Time evolution of electromagnetic wave packets through superlattices: Evidence for superluminal velocities

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We study the space-time evolution of electromagnetic wave packets through optical superlattices. We present rigorous analytical solutions describing the multiple-scattering processes of Gaussian wave packets defined in the band gap and in the resonant energy regions. Following their space-time evolution, we obtain the Maxwell equations prediction for the time spent inside the superlattice. From a close and careful observation of the reflected and transmitted parts of Gaussian packets in a photonic band gap, we conclude unambiguously that the superluminal transmission and the Hartman effect are inherent properties of the electromagnetic theory. It is also shown that the theoretical predictions for the time spent inside an optical superlattice are in good agreement with the experimental results and the phase time predictions.

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I. INTRODUCTION

In the last century the calculation of the time spent by a particle when passing through a potential barrier was, for quite a while, one of the basic and controversial problems. Already in the early days of quantum theory, Pauli suggested the impossibility of constructing a self-adjoint time operator [1]. When the issue of the delay time of a transmitted wave packet through a potential barrier was first approached in 1932 by MacColl [2] and later by Hartman [3] (using the stationary phase time introduced by Wigner in the context of nuclear physics [4]), the striking superluminal effect arose for the first time. In the absence of experimental results, the obvious concern about violations of causality and the special theory of relativity, prevented theorists from accepting the phase time as the tunneling time (TT). Alternative definitions [5–11] for the TT then appeared. However, precise delay time measurements for single photons [12] and electromagnetic pulses in the band gap of optical superlattices [13] (OSL's) upheld the possibility of superluminal motion and the relevance of the phase time. Extremely good agreement [15] with these experiments was found when the phase time $\tau(\omega)$ was evaluated for the actual wave and superlattice parameters [14]. Nevertheless, since a wave packet (WP) moving through an OSL experiences an infinite number of internal reflection and transmission processes, it is often argued that the WP centroid would not determine the WP transmission time, even though the WPs are defined in wide photonic band gaps. Hence alternative theoretical explanations are still pushed forward. It is then pertinent to study the transmission time problem from a different point of view. The idea is to follow the actual WP evolution as determined solely by the Maxwell equations and to search whether the apparent superluminal effect is inherent or not to the electromagnetic theory. In this way we move away from the dichotomy of time definition versus experimental evidence. We turn to the fundamental laws of nature that govern the time evolution of electromagnetic wave packets—i.e., the Maxwell equations—and let them determine the time evolution of the wave packets. To this aim, we offer here rigorous calculations of the space-time evolution of an electromagnetic WP through optical superlattices.

The actual time delay experiments of isolated photons and photonic pulses mentioned before, and those in Ref. [16–24], measure traversal times in a simple and direct way. Most of these results substantiate convincingly the existence of superluminal velocities, the intriguing Hartman effect, the reshaping mechanism of wave packets, and the relevance of the phase time. On the theoretical side, interest in the tunneling time has grown with an increasing diversity of publications. From the point of view of the relation of these theoretical contributions with the experimental results, one can distinguish at least three groups among the most representative contributions. In the first group [25–33], extensions of old approaches and definitions lead to qualitative and approximate new results which, regrettably, are not directly compared with the very precise experimental results. In the second group, exact and approximate phase time calculations [14,34] confirm the experimental results in Refs. [12,13,21]. In the third group, calculations of tunneling times for different kind of particles and systems [35–40] were carried out basically because the TT *per se* became a quantity of interest. Although the superluminal velocities and the associated Hartman effect are frequently and widely invoked, all we know with certainty are the experimental evidence and a couple of phase time calculations verifying these results. Without dismissing the importance of the extensive theoretical work, it is worth noticing that a serious attempt at describing rigorously the space-time evolution of a wave packet, based only on the exact solutions of the Maxwell equations, is clearly lacking. As mentioned before, the aim of this paper is to provide *theoretical evidence* of WP transmission through OSLs. Our results and the formalism developed here will not only show that the superluminal and Hartman effects are inherent to the electromagnetic theory, but also shed light on other important topics of current interest. The formalism can also be used to test whether a given tunneling time definition is compatible or not with well-established electromagnetic and quantum theories. It is worth mentioning again that the previous frequency-dependent phase time calculations [14] already provide essential information to understand qualitatively the reshaping mechanism as well as conditions to observe superluminal phenomenology. It is clear to us that superluminality and the Hartman effect are basically related to the absence of dwelling states in the scattering region. A questionable proposal to explain superluminal mechanism [41] led to the also questionable arguments against any implication of superluminal velocity based on the judgment of the pulse-peak behavior [42]. The criticism assumes that in a photonic band gap [43] (PBG) mainly the leading edge of the incident WP survives the tunneling event, looking hence faster than the incident pulse. One should, however, notice that when a WP is defined in a PBG, the rear and front tails together with the attenuated central part of the incident pulse all contribute to the reshaping of the transmitted pulse. If this is the case, there is no reason why the position of the reshaped-packet peak will be defined only by the front tail (the leading edge). Moreover, most of the tunneling time definitions, including the so-called Büttiker-Larmor traversal time (see Steinberg *et al.* [12]), predict larger tunneling times for WP components with higher transmission coefficients; therefore, the tails cannot be responsible—if anything—for superluminal velocities. To understand this problem, one would like to unravel what the well-established electromagnetic theory determines for the time evolution of the whole WP components.

