# Efficiency of competitions

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League competition is investigated using random processes and scaling techniques. In our model, a weak team can upset a strong team with a fixed probability. Teams play an equal number of head-to-head matches and the team with the largest number of wins is declared to be the champion. The total number of games needed for the best team to win the championship with high certainty $T$ grows as the cube of the number of teams $N$, i.e., $T \sim N^3$. This number can be substantially reduced using preliminary rounds where teams play a small number of games and subsequently, only the top teams advance to the next round. When there are $k$ rounds, the total number of games needed for the best team to emerge as champion, $T_k$, scales as follows, $T_k \sim N^{\gamma_k}$ with $\gamma_k = [1-(2/3)^{k+1}]^{-1}$. For example, $\gamma_k = 9/5, 27/19, 81/65$ for $k = 1, 2, 3$. These results suggest an algorithm for how to infer the best team using a schedule that is linear in $N$. We conclude that league format is an ineffective method of determining the best team, and that sequential elimination from the bottom up is fair and efficient.

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## I. INTRODUCTION

Competition is ubiquitous in physical, biological, sociological, and economical processes. Examples include ordering kinetics where large domains grow at the expense of small ones [1,2], evolution where fitter species thrive at the expense of weaker species [3], social stratification where humans vie for social status [4–6], and the business world where companies compete for market share [7,8].

The world of sports provides an ideal laboratory for modeling competition because game data are accurate, abundant, and accessible. Moreover, since sports competitions are typically head-to-head, sports can be viewed as an interacting particle system, enabling analogies with physical systems that evolve via binary interactions [9–17]. For instance, sports nicely demonstrate that the outcome of a single competition is not predictable [18,19]. Over the past century the lower seeded team had an astounding 44% chance of defeating a higher seeded team in baseball [19]. This inherent randomness has profound consequences. Even after a long series of competitions, the best team does not always finish first.

To understand how randomness affects the outcome of multiple competitions, we study an idealized system. In our model league, there are $N$ teams ranked from best to worst, so that in each match there is a well-defined favorite and underdog. We assume that the weaker team can defeat the stronger team with a fixed probability. Using random walk properties and scaling techniques analogous to those used in polymer physics [20,21], we study the rank of the champion as a function of the number of teams and the number of games. We find that a huge number of games $T \sim N^3$ is needed to guarantee that the best team becomes the champion.

We suggest that a more efficient strategy to decide champions is to set up preliminary rounds where a small number of games is played, and based on the outcome of these games, only the top teams advance to the next round. In the final championship round, $M$ teams play a sufficient number of $M^3$ games to decide the champion. Using $k$ carefully constructed preliminary rounds, the required number of games $T_k$ can be reduced significantly,

$$T_k \sim N^{\gamma_k} \quad \text{with } \gamma_k = \frac{1}{1-(2/3)^{k+1}}. \tag{1}$$

Remarkably, it is possible to approach the optimal limit of linear scaling using a large number of preliminary rounds.

## II. LEAGUE COMPETITION

Our model league consists of $N$ teams that compete in head-to-head matches. We assume that each team has an innate strength and that no two teams are equal. The teams are ranked from 1 (the best team) to $N$ (the worst team). This ranking is fixed and does not evolve with time. The teams play a fixed number of head-to-head games, and each game produces a winner and a loser. In our model, the stronger (lower seed) team is considered to be the favorite and the weaker (higher seed) team is considered to be the underdog. The outcome of each match is stochastic: the underdog wins with the upset probability $0 < q < 1/2$ and the favorite wins with the complementary probability $p = 1 - q$. The team with the largest number of wins is the champion.

We comment that this competition model is based on extensive empirical studies of actual league competitions in the major U.S. sports leagues. These investigations show that the upset frequency is constant throughout the season and, moreover, that the upset frequency has barely changed in over a century [19]. This competition model quantitatively predicts key statistical characteristics of actual sports data including, for example, the distribution of win percentage in leagues [19] as well as winning probabilities in tournaments [16,18].

