times between a message and a response, for all the internet discussions. More precisely, where a message j , posted at real time t_j , receives a response i at real time t_i , we have studied both the distribution of the real-time interval $\tau=t_i-t_j$, $T(\tau)$, and the distribution of network-time interval $i-j$. The distribution $T(\tau)$ is related to the degree distribution at time t , $p(k,t)$ via

$$T(\tau) = \int w(k,t,\tau)p(k,t)dkdt, \quad (3)$$

where $w(k,t,\tau)$ is the probability that a node of degree k at time t waits another τ time steps before gaining an edge. This latter function contains, implicitly, two temporal processes: the natural waiting time for a new edge which exists in all growing network models plus the effect of the aging identified and modeled in Refs. [15–18]. However, for $1 \ll \tau \ll t$, we expect that the effect of the former will be exponential in τ on $T(\tau)$ whereas if there is appreciable aging, this will manifest itself as a fat tail in $T(\tau)$ for large τ .

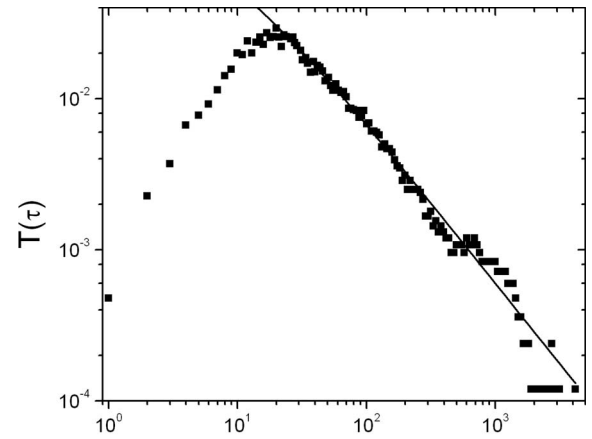
In fact, our results for real time show that in an internet news group messages age and have a power law $T(\tau)$. In Fig. 3 we show that the time-interval distribution

$$T(\tau) \sim [\tau + \tau_0]^{-\delta}. \quad (4)$$

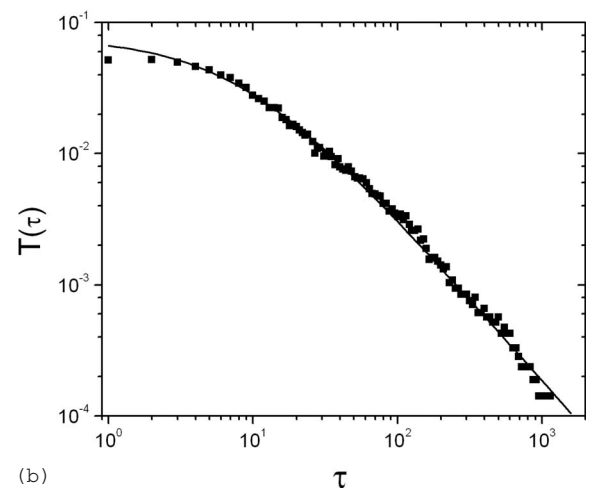
The positive slope of the curve in Fig. 3(a) for small time intervals results from the presence of the moderator in the forum `www.onet.pl`. The moderator has to check each message and this takes some time. Figure 3(b) shows that Eq. (4) gives a good approximation to the empirical measurements.

In Fig. 4 we show the time-interval distribution in network time and this merits two observations. First, there is a change in the time-interval distribution. For all news groups (but not for the forums) we obtained time-interval distributions with two regimes of aging. For each news group there is a characteristic crossover time interval t_c after which messages start aging faster. This characteristic time is different for each network.

Second, the shape of the time-interval distribution for an internet forum is not effected by a moderator and exactly follows Eq. (4). This means that for small time intervals messages age slower and for large intervals faster but the change is smooth and without the critical point observed in news groups.



(a)



(b)

FIG. 3. The time-interval distribution in real time for (a) the forum “Poland in the EU” and (b) the news group “Humor.” The exponent $\delta=1.25$ for (a) and $\delta=1.33$ for (b). The real-time unit is 1 min.