We will present here general results on the space-time evolution of Gaussian electromagnetic WPs through an OSL of length $L=nl_c$, with n the number of cells. We will use these results to determine unambiguously the time spent by the WPs in the OSL. For this purpose we proceed as follows. At $t=0$ we fix the WP peak at $-z_0$, far enough from the OSL. As reference graphs we plot the WP at $t=0$ and at $t_a=z_0/v_g$, when the WP centroid (moving with group velocity $v_g=c$, the speed of light in vacuum) reaches the OSL. To find out when precisely the WP leaves the OSL, one can proceed in different ways. One can, for example, plot the WP at $t_\Delta=t_a+\Delta t$ with various values of Δt, until the WP is found just leaving the OSL. We can also use a proposed OSL transmission time τ to plot the WP at $t_\tau=t_a+\tau$ and to see whether the WP is certainly emerging from the OSL or not. When the WP centroid is defined in the band gap, it is not easy (as will be seen below) to obtain any conclusion from the reflected part at t_τ because of the strong interference with the incoming components, while from the transmitted part is also difficult because it tends to vanish. It would be more convenient to look at the WP at $t_b=2t_a+\tau$, when presumably the reflected WP centroid would be back to $-z_0$ while the transmitted part would be at z_0+L. At these positions there is no more interference, or at least it is negligible. Comparing the plots with the reference graphs one can test not only the proposed tunneling time τ, but also determine the actual time Δt spent in the OSL. Using an OSL with the same parameters as in Ref. [12], we will consider two specific and distinct cases. In the first example the WP is defined with centroid wavelength $\lambda_0=735$ nm, in the center of a photonic band gap. In the second example the WP is defined with centroid wavelength $\lambda_0=400$ nm in the center of an allowed photonic band. In the first case, as shown in Fig. 1, the OSL is opaque. For systems with similar conditions, the experimental results show superluminal velocities and, varying the number of cells n, the Hartman effect. The phase time predicts superluminal velocities, with transmission time τ of 2.32 fs. Hence, in order to provide theoretical evidence of superluminal velocities and the Hartman effect, we will study in this example not only a time series of the WP evolution but also the transmitted WP as a function of n. In the second example, the OSL is highly transparent with almost constant transmission coefficient and phase time τ of 7.2 fs. In this case there will be no superluminal transmission but the theoretical description can be compared with the phase time prediction.

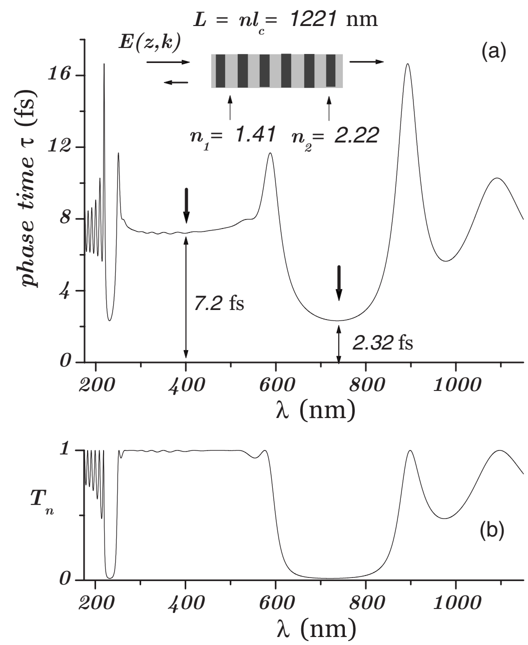

FIG. 1. The phase time τ (a) and transmission coefficient (b) for the OSL shown in the inset. The thick arrows indicate points around which the transmission coefficients are almost constant.

