Interaction of two particles in a shear flow

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Our experimental study investigates the interaction of pairs of nearly spherical solid particles suspended in a stratified shear flow inside a Couette cell. The surfaces of the particles are well characterized, with roughness maps produced by a scanning electron microscope. We measure the degree of irreversibility introduced into the flow by particle interactions and relate it to the subtle differences in the particle surface features.

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For a number of well-established and developing applications, the understanding of the behavior of particles suspended in a shear flow is crucial. The subject is of practical importance to encapsulation of electronic components, transport of sediments, contaminants, and slurries, and secondary oil recovery by hydraulic fracturing, to name just a few areas of interest. Moreover, in viscous fluid flow with particle transport and interaction, the events on the local (i.e., particle) scale influence the large-scale flow properties, thus presenting an example of the multidisciplinary fundamental problem of the relationship between microscale interactions and macroscopic behavior.

Multiple rheological models developed in recent years to describe the particle-laden flow include diffusive flux [1,2], momentum balance [2,3], and two-continuum [4] models. The common problem with these models lies in their failure to predict transient particle concentration profiles. It is noteworthy that the transient properties of a particle-laden flow likely depend strongly on the diffusivity of the particle phase, while the latter is greatly influenced by the irreversibilities present in the system.

Any consideration of particle-laden flow is complicated by the presence of at least two sources of irreversibility due to particle interaction, whose physical mechanisms are distinctly different. First, particle surface roughness has been suggested as the culprit for irreversibility in two-particle interactions [5,6]. On the other hand, interaction of *three* particles, regardless of their surface properties, is reported to be chaotic [7–9]. Further, although two-particle interactions can lead to irreversibility as characterized by their self-diffusivity [6], it has generally been thought that particle migration in nonlinear shear flows results only from multiple-particle interactions [1,10].

We have conducted a careful experimental investigation of the behavior of two particles in shear flow that clearly shows that two-particle interaction is irreversible, with even nearly spherical smooth particles (average roughness $\sim$250 nm, or $\sim 10^{-4}$ of the particle radius) showing a measurable deviation from the initial positions in a nominally reversible, stably stratified shear flow, where the inner cylinder of a cylindrical Couette cell is slowly rotated by the same angle clockwise and counterclockwise, bringing two particles into interaction. Although the final positions of the particles can be predicted only in the statistical sense, the center of mass of the particle pair moves outward toward the low-shear-rate region of the flow field, indicating that particle migration can occur in two-particle systems. We suggest that the variation in these final positions is caused by differences in the microscopic properties of the particle surfaces in the immediate vicinity of the point of closest proximity between the surfaces. Thus the two-particle interaction in shear flow reveals a direct relationship between microscale phenomena near this point and the macroscale behavior of the flow, as manifested by the variation in the final positions of the particles. Moreover, our study shows that the statistical properties of the microscale surface features (average roughness) are directly correlated with the statistics of the particle displacements after the interaction.

The experimental arrangement (Fig. 1) uses a computer-controlled stepper motor to rotate the inner cylinder of a Couette cell with the radius of the inner cylinder $r_i = 5$ cm and the radius of the outer cylinder $r_o = 10$ cm. The cell is 4 cm deep and filled with a stratified viscous fluid (water solution of $ZnCl_2$ and Triton X100). Thus the aspect ratio of the fluid-filled cross section of the cell (height to width ratio) is about 0.8. The refractive index of the fluid (1.49) matches that of the acrylic block from which the outer cylinder of the cell is machined, to facilitate imaging of the side views of the cell. The density and stratification of the fluid are selected to render the polymethyl methacrylate (PMMA) particles used in experiment neutrally buoyant at a depth of 1 cm. At this depth, the fluid density equals the specific den-

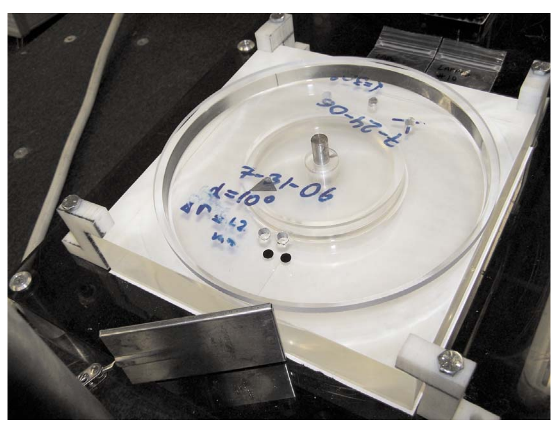

FIG. 1. (Color online) Closeup of the experimental arrangement showing the Couette cell covered with one of the acrylic templates used to deposit the particles and the 45° mirror (bottom of the image).