Since the better team does not necessarily win a game, the best team does not necessarily win the championship. In this study, we address the following questions: How many games are needed for the best team to finish first? What is the typical rank of a champion decided by a relatively small number of games? What is the optimal way to choose a champion?

We answer these questions using scaling techniques. Consider the $n$th ranked team with $1 \leqslant n \leqslant N$. This team is inferior to a fraction $\frac{n-1}{N-1}$ of the $N-1$ remaining teams and superior to a fraction $\frac{N-n}{N-1}$ of the teams. Therefore, the probability $P_n$ that this team wins a game against a randomly chosen opponent is a linear combination of the probabilities $p$ and $q$,

$$P_n = p\frac{N-n}{N-1} + q\frac{n-1}{N-1}. \tag{2}$$

Using $p=1-q$, the probability $P_n$ can be rewritten as follows:

$$P_n = p - (2p-1)\frac{n-1}{N-1}. \tag{3}$$

The latter varies linearly with rank: it is largest for the best team $P_1=p$ and smallest for the worst team $P_N=q$.

Now, suppose that the $n$th team plays $t$ games, each against a randomly chosen opponent. The number of wins it accumulates, $w_n(t)$, is a random quantity that grows as follows:

$$w_n(t+1) = \begin{cases} w_n(t)+1 & \text{with probability } P_n \\ w_n(t) & \text{with probability } 1-P_n. \end{cases} \tag{4}$$

The initial condition is $w_n(0)=0$. The number of wins performs a biased random walk and as a result, when the number of games is large, the quantity $w_n(t)$ is well characterized by its average $W_n(t)=\langle w_n(t)\rangle$ and its standard deviation $\sigma_n(t)$, defined via $\sigma_n^2(t)=\langle w_n^2(t)\rangle-\langle w_n(t)\rangle^2$. Here, the brackets denote averaging over infinitely many realizations of the random process. Since the outcome of a game is completely independent of all other games, the average number of wins and the variance in the number of wins are both proportional to the number of games played.

$$W_n(t) = P_n t, \tag{5a}$$

$$\sigma_n^2(t) = P_n(1-P_n)t. \tag{5b}$$

Both of these quantities follow from the behavior after one game: since $w_n(1)=1$ with probability $P_n$ and $w_n(1)=0$ with probability $1-P_n$, then $\langle w_n(1)\rangle=\langle w_n^2(1)\rangle=P_n$. Moreover, the distribution of the number of wins is binomial and for large $t$, it approaches a Gaussian, fully characterized by the average and the standard deviation [22].

The quantities $W_n$ and $\sigma_n$ can be used to understand key features of this system. Let us assume that each team plays $t$ games against randomly selected opponents and compare the best team with the $n$th ranked team. Since $P_1 > P_n$, the best team accumulates wins at a faster rate, and after playing sufficiently many games, the best team should be ahead. However, since there is a diffusivelike uncertainty in the number of wins, $\sigma_n \sim \sqrt{t}$, it is possible that the $n$th ranked team has more wins when $t$ is small. The number of wins of the $n$th team is comparable with that of the best team as long as $W_1(t)-W_n(t) \propto \sigma_1(t)$, or

$$(2p-1)\frac{n-1}{N-1}t \propto \sqrt{t}. \tag{6}$$

Since the diffusion coefficient $D_n=P_n(1-P_n)$ in Eq. (5b) varies only weakly with $n$, $pq \leqslant D_n \leqslant 1/4$, this dependence is tacitly ignored. When these two teams have a comparable number of wins, they have comparable chances to finish first. Hence, Eq. (6) yields the characteristic rank of the champion $n_*$ as a function of the number of teams $N$ and the number of games $t$,

$$n_* \sim \frac{N}{\sqrt{t}}. \tag{7}$$

Since we are primarily interested in the behavior as a function of $t$ and $N$, the dependence on the probability $p$ is henceforth left implicit. As expected, the champion becomes stronger as the number of games increases (recall that small $n$ represents a stronger team). By substituting $n_* \sim 1$ into Eq. (7), we deduce that the total number of games $t_*$ needed for the best team to win is $t_* \sim N^2$.