II. FORMULATION

For simplicity, let us consider a Gaussian electromagnetic wave packet with parallel polarization and normal incidence from the left, defined as

$$\Psi_E(z,t) = \int dk e^{-\gamma(k-k_0)^2} e^{ikz_0} E(z,k) e^{-i\omega t}, \qquad (1)$$

whose packet peak at $t=0$ is at $-z_0$. To determine the k-component electric field $E(z,k)$ at any point outside and inside the OSL, we use the transfer matrix (TM) method. We assume the OSL to contain n cells where each cell has length $l_c=l_1+l_2$. l_1 and l_2 are the lengths of dielectric layers with the corresponding permittivities ϵ_1 and ϵ_2, refractive indices n_1 and n_2, and permeabilities μ_1 and μ_2. The refractive index outside the OSL is the vacuum's n_0. At any point, the electric and magnetic fields contain right- and left-moving parts. Hence

$$E(z,k_i) = A_r e^{ik_i z} + A_l e^{-ik_i z} = E_r(z,k_i) + E_l(z,k_i),$$

$$H(z,k_i) = \frac{n_i}{\mu_i c}[E_r(z,k_i) - E_l(z,k_i)],$$

where $k_i=n_i k$ $(i=0,1,2)$, $k=\omega/c$, and c is the speed of light in vacuum. In the TM method we write $E(z,k)$ in a vectorlike representation

$$\mathcal{E}(z,k) = \begin{pmatrix} E_r(z,k) \\ E_l(z,k) \end{pmatrix}, \tag{2}$$

which is related to $\mathcal{E}(z',k)$ by

$$\mathcal{E}(z',k) = M(z',z)\mathcal{E}(z,k). \tag{3}$$

Here $M(z',z)$ is a TM. Inside the OSL, a vector $\mathcal{E}_{j+1}(z,k)$ at $z=jl_c+z_p$ in cell $j+1$ [with $j=0,1,\ldots,(n-1)$ and $0<z_p \leq l_c$] is related to $\mathcal{E}(0^-,k)$ at $z=0^-$ by

$$\mathcal{E}_{j+1}(z,k) = M_p(z,jl_c)M_j(jl_c,0^+)M(0^+,0^-)\mathcal{E}(0^-,k).$$

$M_p(z,jl_c)$ is the partial-cell TM and $M_j(jl_c,0^+)$ is the TM for the first j cells. Thus

$$M_j(jl_c,0^+) = [M(l_c,0^+)]^j, \tag{4}$$

where $M(l_c,0^+)$ is the single-cell TM:

$$M(l_c,0^+) \equiv \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} = \begin{pmatrix} \alpha_b e^{ik_1 l_1} & \beta_b \\ \beta_b^* & \alpha_b^* e^{-ik_1 l_1} \end{pmatrix}. \tag{5}$$

In this matrix, assuming $\mu_1=\mu_2=\mu_0=1$, we have

$$\alpha_b = \cos(k_2 l_2) + \frac{i}{2}\left(\frac{n_2}{n_1} + \frac{n_1}{n_2}\right)\sin(k_2 l_2), \tag{6}$$

$$\beta_b = \frac{i}{2}\left(\frac{n_2}{n_1} - \frac{n_1}{n_2}\right)\sin(k_2 l_2). \tag{7}$$

It is known, from the theory of finite periodic systems [44], that the matrix elements of $M_j(jl_c,0^+)$ can be written as

$$\alpha_j = U_j(\alpha_R) - \alpha^* U_{j-1}(\alpha_R), \tag{8}$$

$$\beta_j = \beta U_{j-1}(\alpha_R) = \beta_b U_{j-1}(\alpha_R) \tag{9}$$

where $U_j(\alpha_R)$ are the Chebyshev polynomials of the second kind of order j, evaluated at the real part of α.