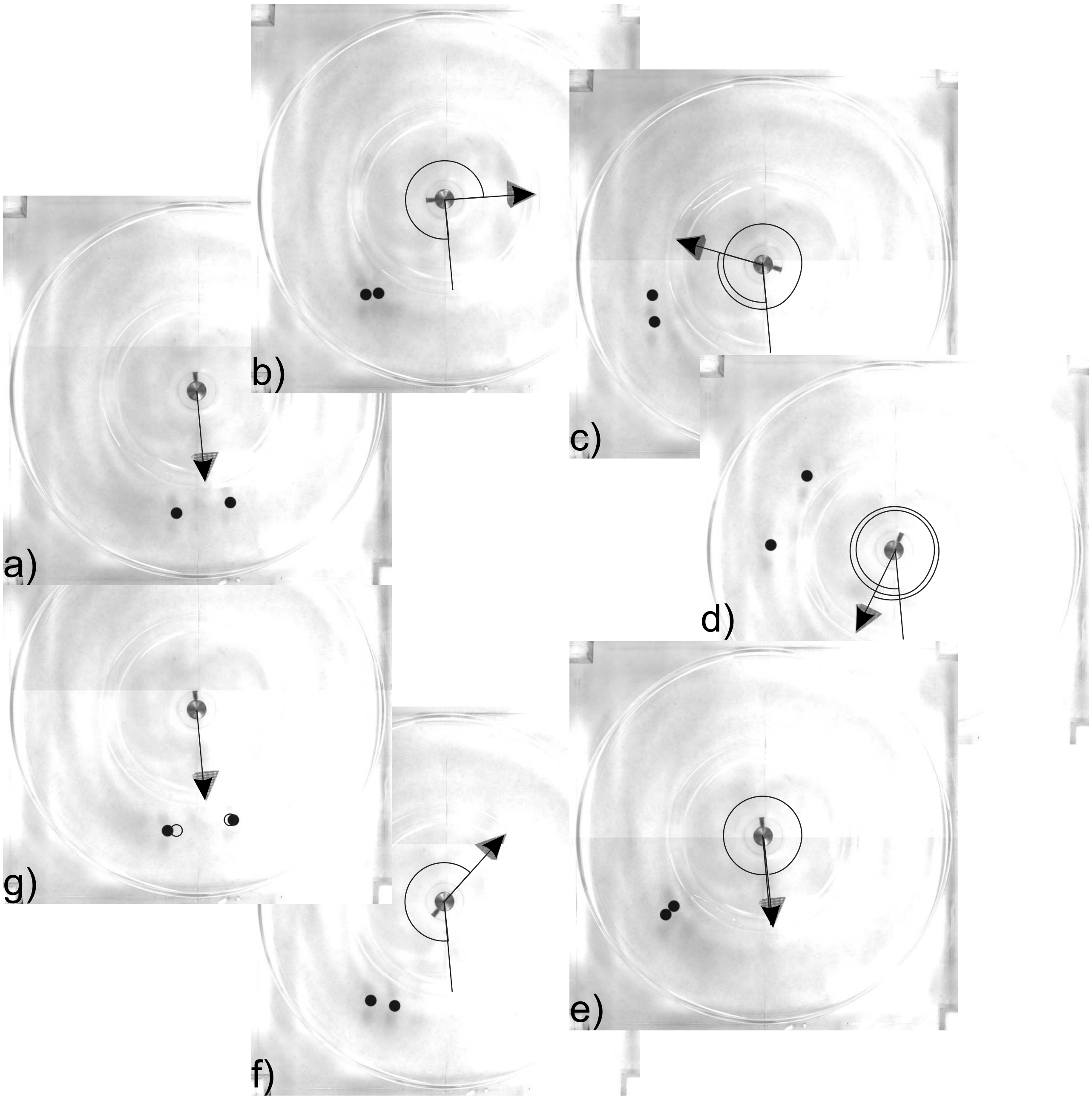

FIG. 2. Sequence of top views of the Couette cell showing a full rotation-counterrotation cycle: (a) initial conditions, (b), (c) clockwise rotation, (d) end of clockwise rotation, (e), (f) counterclockwise rotation, and (g) final conditions. Hollow circles in (g) denote the initial positions of the particles; the spirals represent the total rotation of the inner cylinder from its original position.

sity of PMMA (1.05), with the overall density difference from top to bottom of the cell due to stable stratification being on the order of 1% of this nominal density. Variation in other properties of the fluid (refractive index, viscosity) due to the density gradient is minimal. The surface characteristics of the PMMA particles used in the experiments have been well characterized using a scanning electron microscope, which provides a map of surface nonuniformities for each individual particle.

In the experiments, we use three different types of particles in terms of the surface treatment—original surface with the least roughness (referred to as "smooth" in the subsequent paragraphs), lapped particles, and bead-blasted particles (the last type having the roughest surface). The goal of the experiment is to observe a two-particle interaction in a low-speed, nominally reversible shear flow, and to determine whether addition of the particles renders the flow irreversible, i.e., after the particles are positioned in the fluid and the inner cylinder of the Couette cell is rotated clockwise and then counterclockwise by the same angle, the particles do not return to their original positions after undergoing an interaction.

An arbitrary angular velocity time profile can be specified for the inner cylinder with a computer controller interfaced with a workstation used both to run the experiment and to acquire the data from the top view camera which has a

1536×1024 pixel resolution with the physical image size corresponding to 69 μm per pixel. This camera visualizes the region of the cell where the particles are initially deposited. To ensure the consistency of the initial conditions, the particles are dropped into the cell through holes machined in an acrylic template (Fig. 1). There is still some standard