Since each of the $N$ teams plays $t_* \sim N^2$ games, the total number of games required for the best team to emerge as the champion with high certainty grows as the cubic power of the number of teams,

$$T \sim N^3. \tag{8}$$

This result has significant implications. In most sports leagues, two teams face each other a fixed number of times, usually once or twice. The corresponding total number of $\sim N^2$ games is much smaller than Eq. (8). In this common league format, the typical rank of the champion scales as $n_* \sim \sqrt{N}$. Such a season is much too short as it enables weak teams to win championships. Indeed, it is not uncommon for the top two teams to trade places until the very end of the season or for two teams to tie for first, a clear indication that the season length is too short.

We may also consider the probability distribution $Q_n(t)$ for the $n$th ranked team to win after $t$ games. We expect that the scale $n_*$ characterizes the entire distribution function,

$$Q_n \sim \frac{1}{n_*}\psi\left(\frac{n}{n_*}\right). \tag{9}$$

Assuming $\psi(0)$ is finite, the probability that the best team wins scales as follows, $Q_1 \sim 1/n_*$. This quantity first grows, $Q_1(t) \sim \sqrt{t}/N$ when $t \ll N^2$, and then, it saturates, $Q_1(t) \approx 1$ when $t \gg N^2$.

The likelihood of major upsets is quantified by the tail of the scaling function $\psi(z)$. Generally, the champion wins $pt$ games (we neglect the diffusive correction). The probability that the weakest team becomes champion by reaching that many wins is $Q_N(t) \sim \binom{t}{pt} q^{pt} p^{qt} \sim (q/p)^{(p-q)t}$, where the asymptotic behavior follows from the Stirling formula $t! \sim t \ln t - t$. We conclude that the probability of the weakest team winning decays exponentially with the number of games $Q_N(t) \sim \exp(-\text{const} \times t)$. Yet, from Eqs. (9) and (7), $Q_N(t) \sim \psi(\sqrt{t})$, and therefore, the tail of the probability distribution is Gaussian

$$\psi(z) \sim \exp(-\text{const} \times z^2) \tag{10}$$

as $z \to \infty$ thereby implying that upset champions are extremely improbable. We note that single-elimination tournaments produce upset champions with a much higher probability because the corresponding distribution function has an algebraic tail [16]. We conclude that leagues have a much narrower range of outcomes and in this sense, leagues are more fair than tournaments.

## III. PRELIMINARY ROUNDS

With such a large number of games, the ordinary league format is highly inefficient. How can we devise a schedule that produces the best team as the champion with the least number of games? The answer involves preliminary rounds. In a preliminary round, teams play a small number of games and only the top teams advance to the next round [23].

Let us consider a two-stage format. The first stage is a preliminary round where teams play $t_1$ games and then the teams are ranked according to the outcome of these games. The top $M \ll N$ teams advance to the final round [24], and the rest are eliminated. The final championship round proceeds via a league format with plenty of games to guarantee that the best team ends up at the top.

We assume that the number of teams advancing to the second round grows sublinearly

$$M \sim N^{\alpha_1}, \tag{11}$$

with $\alpha_1 < 1$. Of course, we better not eliminate the best team. The number of games $t_1$ required for the top team to finish no worse than $M$th place is obtained by substituting $n_* \sim M$ into Eq. (7), $t_1 \sim N^2/M^2$. Since each of the $N$ teams plays $t_1$ games, the total number of games in the preliminary round is of the order $Nt_1 \sim N^3/M^2 \sim N^{3-2\alpha_1}$. Directly from Eq. (8), the number of games in the final round is $M^3 \sim N^{3\alpha_1}$. Adding these two contributions, the total number of games $T_1$ is

$$T_1 \sim N^{3-2\alpha_1} + N^{3\alpha_1}. \tag{12}$$

This quantity grows algebraically with the number of teams $T_1 \sim N^{\gamma_1}$ with $\gamma_1 = \max(3-2\alpha_1, 3\alpha_1)$ and this exponent is minimal, $\gamma_1 = 9/5$, when

$$\alpha_1 = 3/5. \tag{13}$$

Consequently, $t_1 \sim N^{4/5}$.

