Multicoaxial cylindrical inclusions in locally resonant phononic crystals

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It is known that the transmission spectrum of the so-called locally resonant phononic crystal can exhibit absolute sharp dips in the sonic frequency range due to the resonance scattering of elastic waves. In this paper, we study theoretically, using a finite difference time domain method, the propagation of acoustic waves through a two-dimensional locally resonant crystal in which the matrix is a fluid (such as water) instead of being a solid as in most of the previous papers. The transmission is shown to be dependent upon the fluid or solid nature of the matrix as well as upon the nature of the coating material in contact with the matrix. The other main purpose of this paper is to consider inclusions constituted by coaxial cylindrical multilayers consisting of several alternate shells of a soft material (such as a soft rubber) and a hard material (such as steel). With respect to the usual case of a hard core coated with a soft rubber, the transmission spectrum can exhibit in the same frequency range several peaks instead of one. If two or more phononic crystals are associated together, we find that the structure displays all the zeros of transmission resulting from each individual crystal. Moreover, we show that it is possible to overlap the dips by an appropriate combination of phononic crystals and create a larger acoustic stop band.

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I. INTRODUCTION

Phononic crystals are heterogeneous materials constituted by a periodical repetition of inclusions in a matrix. Associated with the possibility of absolute band gaps in their band structures [1], these materials have found several potential applications, in particular in the field of wave guiding and filtering [2] (like in their photonic counterpart), as well as in the field of sound isolation. In the latter case, the main interest consists of finding structures that attenuate the propagation of sound over a sample whose thickness remains smaller than, or of the order of, the wavelength in air. A first attempt to find giant acoustic stop bands at low frequencies has been made by considering phononic crystals constituted by air inclusions of cylindrical [3] or spherical [4] shape in water. In these structures, the first band remains very narrow due to the low value of the sound velocity in the inclusions together with the large contrast between the acoustic properties of both constituents, while the next few higher bands are just flat bands associated with the internal resonances of the air inclusions. It has been shown [5] that these useful sound attenuation properties remain valid when the air cylinders are protected from water by surrounding them with a soft solid rubber material, for instance giving rise to a large attenuation between 1 and 10 kHz with the whole sample thickness not exceeding 70 mm. However, most of the recent studies have been directed towards a new class of phononic crystals, the so-called locally resonant materials [6–12], first introduced by Sheng and co-workers [6]. These structures essentially consist [6,7,9–12] of a hard core, such as a metal, surrounded by a soft coating (silicone rubber) and immersed in a polymer such as epoxy. Due to the local resonances associated with the soft coating material, drastic dips can appear in the transmission coefficient at very low frequencies situated about two orders of magnitudes below the Bragg frequency. Such behaviors have been obtained in both three [6–8] and two-dimensional [9–11,13] locally resonant phononic crystals. The dips in the transmission spectrum display an asymmetric line shape typical of Fano resonances [8]. Nevertheless, the width of the attenuation peaks is in general rather small and, in order to obtain a broadband attenuation, it is necessary to combine different locally resonant materials to overlap the dips resulting from several resonant frequencies.

In most of the preceding studies about locally resonant phononic crystals, the structure is made only by solid constituents. One object of this paper is to investigate similar structures when the matrix is made by a fluid such as water whereas the embedded core remains a cylindrical solid. Indeed, it will be shown that the transmission properties and band structure can be significantly affected with the solid or fluid nature of the matrix as well as the nature of the coating material which is in contact with the matrix. The other main object of the paper is to generalize, for two-dimensional phononic crystals, the most studied case of a coated core inclusion to the case of a multilayer cylindrical core constituted by two or several coaxial shells surrounding the internal hard core. The resonant nature of the scatter is obtained by alternating shells of a soft polymer and a hard material such as steel. This new geometry of the scatter yields the possibility of obtaining several dips in the transmission coefficient in a given frequency range. Moreover, we show that by combining two or more phononic crystals of different parameters, it is also possible to overlap some dips and obtain a widening of the frequency gaps.

The organization of the paper is as follows. The geometry of the problem and the method of calculation are presented in Sec. II. In Sec. III, we discuss the behavior of the transmission coefficient as a function of the number and the order of the materials constituting the shells, in particular the uttermost materials respectively in the internal core and in contact with the water matrix. The conclusions are given in Sec. IV.

FIG. 1. Representation of the basic unit structure made of a hard cylinder core covered by a multilayer of rubber and steel. N is the number of shells in the coating ($N=6$ in this figure).

II. GEOMETRICAL AND SIMULATION PARAMETERS

The structural unit used to build the phononic crystal consists of an infinitely long cylinder, composed of multicoaxial shells, embedded in a water matrix. The inner (core) cylinder is made of steel. This core is coated by alternate shells constituted respectively by a thin layer of an elastically soft material and a thin layer of a hard material (steel). In this way, we obtain an alternation of hard and soft materials (Fig. 1). We call N the number of shells coating the core and, depending on whether N is even or odd, the uttermost shell in contact with water will be steel or the polymer. In most of our calculation, the soft polymer is chosen to be the butyl rubber (poly-isobutene-co-isoprene) which has very small elastic constants and consequently very low velocities of sound [14]. The structural parameters and the sound velocities of the used materials are reported in Table I. In our calculations, we fix the outer radius of the cylinder to be equal to 8.4 mm and the thickness of each layer in the coating equal to 1.6 mm. The filling fraction of the whole cylinder, taken to be $f=55\%$, will be kept constant throughout the paper. Finally, the sonic crystal is constituted by six rows of elementary units arranged on a square lattice, with a lattice parameter of $a=20$ mm. The whole size of the sonic crystal is therefore 12 cm.