deviation in the initial coordinates of the particles thus positioned. To ensure that the interaction of the particles is two dimensional, a side view of the cell is captured by the side view camera, which stores a sequence of the images of a composite of two side views of the cell (one direct, one through a 45° mirror) acquired as the inner cylinder of the cell rotates. In the experiments conducted so far, the angular velocity of the inner cylinder ω_i was selected so that the Reynolds number based on the gap of the Couette cell Re $=(r_o-r_i)\omega_i r_i/\nu$ (ν being the kinematic viscosity of the fluid) was 0.1.

Presently, we have been experimenting with particle pairs comprised of particles of the same surface type (smooth, lapped, bead blasted). After the particles are dropped into the fluid using the template shown in Fig. 1, they are allowed to settle in the fluid to a depth of 1 cm, where they reach neutral buoyancy. Then the inner cylinder is subjected to series of clockwise-counterclockwise rotations (total angle 750°), with the top view camera capturing subsequent images of the particles after each rotation-counterrotation cycle, as Fig. 2 shows. The positions of the particles in the horizontal plane are then converted to cylindrical coordinates (r,θ) using a computer program that calculates the centroid of each particle with a standard algorithm [11]. This makes it possible to resolve the positions of the particles with a subpixel accuracy (we estimate our worst-case error as 0.3% of the particle diameter).

A representative cycle of rotation is shown in Fig. 2, with the particle pair coming into interaction twice: during the forward (clockwise) part of the cycle and during the counterclockwise (reverse) part. Comparison of the initial and final positions shows that, after this pair of interactions, the particles do not return to their initial positions. In the "reality check" experiments we conducted, the particles were positioned in the Couette cell so that the rotation of the inner cylinder of the cell did not cause them to interact. In these experiments, we observed the particles return to their initial positions after the cell is rotated clockwise and then counterclockwise with no discernible irreversibility.

