Self-organization of bouncing oil drops: Two-dimensional lattices and spinning clusters

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Multiple oil drops bouncing on the surface of a vertically vibrating bath of the same oil exhibit self-organization behavior in two dimensions [S. Protière, Y. Couder, E. Fort, and A. Boudaoud, J. Phys.: Condens. Matter **17**, S3529 (2005)]. We describe further the morphology and dynamic behavior of stable assemblies of large bouncing oil drops, for which we find that both the spacing and the lattice structure itself change with frequency, with variants of both square and hexagonal structures being observed. Large "rafts" of drops form soft triangular lattices with faceted boundaries. Small clusters of drops are unstable to coherent, collective spinning under certain driving conditions, manifesting spontaneous rotational symmetry breaking.

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Walker reported in 1978 that a drop of soap solution could be bounced repeatedly on a vertically vibrating bath of the same fluid [1]. In 2005, Couder *et al.* showed that similar experiments can be performed using silicon oil, reporting that the drops could be prevented from coalescing almost indefinitely if the bath was vibrated within a specified frequency and amplitude range [2]. They proposed that within this driving regime, which is well below the Faraday instability, the oil drop is given enough acceleration to lift briefly off the surface of the bath. This allows a thin lubricating layer of air trapped between the drop and the bath surface to be repeatedly refreshed between bounces (see [2,3], and references therein).

Couder and co-workers subsequently studied collections of drops of silicon oil with a variety of diameters, showing that they form beautiful triangular lattices ranging from close-packed to remarkably dilute, depending on the forcing amplitude. They mapped out the phase diagram for the bouncing behavior of small oil drops as a function of drop diameter and forcing amplitude, and observed that compact aggregates of large drops sometimes spin when driven at sufficiently high frequency [4,5].

Here we present an independent experimental study of the two-dimensional (2D) ordering of multiple bouncing drops of vegetable oil. The morphologies of 2D droplet assemblies are presented as a function of drop number for different driving conditions. We propose that each droplet generates a solitonlike perturbation of the surface that is rapidly damped as it propagates away from the point of impact. Measurements of drop separation vs driving frequency qualitatively confirm the idea suggested by Couder that droplets interact with their own collective standing wave, adaptively selecting their locations based on the local surface topography. The origins of the apparent long-range attraction between the drops are discussed and the effects of bath diameter and depth on the interactions of the drops described. Finally, observations of the spinning dynamics of small clusters suggests that spinning is a supercritical bifurcation similar to spontaneous parity-breaking rotations seen in other nonlinear condensed systems.

In our experiments, generic vegetable oil was contained in cylindrical glass beakers 4–8 cm in diameter and up to 4 cm in height. These were glued to a thin Perspex disk covering the top of a light metal cylinder about 15 cm in diameter, itself epoxied to the cone of an 8 in. loudspeaker (Roadmaster RSW80). The vibration of the oil bath was controlled using a function generator connected through a 1400 W audio amplifier (Pyle PLA2380). The driving frequency and amplitude were measured with a solid state accelerometer (iMEMS AD22281) attached to the oil bath. The bath was typically driven at frequencies in the range 35–130 Hz and accelerations of 7–30 g, conditions well below the onset of the well-known Faraday instability (see [6,7], and references therein). Drops of uniform size (~2.5 mm in diameter) were deposited on the surface of the bath using a syringe fitted with a 0.4-mm-diam needle. Images were recorded using a Ken-O-Vision KVF7100 video camera and a Kodak DX6490 digital still camera. Spinning drops were visualized with a stroboscope (General Radio Co. Strobotac Type 1531-A). All experiments were performed at room temperature.

When multiple drops bounce simultaneously on a vibrating oil bath they tend to cluster together, forming stable, self-organized structures. The spatial arrangement of the drops in two dimensions depends on the driving conditions, evolving subtly with frequency and the number of drops. At lower frequencies, the drops are far apart, as shown in Fig. 1, and in the $N=4$ case the lattice is almost square. Increasing the frequency causes the drops to move closer together and to form clusters with various interesting shapes that depend on their size. For example, at intermediate frequencies the central drops in the $N=6$ and $N=7$ clusters are clearly more closely spaced than those further out. At high frequencies, all droplet clusters tend to have similar close-packed, hexagonal lattice structures. The structures exemplified in Fig. 1 were stable over long periods of time; they could be obtained repeatably and the morphological transitions with frequency were reversible.