The numerical simulations are performed in the framework of a two-dimensional finite-difference time-domain (FDTD) method allowing the computation of the transmission coefficient, the dispersion curves, and the map of the displacement. The FDTD method solves the elastic wave equations by discretizing both time and space and by replacing derivatives by finite differences. Our calculation is performed on a two-dimensional lattice, repeated periodically along the X direction. For the Y direction, the size of the sample is finite and absorbing Mur boundary conditions are applied at both free ends in order to avoid reflections of the outgoing waves [15]. The structure is supposed to be infinite along the Z direction. In order to calculate the transmission spectrum, the structural sonic material is sandwiched between two homogeneous media used for launching and probing the acoustic waves. Space is discretized in both X and Y directions using a mesh interval equal to $\Delta x=\Delta y=a/100$ where a is the lattice parameter of the crystal. This value insures a minimum of eight intervals of the discretized mesh inside the smallest shell we studied. The equations of motion are solved with a time integration step $\Delta t=\Delta x/4$ and a number of time steps equal to 2^{22} which is the necessary tested time for a good convergence of the numerical calculation.

A broad band wave packet is launched from the left, inside the homogeneous region. The incoming wave is a longitudinal pulse, uniform along the X direction and with a Gaussian profile along the Y axis. The transmitted signal, probed at the end of the waveguide, is recorded as a function of time, integrated over the cross section of the waveguide and finally Fourier transformed to obtain the transmission coefficient versus frequency. All the transmission spectra are normalized with respect to the one corresponding to the homogeneous medium constituted by the host water matrix. More details about the calculation are given in Ref. [5].

TABLE I. Structural parameters of the constituents in the phononic crystal: Mass density (ρ), longitudinal (v_L), and transverse (v_T) velocities of sound.

Material	Water	Steel	Butyl rubber	Silicone rubber
ρ (kg/m^3)	1000	7780	933	1300
v_L (m/s)	1490	5825	55	24
v_T (m/s)	0	3226	19	6

III. RESULTS AND DISCUSSION

This section is divided into two parts dealing respectively with odd and even values of the number N of coating layers. This means the shell in contact with the water matrix will be respectively the polymer or steel. Our emphasis will be put on the latter case where, for the purpose of practical realization, the polymer will be protected from water. Nevertheless, for the sake of comparison and to show a totally different behavior, we also briefly describe the case where the polymer is in contact with water.

A. Odd number of shells

Let us first consider the case where the solid core made of steel is coated only by one layer of elastically soft polymer (butyl rubber) [see inset in Fig. 2(a)]. In Figs. 2(a) and 2(b), we present the complete dispersion curves of the composite crystal and the transmission spectrum along the main directions of the reduced Brillouin zone, i.e., ΓX and ΓM [see inset of Fig. 2(a)]. The first band extends up to 1.7 kHz where it bends in the vicinity of the Brillouin zone. In the long wavelength limit, the average velocity of sound is about 80 m/s which is much smaller than the velocity in water due to the presence of the soft polymer shell. Above the absolute band gap, which extends from 1.7 to 1.85 kHz, most of the dispersion curves are rather flat. The transmission coefficient in the first band displays a set of peaks corresponding to Fabry-Pérot oscillations of the whole phononic crystal. A few oscillations are also visible in the next bands but the transmission drops to negligible values when going to higher frequencies, probably due to the flatness of the dispersion curves.

The addition of supplementary shells, keeping always N as an odd number [see Fig. 2(c)], does not change the above general trends. The effect of increasing N is essentially a broadening of the lowest band gap, which extends, for the ΓX direction, from 1.35 kHz to 2.0 kHz in the example of

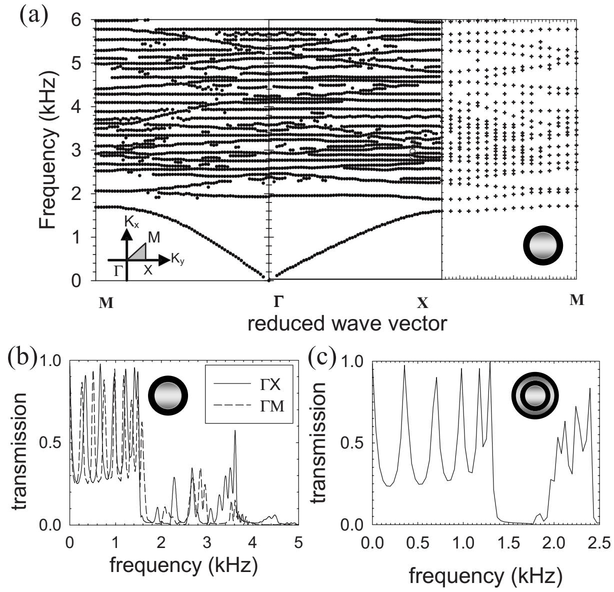

FIG. 2. (a) Dispersion curves and (b) transmission coefficient along ΓX and ΓM for a phononic crystal (f=55% for the whole cylinder) in which the rigid core is coated with a polymer. (c) Transmission coefficient when the core is coated by three shells.