Assuming an incoming wave of unit amplitude, the electric field at $z<0$ is

$$E(z,k) = e^{ikz} + r_T e^{-ikz}. \tag{10}$$

Here $r_T=-\beta_T^*/\alpha_T^*$ is the reflection amplitude and

$$\alpha_T^* = \alpha_{nR} - \frac{i}{2}\left(\frac{n_0}{n_1} + \frac{n_1}{n_0}\right)\alpha_{nI} + \frac{i}{2}\left(\frac{n_0}{n_1} - \frac{n_1}{n_0}\right)\beta_{nI},$$

$$\beta_T^* = \frac{i}{2}\left(\frac{n_0}{n_1} - \frac{n_1}{n_0}\right)\alpha_{nI} - \frac{i}{2}\left(\frac{n_1}{n_0} + \frac{n_0}{n_1}\right)\beta_{nI}. \tag{11}$$

The subindices R and I stand for the real and imaginary parts. The electric field in cell $(j+1)$ takes the form

$$E_{j+1}(z,k) = \frac{1}{2}[(\alpha_p+\gamma_p)\alpha_j + (\beta_p+\delta_p)\beta_j^*]\left[\left(1+\frac{n_0}{n_1}\right) + \left(1-\frac{n_0}{n_1}\right)r_T\right] + \frac{1}{2}[(\alpha_p+\gamma_p)\beta_j + (\beta_p+\delta_p)\alpha_j^*] \times \left[\left(1-\frac{n_0}{n_1}\right) + \left(1+\frac{n_0}{n_1}\right)r_T\right], \tag{12}$$

where α_p, β_p, γ_p, and δ_p are the elements of $M_p(z,jl_c)$. Evaluating at $z>nl_c$ the transmitted electric field becomes

$$E(z,k) = \frac{1}{\alpha_T^*}e^{ik(z-nl_c)}. \tag{13}$$

Using Eqs. (10), (12), and (13) in Eq. (1) we have the WP in space and time described by the Maxwell equations. It is worth mentioning that in these equations, multiple-scattering processes are rigorously taken into account and the results are exact for arbitrary number of cells n. As time increases the WP evolves in a rather complicated *but describable* way. The actual behavior depends critically upon the OSL parameters and the WP characteristics. The basic idea here is to follow the WP evolution and to determine, from crucial snapshots, the time that the WP needs to cross the OSL (or to come back reflected). To illustrate this evolution we will plot the WP, as mentioned above, at (i) $t_0=0$, (ii) $t_a=z_0/v_g$ when the wave packet peak arrives at the left of the OSL, (iii) $t_\tau=t_a+\tau$ when (according to the phase time prediction) the centroid should be just *leaving* the OSL, and (iv) at $t_b=2z_0/v_g+\tau$ when the peak of the reflected WP should be back at $-z_0$ while the transmitted part should be at z_0+L. Plotting the WP at t_τ and t_b allows us to determine whether the time Δt spent by the WP inside the OSL coincides with the phase time τ or not.

III. RESULTS

For the examples discussed below we will consider the superlattice $[(L/2)H(L/2)]^n$ made of silica (L) and titanium oxide (H) with $l_1=124.46$ nm, $l_2=79.06$ nm, and refractive indices $n_1=1.41$ and $n_2=2.22$ as in Ref. [12]. The number of cells will be chosen to be $n=6$, except when the WP behavior is studied as a function of n. To get an insight into the transmission times of the field components, we have plotted in Fig. 1 the transmission coefficient and the phase time as functions of the wavelength $\lambda=2\pi/k$. As is well known [14], the phase time follows the band structure of the transmission coefficient. Wave packets with centroid λ_0 at 400 nm and 735 nm (see arrows) are characterized by transmission coefficients $T_n\approx 1$ and $T_n\approx 0$ and phase times $\tau\approx 7.2$ fs and $\tau\approx 2.32$ fs, respectively; cf. 4.07 fs for $v_g=c$. If the phase time predictions are correct, these values will imply subluminal velocities in the first case and superluminal velocities in the second one.