Thus, it is possible to significantly improve upon the ordinary league format using a two-stage procedure. The first stage is a preliminary round in which each of the $N$ teams plays $t_1 \sim N^{4/5}$ games and then the top $M \sim N^{3/5}$ teams advance to the final round. The rest of the teams are eliminated. The first preliminary round requires $N^{9/5}$ games. In the final round the remaining teams play in a league with each of the possible $\binom{M}{2}$ pairs of teams playing each other $M$ times. Again the number of games is $N^{9/5}$ so that in total,

$$T_1 \sim N^{9/5} \tag{14}$$

games are played. This is a substantial improvement over ordinary $N^3$ league play.

Multiple preliminary rounds further reduce the number of games. Introducing an additional round, there are now three stages: the first preliminary round, the second preliminary round, and the championship round. Out of the first round $N^{\alpha_2}$ teams proceed to the second round and then, $N^{\alpha_1\alpha_2}$ teams proceed to the championship round. The total number of games $T_2$ is a straightforward generalization of Eq. (12)

$$T_2 \sim N^{3-2\alpha_2} + N^{\alpha_2(3-2\alpha_1)} + N^{3\alpha_1\alpha_2}. \tag{15}$$

These three terms account, respectively, for the first round, the second round, and the final round. The first term is analogous to the first term in Eq. (12), and the last two terms are obtained by replacing $N$ with $N^{\alpha_2}$ in Eq. (12). The total number of games is minimal when all three terms are of the same magnitude. Comparing the last two terms gives $3-2\alpha_1 = 3\alpha_1$ and therefore, Eq. (13) is recovered. Comparing the first two terms gives

$$3 - 2\alpha_2 = \alpha_2(3 - 2\alpha_1). \tag{16}$$

Thus, $\alpha_2 = 15/19$ and since $\alpha_2 > \alpha_1$, the first elimination is less drastic than the second one. The total number of games $T_2 \sim N^{27/19}$ represents a further improvement.

These results indicate that it is possible to systematically reduce the total number of games via successive preliminary rounds that lead to the final championship round. In the most general case, there are $k$ preliminary rounds in addition to the final round. The number of teams advancing to the second round, $M_k$, grows as follows:

$$M_k \sim N^{\alpha_k}. \tag{17}$$

From Eq. (16), the exponent $\alpha_k$ obeys the recursion relation $3-2\alpha_{k+1} = \alpha_{k+1}(3-2\alpha_k)$ or equivalently,

$$\alpha_{k+1} = \frac{3}{5 - 2\alpha_k}. \tag{18}$$

By using $\alpha_1 = 3/5$ we deduce the initial element in this series, $\alpha_0 = 0$. Introducing the transformation $\alpha_k = a_k/a_{k+1}$ reduces Eq. (18) to the Fibonacci-type recursion $3a_{k+2} = 5a_{k+1} - 2a_k$. The general solution of this equation is $a_k = Ar_1^k + Br_2^k$, where $r_1 = 1$ and $r_2 = 2/3$ are the two roots of the quadratic equation $3r^2 = 5r - 2$. The coefficients follow from the zeroth element: $\alpha_0 = 0$ implies $a_0 = 0$ and consequently, $a_k = A[1-(2/3)^k]$. Therefore,

$$\alpha_k = \frac{1 - (2/3)^k}{1 - (2/3)^{k+1}}. \tag{19}$$

The exponent $\alpha_k \approx 1 - \frac{1}{3}\left(\frac{2}{3}\right)^k$ (for $k \gg 1$) decreases exponentially to one (Table I). This means that the number of teams advancing from the first to the second preliminary round is increasing with the total number of preliminary rounds played. Nonetheless, the fraction of teams that are eliminated $1 - N^{\alpha_k - 1}$ converges to one as $N \to \infty$. Hence, nearly all of the teams are eliminated.