The power-law behavior of the time-interval distribution was studied by Barabási [6] for an e-mail exchange group. By simulating the types of activity of internet users, it was shown that only the “burst” activity results in power-law distributions $A(\tau) \sim \tau^{-\delta}$, where $\delta=1$. Figure 4(b) shows that for small network time intervals the index δ is close to 1. For all news groups $\delta \in (1.0, 1.5)$. Because of the moderator the results for internet forums are disturbed, however, the value of $\delta=1.37$ is still close to 1 [Fig. 4(a)].

We also studied the relationship between the network-time interval and the real-time interval. Of course these are related by the fact that the activity $n(t_i)$, which is the number of messages that were posted in time t satisfying $t_i < t < t_{i+1}$, can be approximated by $n(t_i) \approx (i-j)/(t_i-t_j)$. Our empirical results show that, as would be expected, on average the relation is linear with

$$n(t_i)(t_i-t_j) \sim \varepsilon(i-j) \quad (5)$$

with $\varepsilon=1.04 \pm 0.02$ for the internet forum “Poland in the EU” Fig. 5(a) and $\varepsilon=0.96 \pm 0.02$ for the news group “Humor” Fig. 5(b).

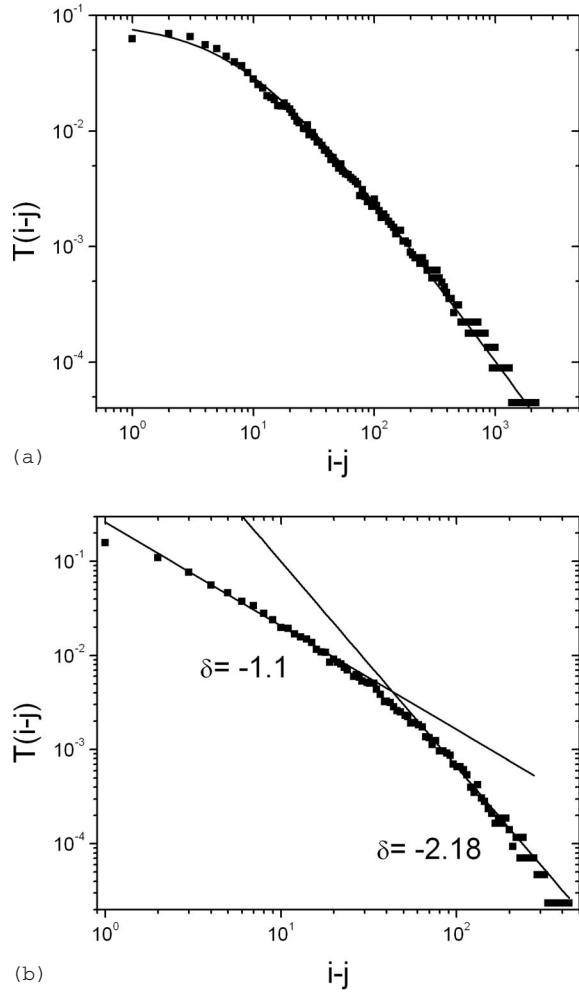


FIG. 4. The time-interval distribution in network time for (a) the forum “Poland in the EU” and (b) the news group “Humor.” The shape of Fig. (a) follows Eq. (4) with $\delta=1.37$. (b) is described by composite power laws with exponents $\delta=1.1$ and $\delta=2.18$.

C. Activity

We define the activity of a news group as the number of messages posted in a given time interval. In Fig. 6 we show the activity time series and the distribution of activity for the discussion forum “Poland in the EU.” Here we have measured the number of messages posted in one hour. As one can see, there is a variation of activity over a wide range of scales.

The peak, in which 898 messages were posted in a single hour, corresponds to the time when Poland was voting in the accession referendum to the EU. This hiatus can be seen in the activity distribution, corresponding to the points to the right in Fig. 6(b), away from the main curve. We examined the distribution of activity for all our news groups, and found that all the distributions were fat tailed, with distributions that ranged from power law to Kohlrausch, $\sim \exp(-\tau^a)$, with $0 < a < 1$.