In Fig. 2 we present a time series for the WP with $\lambda_0=735$ nm. The WP width is chosen such that the main part of the WP lies inside a region of almost constant phase time $\tau\approx 2.32$ fs, which is close to the experimental [12] tunneling time $\tau_{ex}\simeq 2.1\pm 0.2$ fs, and the predicted phase time [14]

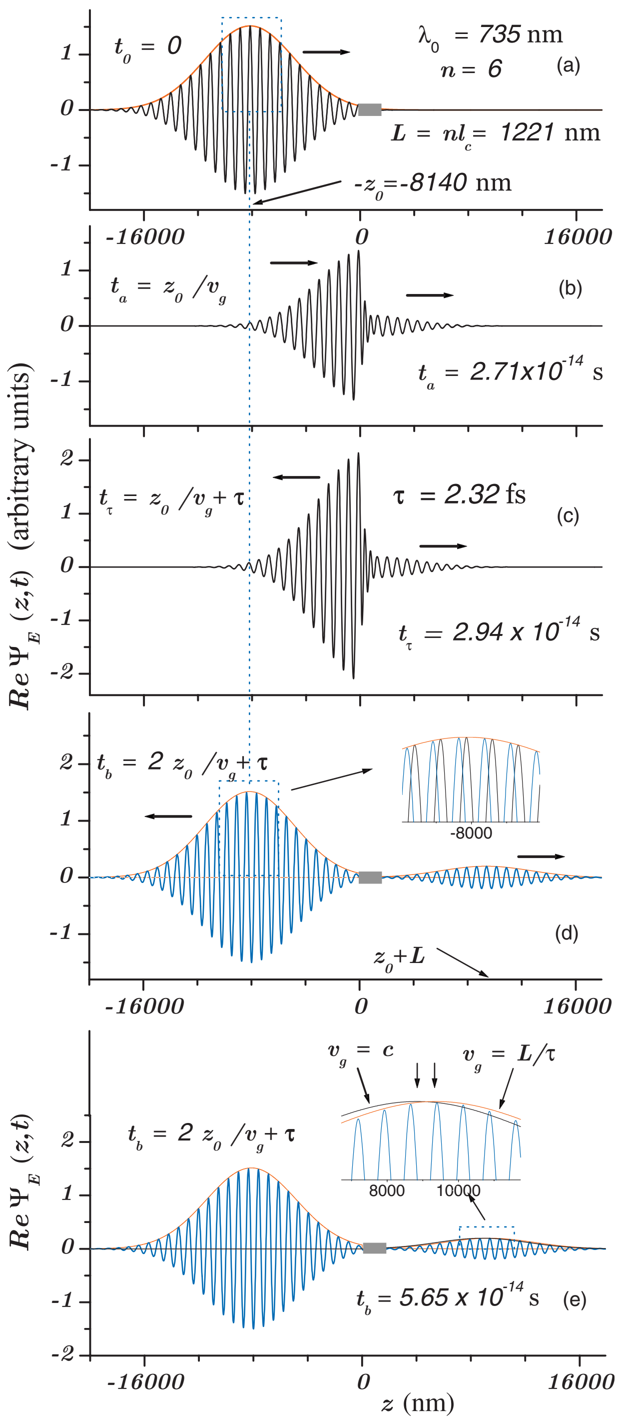

FIG. 2. (Color) Time series of a WP with centroid wavelength $\lambda_0=735$ nm, located at $-z_0=-40l_c$ at $t=0$. The optical superlattice $[(L/2)H(L/2)]^6$ is made of silica (L) and titanium oxide (H) with $l_1=124.46$ nm, $l_2=79.06$ nm, and refractive indices $n_1=1.41$ and $n_2=2.22$ as in Ref. [12]. In (b) the WP is at $t_a=z_0/v_g$ while in (c) is at $t_\tau=z_0/v_g+\tau$ with $\tau=2.32$ fs as the phase time. The WP at $t_b=2z_0/v_g+\tau$ in (d) and (e) have the same enveloping curve as that at $t=0$. The red Gaussian curve in the inset of (e) is drawn assuming group velocity $v_g=L/\tau$ and the black curve for $v_g=c$. Despite the phase shift shown in the inset of (d), these figures show that the actual time Δt spent in the OSL, described by the Maxwell equations, coincides with the phase time τ and agrees with superluminal velocities.