Fig. 2(c). The main conclusion is the possibility of opening an absolute band gap in low frequency range when the polymer is in contact with the water matrix, while the effect is not related to the existence of a local resonance that cuts a band of propagating modes.

B. Even number of shells

1. N=2

First, we assume that the inner core is coated with only one double layer, made of the soft polymer and steel, respectively [see inset in Fig. 3(a)]. With a filling fraction f=55%, the diameter of the inner core is 5.2 mm and the thickness of each layer has been chosen to be 1.6 mm. Figure 3(a) shows the transmission curve of the corresponding phononic crystal along ΓX. For the sake of comparison we also present [Fig. 3(b)] the transmission coefficient of the conventional crystal without coating layers. It can be seen that in both cases, the first Bragg gap appears around the frequency of 35 kHz, i.e., $f=v/2a$ where v is the velocity of sound in water and a the period. Due to the coating layers, there are two main differences occurring in the transmission curve of Fig. 3(a) with respect to that of Fig. 3(b): (i) Several dips in the first pass band that extends up to 30 kHz; (ii) a new pass band inside the Bragg gap that is also an effect of the coating (see also Ref. [5]). The occurrence of low frequency dips, about one to two orders of magnitude lower than the Bragg gap, is a characteristic feature of the locally resonant sonic materials constituted by solid media [6–12], as first obtained by Liu *et al.* in a 3D phononic crystal [6].

The first two dips appearing at 1.45 and 6.65 kHz in the transmission coefficient for both ΓX and ΓM directions of the Brillouin zone [Fig. 3(d)] can also be seen in the dispersion curves [Fig. 3(c)]. The linear dispersion curve of the propagating modes is cut by two flat dispersion curves, giving rise to the opening of small low frequency gaps. Thus such phononic crystal leads to absolute acoustic stop bands in the sonic range. It is worthwhile to notice that the results obtained here when a steel layer is in contact with the water matrix are quite at variance with those obtained in Sec. III A where the outermost layer in the coating was the polymer.

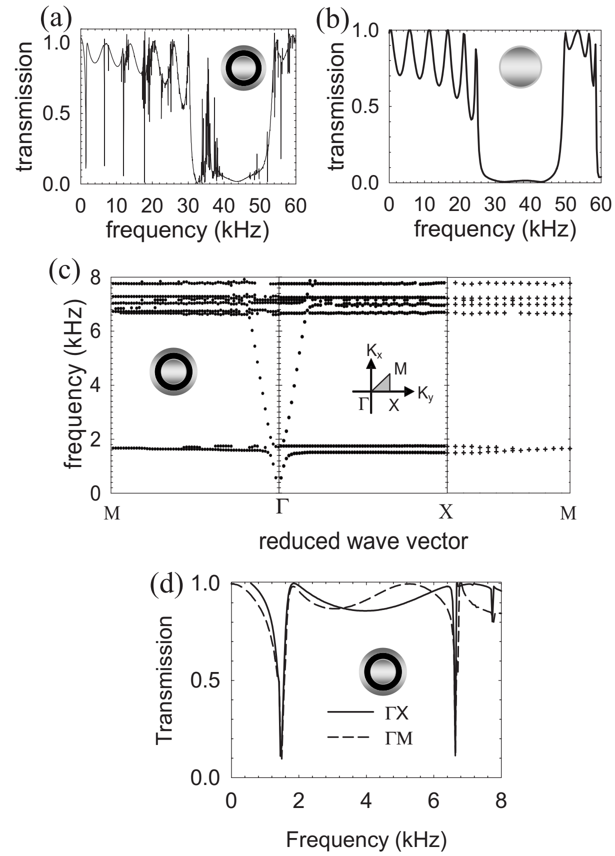

FIG. 3. (a) Transmission spectrum of the phononic crystal (f=55% for the whole cylinder) where the rigid cores are coated with a double shell of polymer and steel. (b) Same as (a) for a conventional crystal without coating of the cores. (c) Band structure of the phononic crystal of (a). (d) Magnification of the transmission coefficient for both ΓX and ΓM directions.

In the following, we discuss the behavior of the displacements fields associated with the first two dips at 1.45 kHz (Fig. 4) and 6.65 kHz (Fig. 5), assuming that the propagation is along the Y direction. At 1.45 kHz, we represent the displacement components U_Y [Fig. 4(a)] and U_X [Fig. 4(b)] both with a three-dimensional map and along a cross section in the X direction. We note that the hard (steel) materials, both in the inner core and in the outside shell, move as rigid bodies essentially parallel to the Y direction [see also Fig. 4(c)], whereas the soft rubber undergoes an elastic deformation with both components of the displacement field. Thus this first low frequency resonance (1.45 kHz) can be understood as an oscillation in which the core and the shell move along the direction of propagation in opposite phase with respect to each other, whereas the polymer acts as a spring