We have measured the correlation function $C(\tau^*)$ of the activity time series $n(t)$, defined by

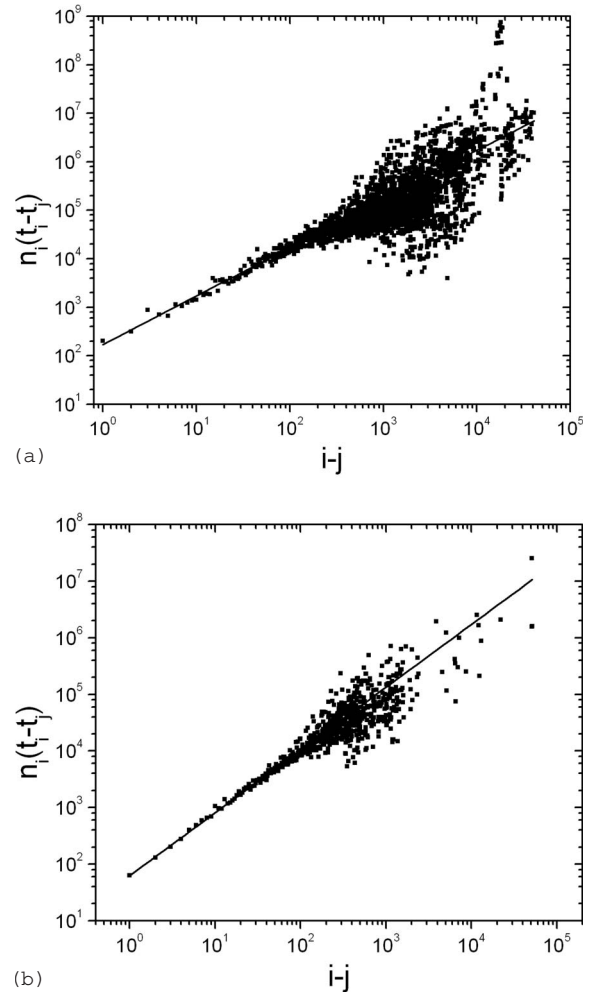


FIG. 5. The average value of the real-time interval multiplied by the activity as a function of the network-time interval for (a) the forum “Poland in the EU” and (b) the news group “Humor.”

$$C(\tau^*) = \sum_{i=0}^{i_{\max}} [n(t_i) - \langle n \rangle][n(t_i + \tau^*) - \langle n \rangle], \quad (6)$$

where $t_i = t_0 i$ and $\langle n \rangle$ is the mean number of messages posted per time t_0 over the whole time series. We studied $t_0 = 1$ h and 1 day.

All the internet discussions indicate a correlation for $\tau^* = 24$ h, which shows the daily routine of the internet discussion users [see, for instance, Fig. 7(b)]. We also found a weak correlation for news groups on the time scale of one week, which is probably connected to the higher activity over a weekend. This is somewhat less pronounced, as Fig. 7(a) illustrates. Some news groups also show correlations for very long times, for instance, for τ^* equal to 180, 270, and 365 days. These were seen in news groups that are only used by students and these long correlations are connected to the academic holiday and semester structure. There is an interesting correlation for $\tau^* = 12$ h in the forum “Poland in the EU.” This correlation is generated by the before-after work activity of the discussion users.

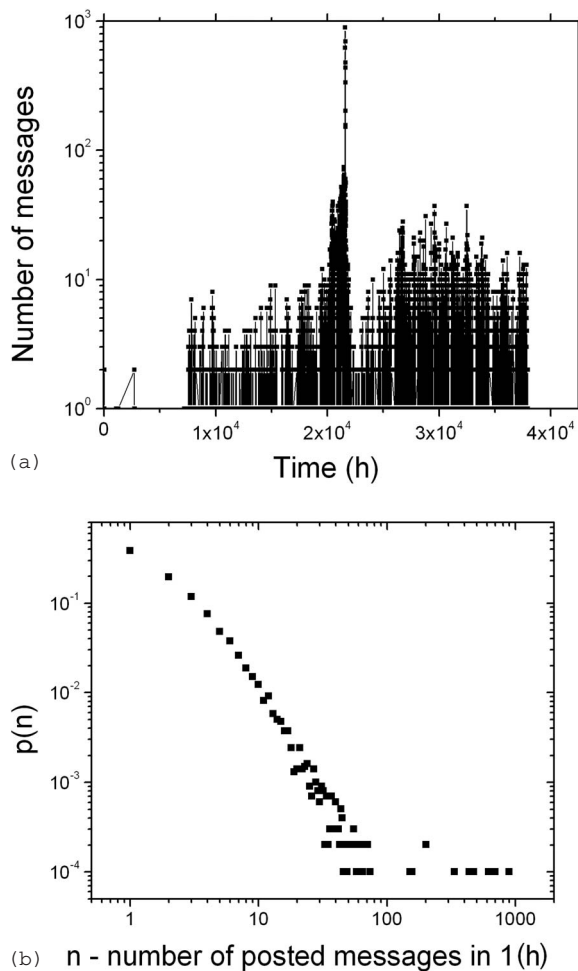


FIG. 6. The activity time series (a) and the activity distribution function (b) for the forum “Poland in the EU.”