In sheared particle-laden flows, the concentration of particles is known to change in the direction of the areas with minimal shear [12]. The particle interaction in our experiments produces the same trend, as Fig. 3 shows. Figure 3 gives the initial radial positions of the center of mass of the two-particle system $\bar{r}=(r_1+r_2)/2$, where r_i, $i=1,2$, are the initial distances of the centers of mass of the individual particles from the center of the cell, as well as the corresponding distances after six rotation-counterrotation cycles. The data are averaged over ten experimental runs for each particle pair, with the error bars representing the standard deviation. In all cases, there is a clear trend for the particles to move away from the rotating inner cylinder of the cell (in the direction of lower shear). Thus one could argue that, in multiple-particle systems, it is the irreversibility of two-particle interaction (and not more complicated multiparticle effects) that causes the particles to migrate toward lower-shear areas. The influence of the cylinder walls might affect this migration. We conducted some exploratory experiments where the particles were positioned further apart in the radial direction, so that the pair interaction was weaker. The main influence of the distance from the rotating inner cylinder was to alter the final radial spread of the two particles, with no statistically relevant effect on the final radial location of the center of mass.

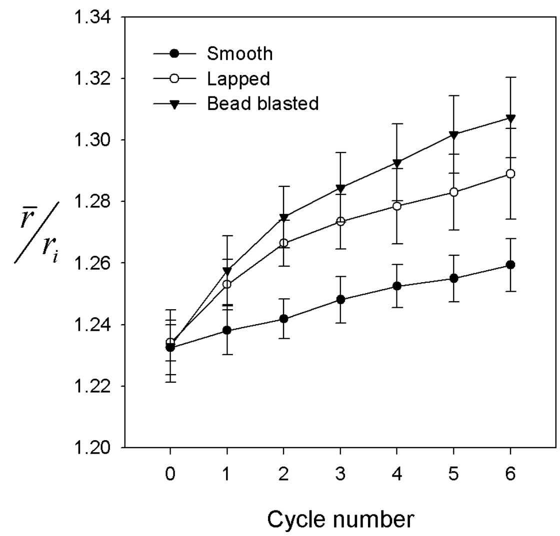

FIG. 3. Evolution of the position of the center of mass of the system of two particles $\bar{r}$ after six successive rotation-counterrotation cycles of 750°. 0 denotes the initial conditions; $\bar{r}$ is normalized by the radius of the rotating cylinder, r_i.

To provide an explanation for this two-particle migration, consider a sequence of snapshots of two particles suspended in a shear flow with a fixed center of gravity but with a variety of surface to surface separations δ. In a numerical simulation we performed, the center of gravity of the particle pair migrated toward the low-shear-rate region of the flow field. We took snapshots, perturbing the particles inward to-

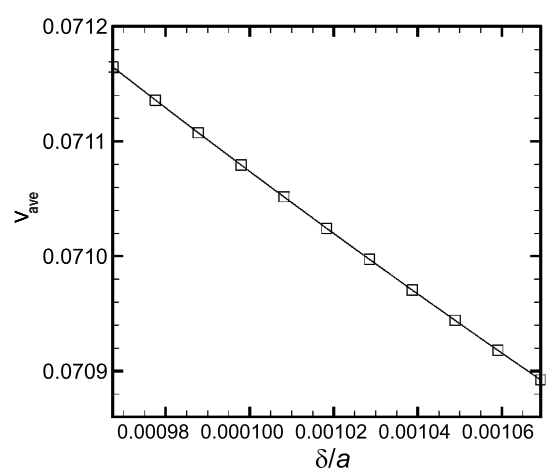

FIG. 4. Average dimensionless velocity of the center of gravity of a pair of particles in nonlinear shear flow as a function of particle separation δ/a (numerical simulation results).