The number of games played by a team in the first round, $t_k$, follows from Eq. (17),

TABLE I. The exponents $\alpha_k$, $\beta_k$, and $\gamma_k$ characterizing $M_k$, the number of teams advancing from the first round, $t_k$, the number of games played by a team in the first round, and $T_k$, the total number of games, as a function of the number of preliminary rounds $k$.

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | $\infty$ |
|---|---|---|---|---|---|---|---|
| $\alpha_k$ | 0 | $\frac{3}{5}$ | $\frac{15}{19}$ | $\frac{57}{65}$ | $\frac{195}{211}$ | $\frac{633}{665}$ | 1 |
| $\beta_k$ | 1 | $\frac{4}{5}$ | $\frac{8}{19}$ | $\frac{16}{65}$ | $\frac{32}{211}$ | $\frac{64}{665}$ | 0 |
| $\gamma_k$ | 3 | $\frac{9}{5}$ | $\frac{27}{19}$ | $\frac{81}{65}$ | $\frac{243}{211}$ | $\frac{729}{665}$ | 1 |

$$t_k \sim N^{\beta_k}, \quad \beta_k = 2(1-\gamma_k). \tag{20}$$

Since $\beta_k \to 0$ as $k \to \infty$, only a small number of games is played in the opening round. Using $T_k \sim N t_k$, we arrive at our main result (1) where $\gamma_k = 3 - 2\alpha_k$. Surprisingly, the total number of games is roughly linear in the number of teams

$$T_\infty \sim N, \tag{21}$$

when a large number of preliminary rounds is used, i.e., $k \to \infty$ [25]. Clearly, this linear scaling is optimal since every team must play at least once. The asymptotic behavior $\gamma_k \approx 1 + (\frac{2}{3})^{k+1}$ implies that in practice, a small number of preliminary round suffices. For example, $\gamma_4 = \frac{243}{211} = 1.151\,65$ (Table I).

We emphasize that in a $k$-round format, the top $N^{\alpha_k}$ teams proceed to the second round, out of which the top $N^{\alpha_{k-1}\alpha_k}$ teams proceed to the third round, and so on. The number of teams proceeding from the $k$th round to the championship round is $M \sim N^{\alpha_1 \alpha_2 \cdots \alpha_k}$. From Eq. (21) and $T_\infty \sim M_\infty^3$, the size of the championship round approaches

$$M_\infty \sim N^{1/3} \tag{22}$$

as $k \to \infty$. This is the optimal size of a playoff that produces the best champion using the least number of games.

## IV. NUMERICAL SIMULATIONS

Our scaling analysis is heuristic: we assumed that $N$ is very large and we ignored numerical constants. To verify the applicability of our asymptotic results to moderately sized leagues, we performed numerical simulations with $N$ teams that play an equal number of $t$ games against randomly selected opponents. The outcome of each game is stochastic: with probability $p$ the favorite wins and with probability $q = 1 - p$, the underdog wins. We present simulation results for $q = 1/4$.

The most important theoretical prediction is the relation (7) between the rank of the winner, the number of games, and the size of the league. To test this prediction, we measured the average rank of the winner as a function of the number of games $t$ for leagues of various sizes. In the simulations, it is convenient to shift the rank by one: the teams are

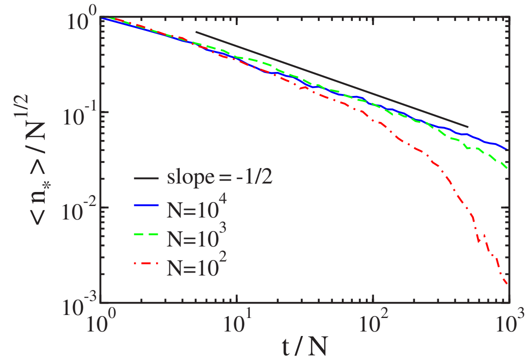


FIG. 1. (Color online) The average rank of the champion $\langle n_* \rangle$ of a league with $N$ teams after $t$ games. The simulation results represent and average over $10^3$ independent realizations with $N = 10^2$, $10^3$, and $10^4$. A straight line of slope $-1/2$, predicted by Eq. (7), is plotted as a reference.

ranked from $n = 0$ (the best team) to $n = N - 1$ (the worst team). With this definition, the average rank decreases indefinitely with $t$. The simulations show that $n_*/N^{1/2} \sim (t/N)^{-1/2}$, thereby confirming the theoretical prediction (Fig. 1).