D. The distance distribution $D(r)$

$D(r)$ is the distribution of the number of edges between each node in the network and its root node. For all the networks the maximum distances are small. Almost all the news groups exhibit an exponential $D(r)$, such as that illustrated for the news group “Electronics” in Fig. 8(b). Of the news groups, only “Humor” has a distance distribution close to a power law.

The distance distributions for forums are modified by the software used to manage the forum, which only allows a maximum distance of $r=13$. A message that somebody wants to post to a message with $r=13$ is added prior to the message with $r=12$. This results in the large value of $D(13)$ seen in Fig. 8(a). Nevertheless, despite this limitation the distance distributions can show power-law behavior, as Fig. 8(a) illustrates.

The ratio n_1/N (Table I) shows how many threads are created as a fraction of all posted messages. A small value indicates that internet users are focused on the existing threads and they are prone to continuing the previous discussions. Large values show that there is almost no discussion, users place an offer or question and expect only answers to them. A related parameter that describes a discussion is the

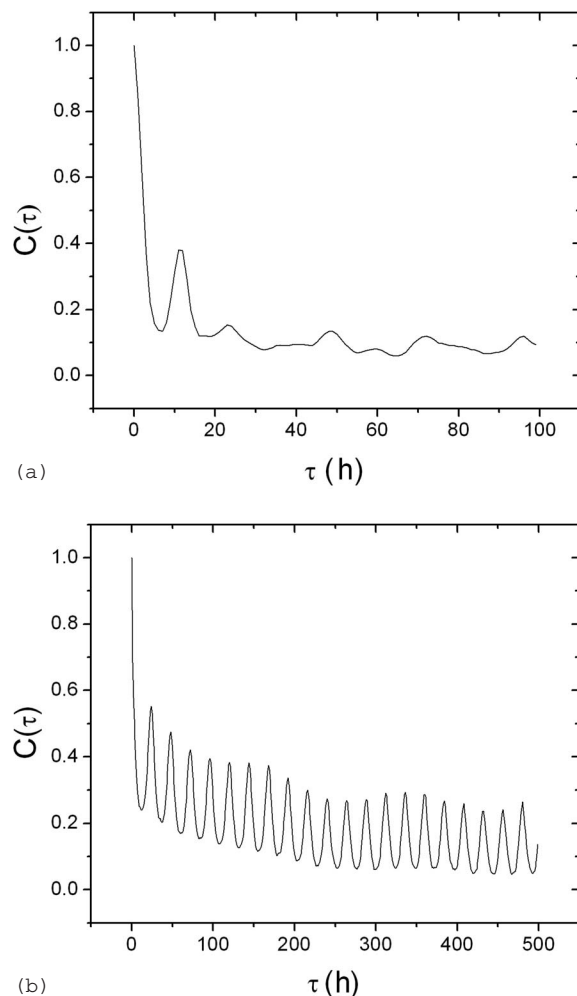
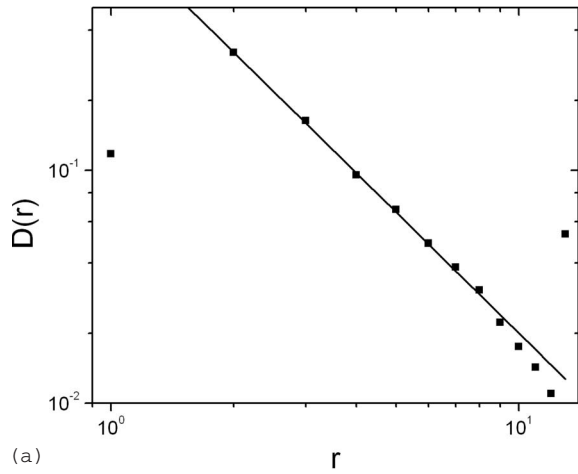


FIG. 7. The correlation function $C(\tau^*)$ for (a) the forum “Poland in the EU” and (b) the news group “Humor,” with a time step $t_0=1$ h.