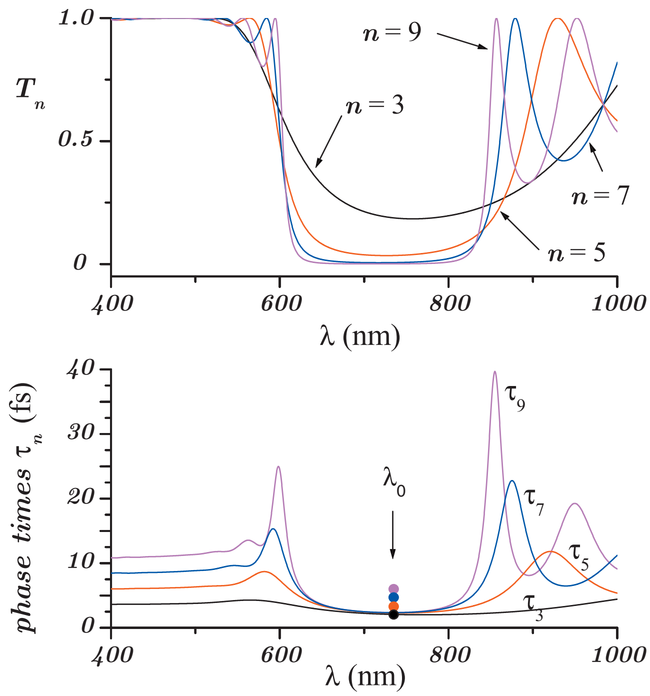

FIG. 3. (Color) Transmission coefficients T_n and phase times τ_n as functions of the wavelength λ, for OSLs with different numbers of cells. For small n the transmission coefficients are more sensitive to n in the gap than in resonant regions. In the band gap the phase time remains almost constant. The color dots indicate the time $t_n=nl_c/c$ that would be spent by the WP if it were to move inside the OSL with the speed of light c.

($\tau\simeq2.3$ fs), for a slightly different system. In Fig. 2(a) the WP is plotted at $t=0$ with the wave-packet peak at $-z_0=-40l_c=-8140$ nm. In Fig. 2(b) we have the WP at $t_a=z_0/v_g=z_0/c$. At this time the wave packet peak is touching the left side of the OSL. In Fig. 2(c), the WP is plotted at $t_\tau=z_0/v_g+\tau$. As expected, the WP peak is apparently leaving the OSL. It is, however, somewhat uncertain to establish any conclusion in this sense because of the phase shift [see inset Fig. 2(d)] and the strong interference between the "arriving" and "leaving" packets. For this reason it is convenient to look at the reflected and transmitted wave packets farther away from the scattering region. In Fig. 2(d) we plot the WP at $t_b=2z_0/v_g+\tau$ when, according to the phase time prediction, the reflected and transmitted WP should be back at $-z_0$ and passing by z_0+L, respectively. To visualize the WP positions better we plot also the red Gaussian curves centered at $-z_0$ and z_0+L. It is interesting to see that these Gaussian curves, properly normalized, are precisely the enveloping curves of the reflected and transmitted WPs at $t_b=2z_0/v_g+\tau$ and of the incoming WP at $t=0$. This absolute conjunction in space shows that the actual time spent by the WP inside the OSL, $\Delta t=t_\tau-t_a$, coincides with the phase time prediction. Since the time spent by the WP, $\Delta t=2.3$ fs, is smaller than the time $L/c\simeq4.07$ fs that would be spent if it were to move with the speed of light c, we conclude that the superluminal velocity of the WP through the OSL is an inherent property of the electromagnetic theory. To strengthen this observation, we consider again the WP at $t_b=2z_0/v_g+\tau$