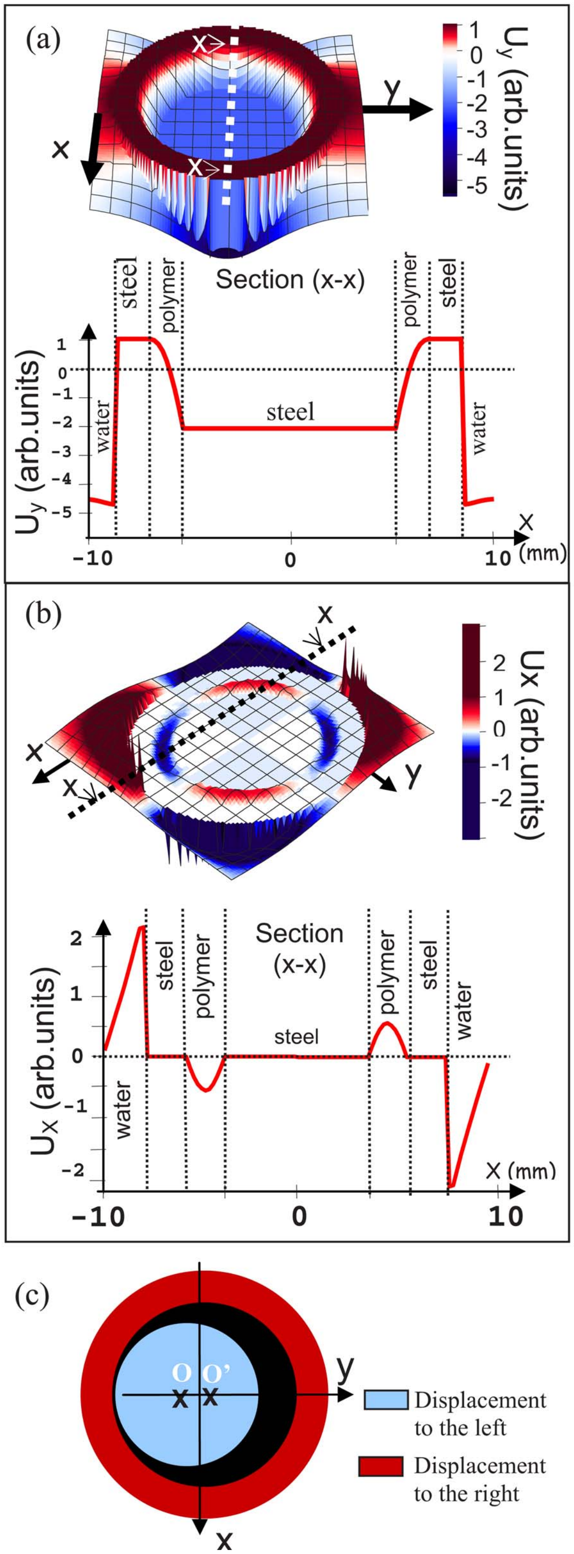

FIG. 4. (Color online) (a) and (b) Components U_Y (parallel to the propagation direction) and U_X (perpendicular to the propagation direction) of the displacement vector for the frequency of 1.45 kHz associated to the first dip in Fig. 3(a). (c) Another representation of the displacement field showing that the hard materials constituting the inner core and the outside shell move as rigid bodies in opposite phase with respect to each other.

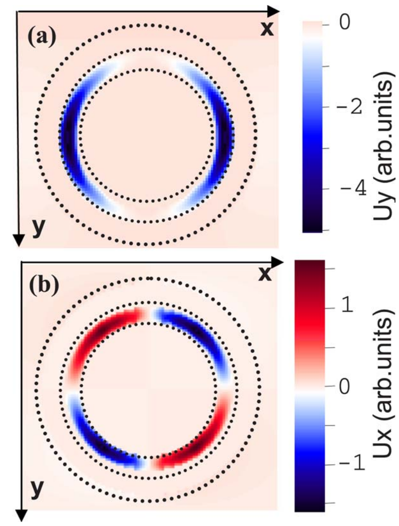

FIG. 5. (Color online) Displacement vector for the frequency associated to the second dip in Fig. 3(a) at 6.65 kHz. The dotted lines represent the position of the core and the shells. The displacement is localized inside the polymer shell.

linking the two hard materials. This scheme is displayed in Fig. 4(c) where O and O' are, respectively, the centers of the core and the shell. Finally, we can see the presence of a displacement field outside the cylinders in the water matrix, for both X and Y directions. In particular, the displacement along X implies an interaction between cylinders in neighboring cells, as will be discussed below.

Figures 5 present the displacement field associated with the second dip in Fig. 3(a) occurring at 6.65 kHz. It can be observed that the displacement is now totally localized inside the polymer shell, whereas the hard materials as well the matrix are at rest. Therefore, this dip originates from a localized mode of the polymer shell sandwiched between two hard materials. This mode is comparable to the one described with an epoxy matrix [6].