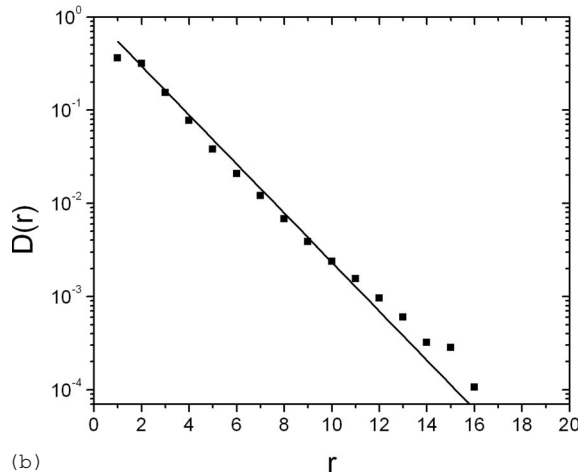
average distance from the root node $\langle r \rangle$ (Table I). For small values of $\langle r \rangle$ the discussion is not engaging and users probably just exchange information. For large $\langle r \rangle$ vigorous discussions are taking place. The ratio n_1/N describes the behavior of the internet users and the average distance $\langle r \rangle$ describes the topological consequences of this behavior. There is a functional dependence between them as Fig. 9 demonstrates. The values of n_1/N and $\langle r \rangle$ show the kind of discussion we examined: technical, where people are interested only in exchanging goods, information, and look for help or theoretical, where people introduce ideas, share opinions, and argue with others. A good example are the two news groups “Games” and “Games.CS.” The “Games” news group is a general discussion about games, where $\langle r \rangle$ is rather small. The news group “Games.CS” is dedicated to only fans of the game *Counter Strike* and its value of $\langle r \rangle$ is much higher, which suggests that the fans are more actively engaged in the discussions.

E. The supremacy function $s(k)$

The supremacy s_i of node i is defined as the total number of all nodes that are not older than i and can be linked to it by



(a)



(b)

FIG. 8. The distance distribution $D(r)$ for (a) the forum “Poland in the EU” and (b) the news group “Electronics.” The distribution for the forum has a power-law behavior $D(r) \sim r^{-\nu}$ with exponent $\nu=1.73$ and the distribution for the news group has an exponential behavior.

a directed path (including the node i). For treelike networks this means that the supremacy s_i of node i is the total number of nodes that are under the node i , including node i . In other words the supremacy s_i is the total number of nodes in the subtree started by node i . The supremacy function $s(k)$ is the average supremacy of all nodes of degree k . In Ref. [19] it was shown that for the Barabási-Albert model [14],

$$s(k) = \frac{m}{m+1} \left(\frac{k}{m}\right)^{m+1} + \frac{1}{m+1}, \quad (7)$$

where m is a number of links created by an incoming node, and for trees, when $m=1$

$$s(k) = \frac{1}{2}k^2 + \frac{1}{2}. \quad (8)$$

For each network we measured the average value $s(k)$ for a particular degree k . Figure 10 shows that for the internet discussions relation $s(k)$ is not $s \sim k^2$, but relation is linear $s \sim k$. The result $s \sim k^2$ was obtained for the Barabási-Albert