To validate Eq. (8), we simulated leagues with a large enough number of games, so that the best team wins with certainty. For every realization there is a number of games $T$ after which the champion takes the lead for good. The average of this random variable $\langle T \rangle$ measured from the simulations is in excellent agreement with the theoretical prediction (Fig. 2).

The simulations also confirm that the scale $n_*$ characterizes the entire distribution as in Eq. (9). Numerically, we find that the tail of the scaling function is superexponential, $\psi(z) \sim \exp(-z^\mu)$ with $\mu > 1$. The observed tail behavior is consistent with $\mu = 2$, although the numerical evidence is not conclusive.

To verify our prediction that multiple elimination rounds, following the format suggested above, reduce the number of games, we simulated a single elimination round ($k = 1$). In the first stage, a total of $N^{9/5}$ games are played. All teams are then ranked according to the number of wins and the top

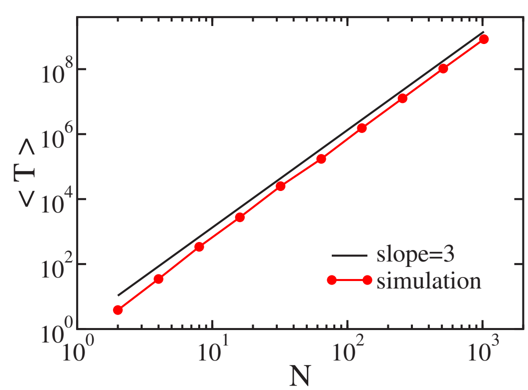


FIG. 2. (Color online) The average number of games $\langle T \rangle$ needed for the best team to emerge as the champion of a league with $N$ teams. The simulation results, representing an average over $10^3$ independent realizations, are compared with the theoretical prediction (8).

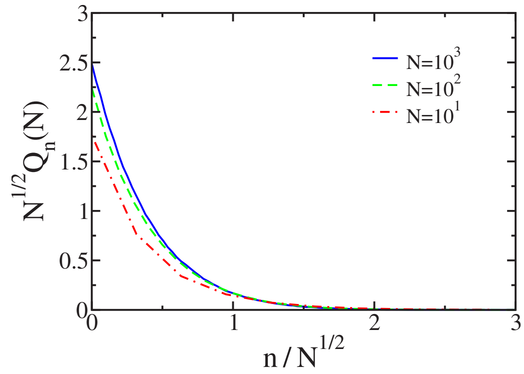


FIG. 3. (Color online) The rank distribution of the league winner for ordinary league format ($t=N$). Shown is the scaled distribution $\sqrt{N}Q_n(t=N)$ versus the scaling variable $n/\sqrt{N}$. The simulation data were obtained using $10^6$ independent Monte Carlo runs.

$M=N^{3/5}$ teams proceed to the championship round. This final round has an ordinary league format with a total of $M^3$ games. We simulated three leagues of respective sizes $N=10^1$, $N=10^2$, and $N=10^3$, and observed that the best team wins with a frequency of 70% (Fig. 3). The champion is among the top three teams in 98% of the cases (these percentages are independent of $N$). As a reference, in an ordinary league with a total of $N^3$ games, the best team also wins with a likelihood of 70%. Remarkably, even for as little as $N=10$ teams, the one preliminary round format reduces the number of games by a factor $>10$. We conclude that the scaling results are useful at moderate league size $N$.