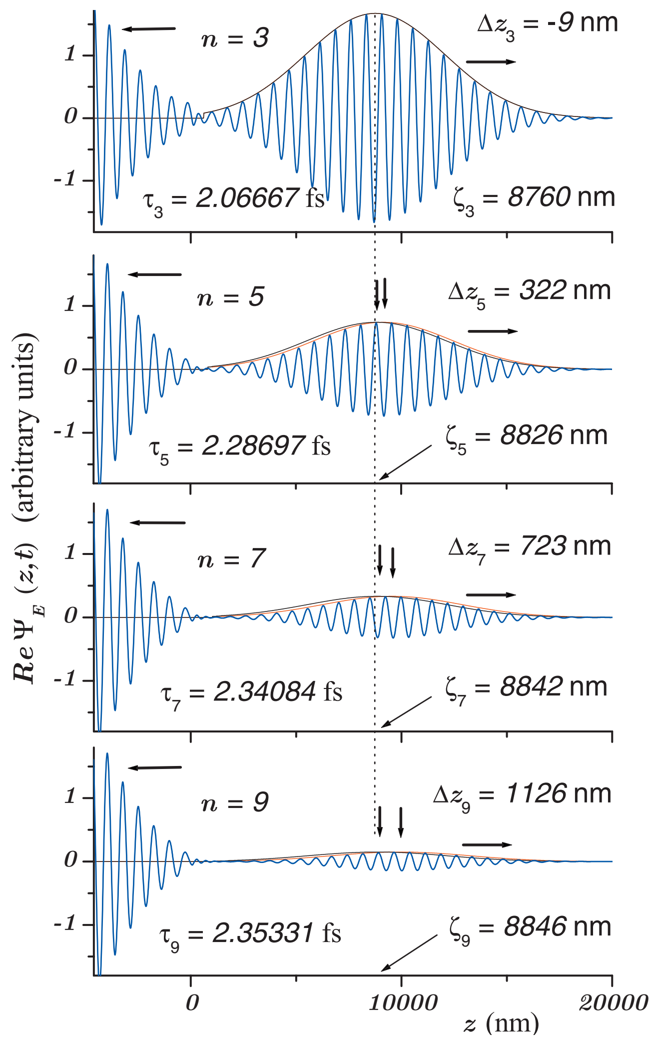

FIG. 4. (Color) Transmitted wave packets with centroid wavelength λ_0=735 nm evaluated at $t_n = 2z_0/c+\tau_n$ with τ_n the phase time for a WP peak to travel through the OSL $[(L/2)H(L/2)]^n$ (defined in Fig. 2) and n the number of cells. The dotted line, the left arrows pointing downward, and the black Gaussian curves indicate the positions $\zeta_n = z_0 + c\tau_n$ at which the WP peak would appear if it were to move inside the OSL with the speed of light c. The right arrows pointing downward and red Gaussian curves indicate the positions $z_n = z_0 + nl_c$ for the peak of the transmitted WP as predicted by the phase time. The lengths $\Delta z_n = z_n - \zeta_n$ indicate the distance between the left and right arrows pointing downward. Good agreements are found between the WP described by the Maxwell equations and the phase time predictions.

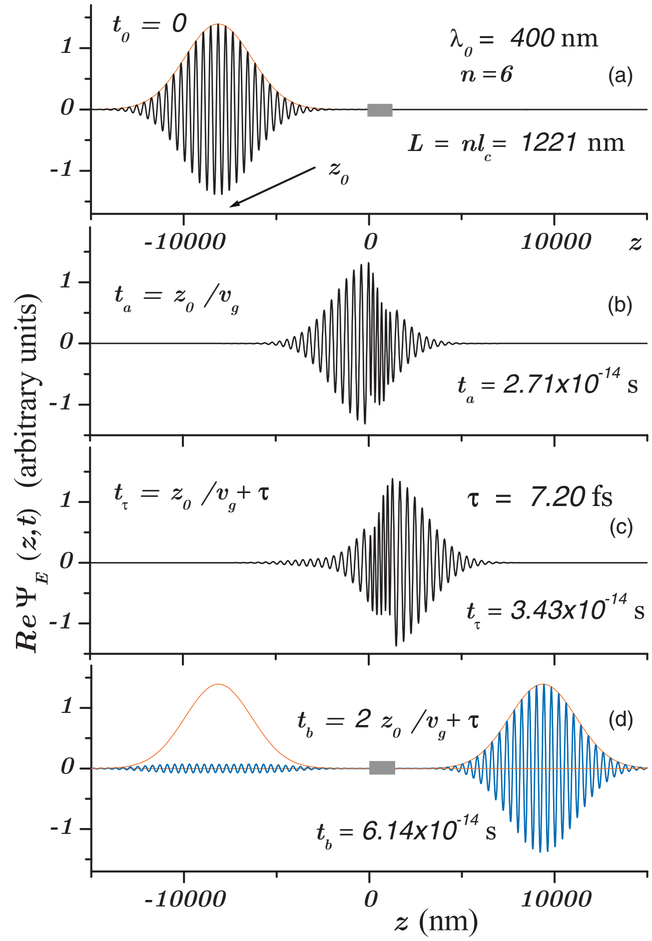

FIG. 5. (Color) Time series of a WP with centroid wavelength λ_0=400 nm. (a) The approaching WP located at $-z_0 = -40l_c$ at t =0. (b) The WP at $t_a = z_0/v_g$ when it just arrives at the OSL. (c) The WP when it just leaves the OSL of length L at $t_\tau = z_0/v_g + \tau$. (d) The WP at $t = 2z_0/v_g + \tau$ when it arrives at $L + z_0$.