We now discuss the evolution of the dip frequencies, especially the lowest one, as a function of different geometrical and physical parameters of the multicoaxial cylinders. Indeed, these frequencies can be changed with the thickness of the coating layers, the nature of the polymer material and the separation between the neighboring cylinders. In Fig. 6(a) we decrease the radius of the inner hard core and increase by the same amount the thickness of the polymer layer, the other parameters being kept fixed. This produces a downward shift of the dip frequencies which mainly results from the increase in the amount of the soft material. The effect is weak for the first dip, while it becomes more significant for the second dip for which the displacement field is localized inside the polymer. In Fig. 6(b), the parameters of the cylindrical inclusions are kept constant, while we change the period a of the crystal and therefore the separation between cylinders. The change in the frequency of the dip indicates that the interaction between neighboring cylinders is not negligible for a relatively high filling fraction. This result is at variance with respect to the case of locally resonant phononic crystals with only solid constituents. In Fig. 6(c), we decrease the thickness of the

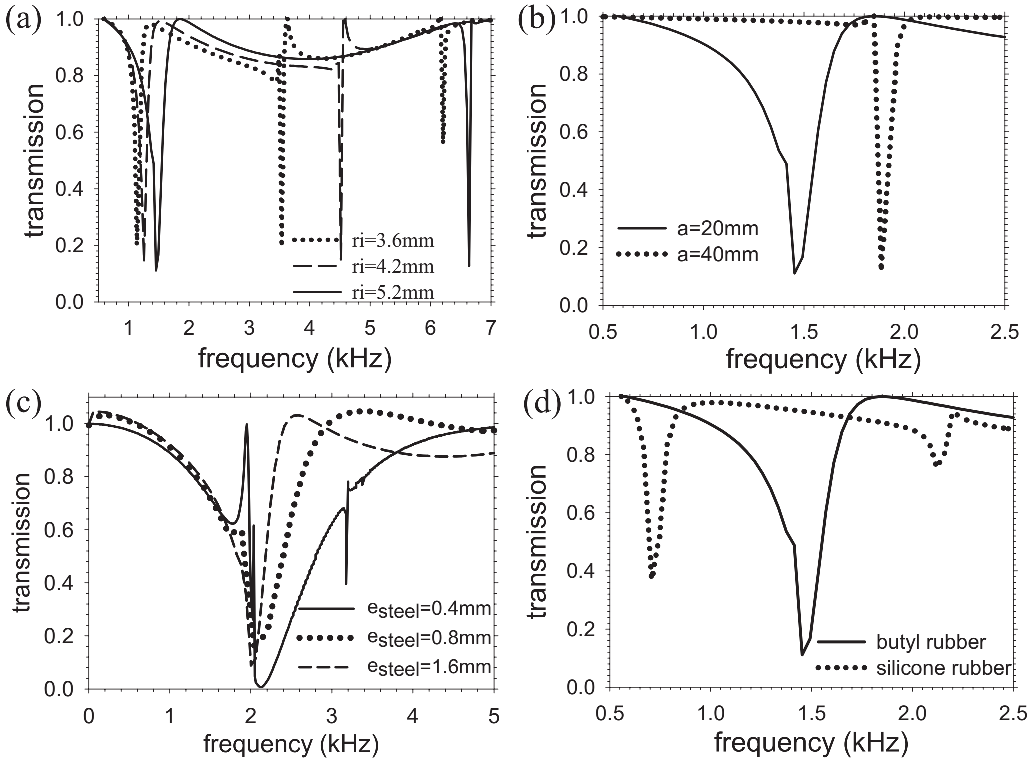

FIG. 6. Transmission coefficient through a sonic crystal with double-shell coating as a function of various geometrical and physical parameters (see text for details). (a) The radius of the inner core is decreased while the thickness of the polymer is increased by the same amount. (b) The period of the phononic crystal is twice as big. (c) The thickness of the outer steel shell is decreased while the radius of the inner core is increased by the same amount. (d) The polymer is changed from butyl rubber to silicone rubber.

outside steel shell and increase by the same amount the radius of the inner core. The decrease in the thickness of the steel shell increases the interaction between the polymer and the incoming wave from water and results in a broadening of the dip. This may be useful for the realization of a broadband sonic shield. In Fig. 6(d), we keep constant the geometrical parameters and change the nature of the polymer in the coatings, namely butyl rubber is replaced by silicone rubber (see Table I and Ref. [6]) which has lower velocities of sound. This results in a downward shift of the dip frequency, from 1.45 to 0.7 kHz.

Finally, to widen the phononic band gap, we make a combination of two phononic crystals in which we have respectively an outer shell thickness of 0.4 mm (0.6 mm) and a core radius of 3.6 mm (4.0 mm) (Fig. 7). These phononic crystals (A and B) display attenuation dips centred at the frequencies of 1.0 kHz and 1.06 kHz, respectively. The association in tandem of both phononic crystals leads to a wider