## V. IMPERFECT CHAMPIONS

Let us relax the condition that the best team must win and implement a less rigorous championship round. Given a total of $T\sim M^c$ games with $1\leqslant c\leqslant 3$, each team plays $t\sim M^{c-1}$ games. From Eq. (7), the typical rank of the winner scales as

$$n_*\sim M^{(3-c)/2}. \tag{23}$$

Suppose that there are infinitely many preliminary rounds. The analysis in Sec. III reveals that the total number of games scales linearly, $T\sim M^c\sim N$, and consequently, $M\sim N^{1/c}$. Therefore, there is a scaling relation between the rank of the winner and the number of teams $n_*\sim N^{(3-c)/2c}$. Indeed, the value $c=3$ produces the best champion. The common league format ($c=2$) leads to $n_*\sim N^{1/4}$, an improvement over the ordinary $N^{1/2}$ behavior.

If there is one preliminary round, Eq. (12) becomes $T_1\sim N^{3-2\alpha_1}+N^{c\alpha_1}$ and therefore, $\alpha_1=3/(2+c)$. Generally for $k$ preliminary rounds, the exponent $\alpha_k$ satisfies the recursion relation (18), and the scaling relations $\gamma_k=3-2\alpha_k$ and $\beta_k=2(1-\alpha_k)$ remain valid. We quote the value

$$\gamma_k=\frac{1}{1-\frac{c-1}{c}\left(\frac{2}{3}\right)^k} \tag{24}$$

that characterizes the total number of games, $T\sim N^{\gamma_k}$. From $T\sim M^c\sim N^{\gamma_k}$, we conclude $M\sim N^{\gamma_k/c}$. Substituting this relation into Eq. (23) yields

$$n_*\sim N^{\nu_k},\qquad \nu_k=\frac{\gamma_k(3-c)}{2c}. \tag{25}$$

Using ordinary league play ($c=2$) and one preliminary round, $N^{3/2}$ games are sufficient to produce an imperfect champion of typical rank $n_*\sim N^{3/8}$. Finally, we note that if each team plays a finite number of games ($c=1$), all of the teams have a comparable chance of winning because $\nu_k=\gamma_k\equiv 1$.

## VI. CONCLUSIONS

In summary, we studied dynamics of league competition with fixed team strength and a finite upset probability. We demonstrated that ordinary league play where all teams play an equal number of games requires a very large number of games for the best team to win with certainty. We also showed that a series of preliminary rounds with a small but sufficient number of games to successively eliminate the weakest teams is a fair and efficient way to identify the champion. We obtained scaling laws for the number of advancing teams and the number of games in each preliminary round. Interestingly, it is possible to determine the best team by having teams play, on average, only a finite number of games (independent of league size). The optimal size of the final championship round scales as the one-third power of the number of teams.

Empirical validation of these results with real data may be possible using sports leagues, for example. The challenge is that the inherent strength of each team is not known. In professional sports, a team's budget can serve as a proxy for its strength. With this definition, the average rank of the American baseball world series champion, over the past 30 years, equals six. There are, however, huge fluctuations: while the top team won seven times, a team ranked as low as 26 (2003 Florida Marlins) also won.

The results in this paper can be generalized in a number of ways. For example, one can use competitions to sort all teams by strength, not merely find the best one. We find that $T_{\rm sort}$, the time needed to sort all teams through ordinary league play grows as $T_{\rm sort}\sim N^3\ln N$ [26]. One may also introduce upset frequencies that depend on the difference in strengths between two teams. Empirical studies show that a single effective upset frequency is adequate to capture key characteristics of sports leagues such as the standard deviation in win percentage. Of course, strength-dependent and empirically based upset frequencies can be used as a more realistic model. An interesting question to answer is under what conditions, i.e., general pairwise (underdog-favorite) assignments, can the teams be sorted by league play? Finally, one can investigate the effects of evolving team strengths. Clearly, the cubic growth law (8) provides a lower bound on the number of games needed to choose the champion.

With wide ranging applications, including, for example, evolution [27,28], leadership statistics is a challenging extreme statistics problem because the record of one team constrains the records of all other teams. Our scaling approach, based on the record of a fixed team, ignores such correlations. While these correlations do not affect the scaling laws, they do affect the distribution of outcomes such as the distribution of the rank of the winner, and the distribution of the number of games needed for the best team to take the lead for good. Other interesting questions include the expected number of distinct leaders, and the number of lead changes as a function of league size [29,30].

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