in Fig. 2(e) with the purpose of looking closer at the transmitted part and its relation with the two Gaussian curves drawn on it [see inset Fig. 2(e)]: while the black curve peaks at $z_0 + c\tau$, the red curve does so at $z_0 + L$. If the WP moved inside the OSL with the speed of light c, the WP would coincide with the black curve. Otherwise, the WP will coincide with the red curve if it moves with the group velocity L/τ, as predicted by the phase time. It turns out that the latter is precisely the case; i.e., the phase time is compatible with the description of the electromagnetic theory. Besides all these, one can also look at the WP reshaping. Since the transmission times of the field components are almost constant, the scattered WP components are reshaped in only one reflected and one transmitted WP. When the WP is defined in a different frequency domain, different reshaping processes might occur [45].

We shall now study this example from the point of view of the OSL size. In Fig. 3 we show the transmission coefficients and phase times τ_n for n=3, 5, 7, and 9, for a WP and OSL as in Fig. 2, and in Fig. 4 we present snapshots of the corresponding transmitted WPs at $t_n = 2z_0/c + \tau_n$. For wavelengths in the band gap and for small number of cells, the transmission coefficients and the reshaped wave packets change rapidly with n, but the phase times remain practically constant. This is related to the Hartman effect. The color dots in Fig. 3 represent the times that would be spent inside the OSL by the WP peak if it were to move with the speed of light c. This time would increase linearly with n. The phase time predictions are smaller. Their precise values are indicated in each graph of Fig. 4.

Since the main purpose is to see the OSL size effect on the WP evolution, we consider the same WP and OSL as in Fig. 2, with the number of cells n as a varying parameter. For each value of n we evaluate the phase time τ_n and plot the transmitted WPs at $t_n = 2z_0/c + \tau_n$. We also plot in each graph two Gaussian curves: a black curve and a red curve. If the WPs moved inside the OSL with the speed of light c, they would have the black enveloping Gaussians curves located at $\zeta_n = z_0 + c\tau_n$, indicated by the dotted line for $n=3$ and by the left arrows pointing downward for other values of n in Figs. 4(b)–4(d). However, it is evident that the WPs are located farther away from ζ_n which means that (inside the OSL) the WPs move faster than the speed of light c. How fast do the WPs move? The red Gaussian curves are drawn by assuming that the WPs spent the phase time τ_n inside the OSL. It turns out that these curves, whose peaks are indicated by the right arrows pointing downward, at $z_n = \zeta_n + \Delta z_n$, are precisely the envelopes of the WPs described by the Maxwell equations. We can then conclude that by increasing the OSL size—i.e., increasing the number of cells—the WP velocity also increases. This is the Hartman effect observed by Spielman *et al.*

Finally, let us consider an example with physical conditions qualitatively different from the previous one. In Fig. 5 we have a WP with centroid wavelength $\lambda_0 = 400$ nm, at $t = 0$, t_a, t_τ, and t_b. As can be seen in Fig. 1, the OSL is transparent for $\lambda_0 = 400$ nm and wavelengths around this value. The transmission coefficients are close to 1 and the phase times of the order of 7.2 fs. This transmission time predicted by the phase time is much larger than the time that would be spent if the WP were to move with the speed of light c; i.e., from the point of view of the phase time, we do not expect superluminal velocities. The Gaussian curves on the left and right are drawn by assuming that the WPs move with the group velocity $v_g = L/\tau$, τ being the phase time. Again, these curves are the enveloping curves of the WPs described by the Maxwell equations. We conclude once more that the time spent in the OSL, predicted by the Maxwell equations, coincides with the phase time.

IV. CONCLUSION

We have presented a formalism for a precise evaluation of the time evolution of Gaussian wave packets through optical superlattices. We have shown that the time spent by a wave packet in the superlattice according to the Maxwell equations agrees perfectly with the phase time prediction and with the experimental delay time, implying superluminal velocities for wavelengths in a gap.

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