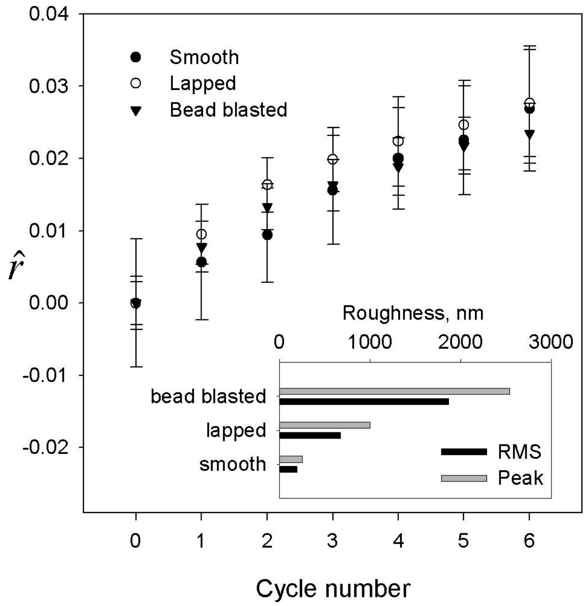

FIG. 5. Positions of the center of mass of the system of two particles normalized by the square root of the peak surface roughness. The inset shows the peak and average (root mean square, labeled rms) roughness in nanometers for each particle type.

ward each other along the line of centers. A plot of the velocity of the center of gravity V_{av} as a function of separation δ/a (a being the particle size) is shown in Fig. 4. The migration velocity is seen to decrease with increasing separation.

This simulation illustrates why rough particles migrate toward the low-shear-rate region of the flow field. If the particles were perfectly smooth, then the particle trajectories would be perfectly symmetric upon approach and separation. At a certain location upon approach, the particles would move toward the low-shear-rate region of the flow field with a given speed. At the symmetric point during separation, the particles would be exactly the same distance apart, and hence the particle pair would move toward the high-shear-rate region of the flow field with the exact same speed as upon approach. Hence, after separation, there would be no net migration of the particle pair. However, for rough particles, the distance between the particles on separation is greater than on approach, and hence the speed at which the particles move toward the low-shear-rate region on approach is faster than the speed at which the particles move toward the high-shear-rate region on separation. Therefore, the net effect is the observed migration of the particle pair toward the low-shear-rate region of the flow field. How would this result compare with particle migration according to the rheological models? If a repulsive interparticle force [13] were considered, a finite normal stress would arise in particle interaction. Hence, using the suspension balance model of Nott and Brady [14], particle migration would also ensue. The argument for the finite normal stress, however, relies on the presence of *multiple* interacting particles, and would not necessarily apply to a two-particle system, whereas in our experiments, the effects of the nonconstant shear rate are the apparent cause of the migration of the particle pair. Moreover, the suspension balance model predicts the particle migration rate to scale as the particle radius squared, like the particle self-diffusivity. Experiments rule out the possibility that this scaling is realized [12,15,16]. In this context, the current experimental results strongly suggest that rheological models for suspension flows can be improved by giving more consideration to the nonlinearity of the flow field.

It is also noteworthy that, while the particle pairs with all types of surface treatment (smooth, lapped, bead blasted) exhibit a similar trend in moving away from the higher-shear-rate zone near the rotating inner cylinder, the extent of this radial motion appears to be related to the surface roughness, with the rougher particles exhibiting the most prominent movement. The error bars of the $\bar{r}$ plots in Fig. 3 do not overlap; however, these plots can be collapsed by the appropriate scaling of $\bar{r}$. In Fig. 5, we plot

$$\hat{r} = \frac{\bar{r} - \bar{r}_{\text{initial}}}{r_i\sqrt{\delta_q/\delta_{q,\text{sm}}}},$$

where $\bar{r}_{\text{initial}}$ is the initial position of the center of mass (cycle 0 in Fig. 3), δ_q is the known maximum surface roughness for the particular particle pair, and $\delta_{q,\text{sm}}$ is the corresponding maximum roughness for the smooth particles. This scaling is effectively a normalization of the radial displacement by the square root of the peak roughness. Based on the data presently at our disposal, we cannot yet ascribe any specific physical meaning to the result, beyond demonstrating that there is a statistically relevant relationship between the extent of the irreversibility of the system (comprised of two particles in shear flow) and the surface properties of the particles.

In summary, we have experimentally demonstrated that the effectively two-dimensional interaction of a pair of nearly spherical particles in nonlinear shear flow is irreversible. As the result of the two-particle interaction, the particle pair moves in the direction of lower shear. The extent of this motion is strongly correlated with the particle roughness.

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