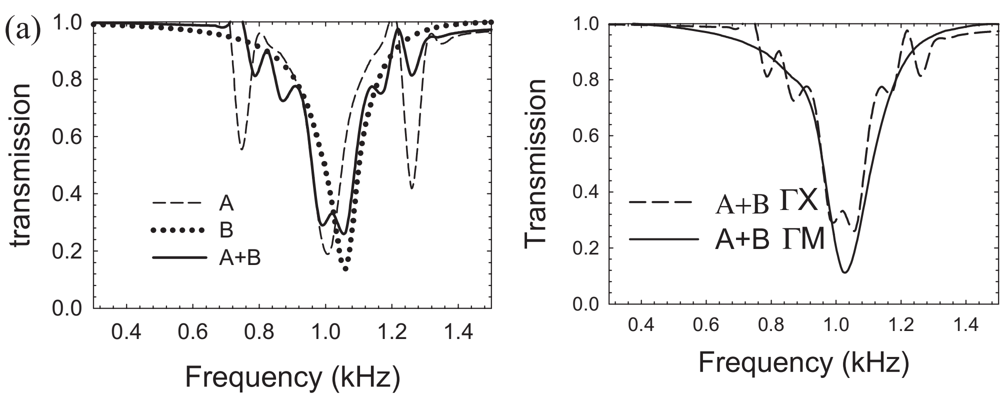

FIG. 7. (a) Transmission spectrum in the ΓX direction for two phononic crystals A and B and for the associated phononic crystal ($A+B$). The phononic crystals A and B are constituted respectively by an outer shell thickness of 0.4 mm (0.6 mm) and a core radius of 3.6 mm (4.0 mm). (b) Transmission spectrum for the ($A+B$) phononic crystal for the two directions ΓX and ΓM.

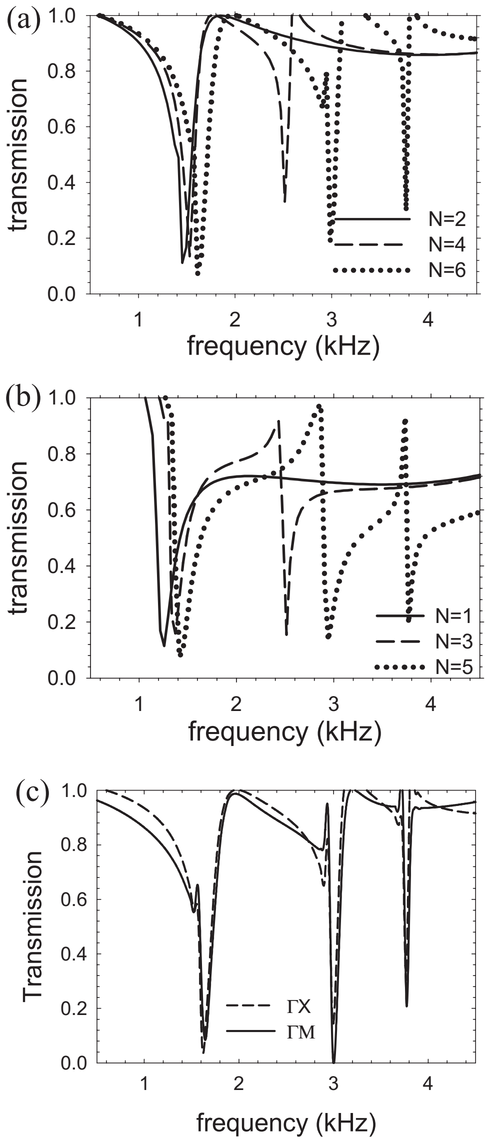

FIG. 8. Transmission curves through a phononic crystal where the coating contains N shells of polymer and steel, embedded in (a) water and (b) epoxy. (c) Transmission spectrum for $N=6$ shells of polymer and steel, embedded in water, for the ΓM direction compared to the ΓX direction.

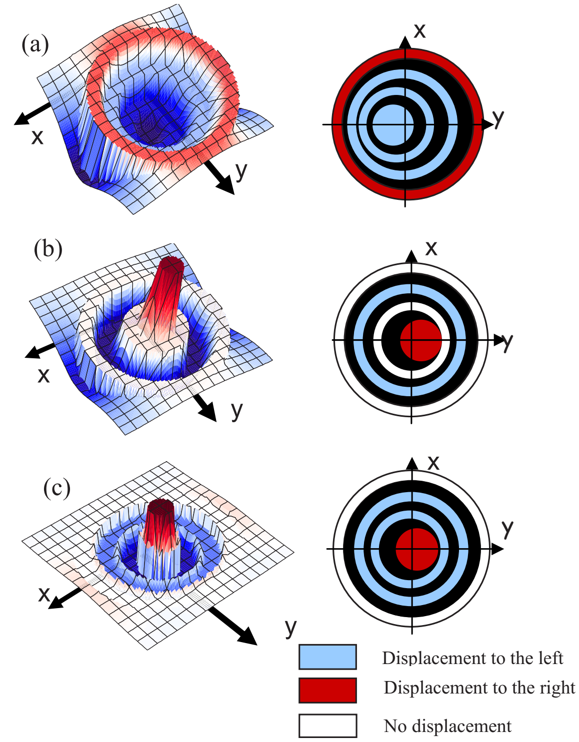

FIG. 9. (Color online) Map of the displacement component U_Y (parallel to the propagation direction) and schematic view of the motion in the (X,Y) plane for the first three dip frequencies when the coating contains $N=6$ shells. (a) $f=1.61$ kHz, (b) $f=3.0$ kHz, and (c) $f=3.77$ kHz.

gap resulting from the overlap of the two initial dips. In Fig. 6(f), we observe the same behavior for the ΓM direction that confirms the absolute character of the acoustic stop band.