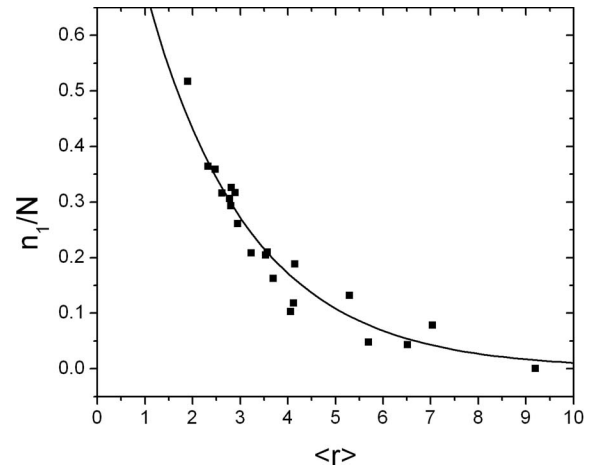
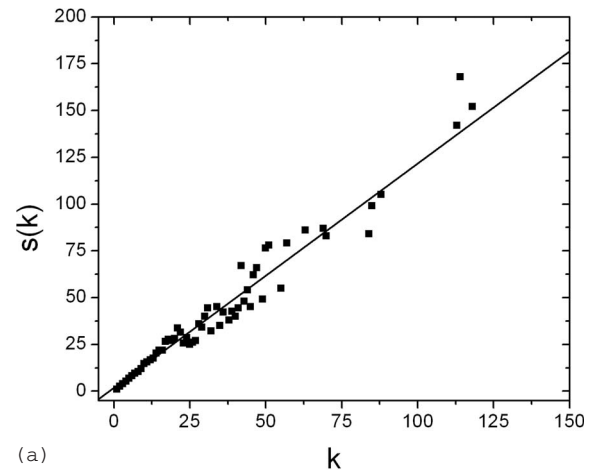
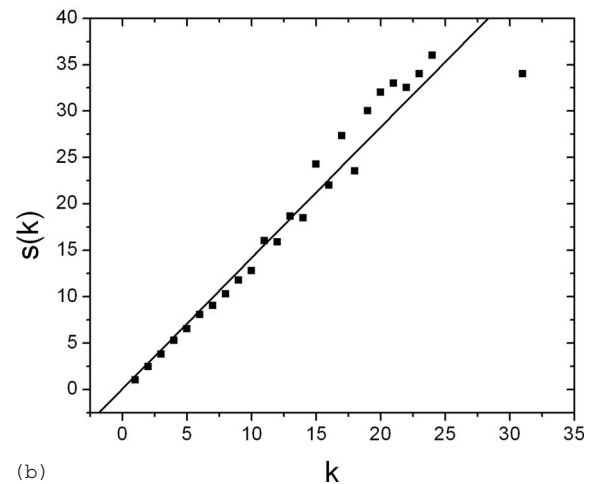


FIG. 9. The ratio of the number of threads n_1 to the total number of messages N as a function of the average distance from the root $\langle r \rangle$. The curve is fitted an exponential function $f(\langle r \rangle) \sim e^{-\langle r \rangle / \langle r_0 \rangle}$, where $\langle r_0 \rangle \approx 1.58$.



(a)



(b)

FIG. 10. Average supremacy $s(k)$ against degree k for (a) “the forum Poland in the EU” and (b) the news group “Humor.” (a) and (b) both follow linear functions with slopes 1.19 and 1.41, respectively.

model, which does not include aging of nodes. This suggests that the linear dependence between supremacy s and degree k could be triggered by the aging of nodes.

IV. SUMMARY

Internet discussions are treelike networks whose degree distributions are described by a power-law function. The networks are growing in time and because the posted messages become out of date naturally, the nodes are aging. For news groups the distribution of the network time interval between a message and a response has two scaling regimes. The small time-interval regime probably corresponds to responses within one session of the discussion, from people currently online, which corresponds to the burst activity studied in Ref. [6], and the behavior for large time intervals is generated by messages posted later by new users arriving online. For the internet forums the time-interval distribution is described by $T(\tau) \sim [\tau + \tau_0]^{-\delta}$ and shows a smooth behavior.

The time correlations within the activity time series show that the activity of internet discussion users is integrated with users' daily routines on both 12 and 24 h scales (Fig. 7). These measurements could help us define an optimal time of operation for people interested in marketing goods or services to internet users.

The distance distribution exhibits an exponential character for most news groups, which means that discussions are not deeply embedded within larger tree structures. The results for

internet forums on www.onet.pl show the intervention of the software employed, which only allows a maximum distance $r=13$ in its forums. However, the distance distributions for these groups exhibit a power-law behavior. These results can be understood by considering the topics of these discussions. The news groups mostly contain closely defined, themed, discussions which are often very technical and frequented by experienced users. Consequently, answers are very short and directly address the problem. Thus, the average distance $\langle r \rangle$ is small. In contrast, the internet forums have a wide range of the users who usually want to discuss and argue with others. This attitude toward discussion creates large and deep tree structures.

Internet discussions are an important source of data within social sciences. They allow the study of the topology of social connections and their temporal statistics [1–5]. Our study is focused on the growing trees of messages, whose structure and temporal statistics, as we have shown, are related to the subject of the discussion and the day-to-day activities of users. Investigating the emergence, aging, and dying of topics in discussion networks should yield data on people's interests—what people like reading or commenting on. This should give insight into the real dynamics of people's opinion change and exchange.

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