2. $N>2$

In this subsection, we consider the case of multicoaxial cylindrical inclusions containing an even number of shells. Figure 8(a) presents the low frequency transmission curves for $N=2$, 4, and 6. The number of dips evolves in relation with N, namely in comparison to the case $N=2$ discussed above; each dip is divided into two (respectively three) dips when N becomes equal to 4 (respectively 6). As a matter of comparison, we also give in Fig. 8(b) the transmission coefficient for a phononic crystal in which the matrix is a stiff solid such as epoxy and the number of shells is 1, 3, or 5. Again, the number of dips increases according to the number of shells. The main difference with the former case is a stronger asymmetry in the line shape of the dips. Figure 8(c) shows, in the case of a water matrix, the transmission behavior obtained in the ΓM direction. The acoustic stop bands overlap perfectly the dips obtained in the ΓX direction, showing that the band gaps should be absolute. Coming back to the water matrix and choosing $N=6$, we illustrate in Figs. 8 the displacement fields of the first three resonance modes. For each frequency, we give the component U_Y of the displacement along the direction of propagation, as well as a schematic view of the vibrations in the (X,Y) plane. The common feature to all these three modes is the fact that the hard parts of the inclusion, namely the inner core and the three steel cylindrical shells, vibrate as rigid bodies linked together through the polymer shells that act as springs. In the lowest mode, occurring at $f=1.61$ kHz, the inner core and the two following steel shells vibrate in phase along the propagation direction (i.e., Y), while the outer steel shell moves with the opposite phase [see Fig. 9(a)]. This behavior is somewhat similar to the one obtained in Fig. 4 with $N=2$ if we assume that the core is now constituted by the inner core plus the two following steel shells. The displacement fields of the second ($f=3.0$ kHz) and third ($f=3.77$ kHz) resonant modes are represented in Figs. 9(b) and 9(c). They correspond to other vibrational states of four rigid bodies linked together through the polymer shells. These modes

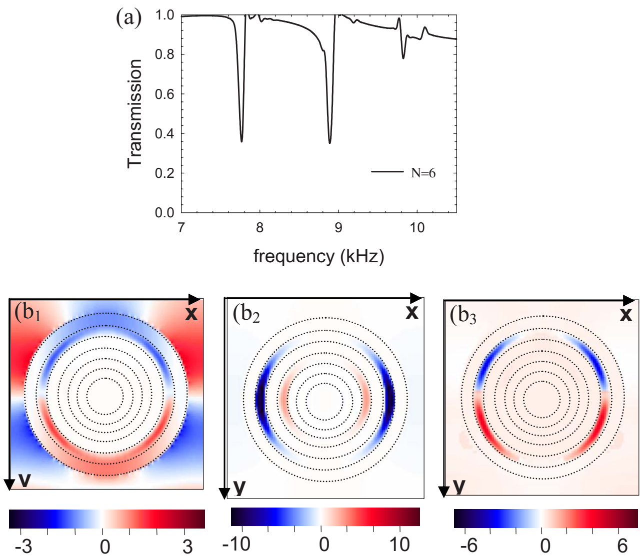

FIG. 10. (Color online) (a) Transmission coefficient and displacement component U_Y (parallel to the propagation direction) when $N=6$ for the dip frequencies (b_1) $f=7.8$ kHz, (b_2) $f=8.9$ kHz, and (b_3) $f=9.8$ kHz.

seem to be more localized inside the inclusion than the first resonant mode since the outer steel shell remains almost at rest.

In Fig. 10(a), we give the transmission coefficient of the phononic crystal at higher frequencies (between 6 and 10 kHz) showing the occurrence of the next set of three dips. The displacement fields associated with these dips are sketched in Figs. 10 (b1, b2, and b3). The common feature to all these modes is their localization in the polymer shells while the hard materials and the matrix remain almost at rest.

Finally, we illustrate in Fig. 11 the transmission through a structure constituted by a combination of three different phononic crystals (labeled *A*, *B*, and *C*), embedded in a water matrix. The crystals differ from each other by the radius of the inner core which affects each resonance mode differently. We choose the values 2.6, 3.0, and 3.4 mm for the crystals *A*, *B*, and *C*, respectively. The thicknesses of the steel shells surrounding the core have been chosen identical in the three crystals: The outer shell has a thickness of 0.4 mm to ensure the wideness of the lowest frequency attenuation peak, as illustrated Fig. 7(c); the two other shells have a thickness of 0.6 mm. Finally, the thicknesses of the polymer shells have been adjusted in each crystal in such a way as to keep constant the whole radius of the cylinder inclusions (namely 8.4 mm). Figure 11 presents the transmission through the associated structure compared to the transmissions of the individual crystals. For the lowest gap, since the frequencies and widths of the initial gaps are almost identical, we find a similar behavior for the combined structure. However, the second gap results from the overlap of three close attenuation peaks in the individual crystals. Therefore, these dips can overlap together and form a larger forbidden frequency gap. The next three frequency gaps result from the superposition of the individual dips coming from each phononic crystal involved in the structure. Since the attenuation peaks associated to each component are rather separated from each other, one can observe an increase in the number of dips in the transmission spectrum without a widening. As a conclusion, the association of several phononic crystals that differ from each other by their geometrical parameters leads to the superposition of the individual gaps and could modify the transmission spectrum in three different ways, namely keeping a dip in size and position, widening a gap, and increasing the number of dips.

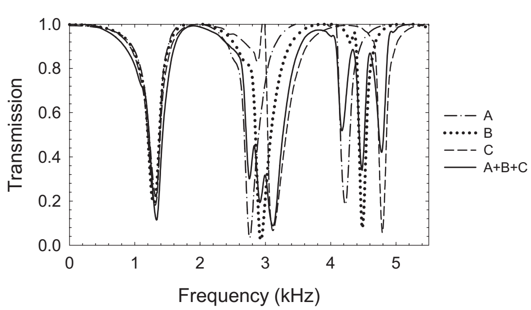

FIG. 11. Calculated transmission spectrum through a structure resulting from the combination of three different phononic crystals *A*, *B*, and *C* which differ from each other by their inner core radius (2.6, 3.0, and 3.4 mm, respectively). In the associated structure ($A+B+C$), the resulting gaps correspond to the superposition of each individual gap.

IV. SUMMARY

In this paper we studied the transmission coefficient and dispersion curves of phononic crystals in which the matrix is a fluid such as water and the inclusions are constituted by multicoaxial cylindrical shells. Each inclusion is constituted by a hard (steel) core coated with alternate shells of a soft polymer and steel. Totally different behaviors are obtained depending on whether the outer shell in contact with water is the soft polymer (odd number of shells) or steel (even number of shells). In the first case, low frequency gaps can appear as a result of the very low average velocity of sound in the phononic crystal. In the second case, several dips appear in the transmission coefficient, inside the pass band of the crystal below the first Bragg gap. These dips correspond to the interaction of flat bands with the linear dispersion curve of the propagating modes and result from the existence of quasilocal resonances in the unit cell of the phononic crystal. This result is similar to the one obtained in locally resonant sonic materials with solid constituents. However, in the structures with a fluid matrix, the interaction between neighboring unit cells is not totally negligible and the positions of the dip frequencies are dependent upon the filling fraction, i.e., the separation between neighboring cylinders.

By increasing the number of shells in the coating, the number of dips is increased, for example they are multiplied by 2 (or 3) when N is changed from 2 to 4 (or 6). We have discussed the displacement field of the resonant modes and generalized the behaviors obtained in the case of a single shell coating. Finally, we have analyzed the evolution of the lowest dips with the relevant geometrical and physical parameters of the phononic crystal. Values of the dip frequencies can be tuned using various parameters of the crystal as the inner core radius, the shell thicknesses, the period or the nature of the soft rubber. By associating several phononic crystals that differ from each other by their geometrical or physical parameters, one can keep unchanged or enlarge a phononic band gap or increase the number of dips, depending on the degrees of the overlap between the single dips.

[1] M. S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. Lett. **71**, 2022 (1993); M. M. Sigalas and E. N. Economou, Solid State Commun. **86**, 141 (1993).

[2] See, for instance, Y. Pennec, B. Djafari-Rouhani, J. O. Vasseur, A. Khelif, and P. A. Deymier, Phys. Rev. E **69**, 046608 (2004), and references therein.

[3] M. S. Kushwaha and B. Djafari-Rouhani, J. Appl. Phys. **84**, 4677 (1998).

[4] M. S. Kushwaha, B. Djafari-Rouhani, and L. Dobrzynski, Phys. Lett. A **248**, 252 (1998).

[5] P. Lambin, A. Khelif, J. O. Vasseur, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. E **63**, 066605 (2001).

[6] Z. Liu, X. Zhang, Y. Mao, Y. Y. Zhu, Z. Yang, C. T. Chan, and P. Sheng, Science **289**, 1734 (2000).

[7] Z. Liu, C. T. Chan, and Sheng Ping, Phys. Rev. B **65**, 165116 (2002).

[8] Z. Liu, C. T. Chan, and P. Sheng, Phys. Rev. B **71**, 014103 (2005).

[9] C. Goffaux, J. Sanchez-Dehesa, A. Levy Yeyati, Ph. Lambin, A. Khelif, J. O. Vasseur, and B. Djafari-Rouhani, Phys. Rev. Lett. **88**, 225502 (2002); C. Goffaux and J. Sánchez-Dehesa, Phys. Rev. B **67**, 144301 (2003); C. Goffaux, J. Sánchez-Dehesa, and Ph. Lambin, *ibid.* **70**, 184302 (2004).

[10] M. Hirsekorn, Appl. Phys. Lett. **84**, 3364 (2004); M. Hirsekorn, P. P. Delsanto, N. K. Batra, and P. Matic, Ultrasonics **42**, 231 (2004).

[11] Q. Bo, C. Jiu-Jiu, and C. Jian-Chun, Chin. Phys. **14**, 2522-07 (2005).

[12] G. Wang, D. Yu, J. Wen, Y. Liu, and X. Wen, Phys. Lett. A **327**, 512 (2004).

[13] G. Wang, X. Wen, J. Wen, L. Shao, and Y. Liu, Phys. Rev. Lett. **93**, 154302 (2004); G. Wang, L. Shao, Y. Liu, and J. Wen, Chin. Phys. **15** (8), 1843 (2006).

[14] *Encyclopedia of Polymer Science and Engineering*, 2nd ed. (Wiley-Interscience, New York, 1987).

[15] G. Mur, IEEE Trans. Electromagn. Compat. **23**, 377 (1981).