As the number of drops increases, higher driving frequencies are required to stabilize the resulting "raft." Large clusters are observed to form beautiful, crystal-like structures exhibiting facets (Fig. 2), but are soft in the sense that new drops can be added at the boundaries, or old ones translated by directed puffs of air, in order to change the shape of the raft. If the driving frequency is lowered or the raft is made too large by adding more drops, the structure may become unstable, with one or more of the drops undergoing large, lateral vibrations about its former equilibrium position [leading, for example, to the blurred image in Fig. 2(c)]. This

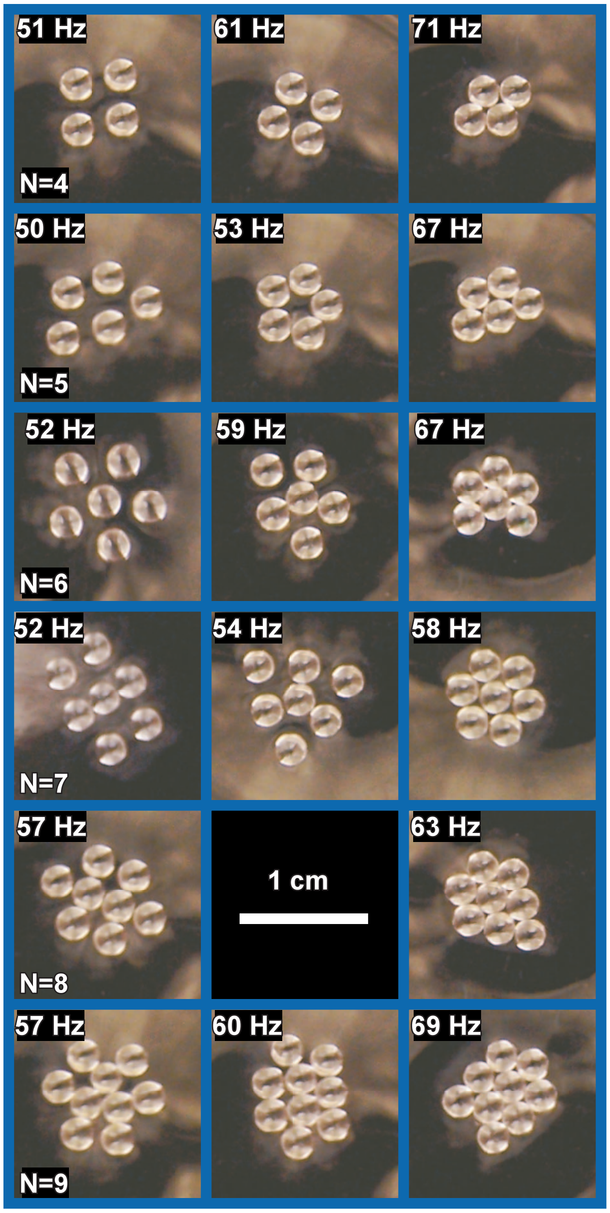

FIG. 1. (Color online) Stable clusters of bouncing oil drops. The spatial arrangement depends on drop number (increasing from top to bottom) and driving frequency (increasing from left to right). With increasing frequency, the drops tend to pack closer together, eventually forming a regular hexagonal lattice. These images show drops ($d=0.25$ cm) on a bath of vegetable oil ($H=2$ cm, $D=3$ cm), driven at 13.5 g.

frustration usually results in the coalescence of one or more drops with the bath, after which the remaining drops rearrange into a smaller, stable raft. The maximum raft size depends on bath depth and diameter as well as driving frequency, with larger rafts being easier to make on deeper, larger diameter baths.

Analogous structural changes of self-organized excitations when the control parameter is varied have been observed in a wide range of driven nonlinear condensed systems, from the surface patterns of a driven air-fluid interface [8] and oscillons in vibrated granular layers [9], to gas discharge systems [10]. The spatial extent of stable excitations in other fluid and granular systems similarly depends on the driving signal, with Faraday waves and oscillons in granular systems, for example, stable only in finite domains of the sample, behavior that has been reproduced theoretically [11].

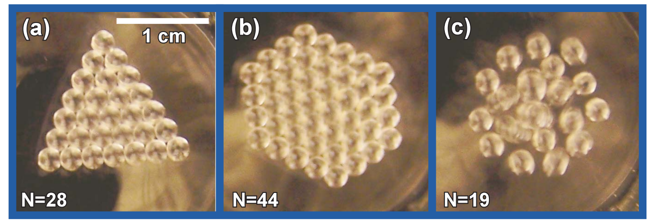

FIG. 2. (Color online) Self-organization of large rafts of bouncing oil drops packed in a hexagonal lattice and exhibiting crystal-like facets for (a) $N=28$ and (b) $N=44$ drops. (c) Local "melting" of the ordered structure occurs when the number of drops in the cluster exceeds a critical number ($N=19$ in this case), with several drops becoming violently agitated. As a result of this frustration, several of the drops soon coalesce with the bath, after which a smaller, more stable raft is formed. The rafts were created in the frequency range 70–100 Hz with a peak oil bath acceleration of 6–8 g ($d=2.5$ mm, $H=2$ cm, $D=3$ cm).

The preferred location of individual drops and clusters depends on the depth of oil in the container. With extremely shallow "baths" ($H\lesssim 2$ mm, for example, a glass coverslip coated with a thin film of oil), the drops hardly interact with each other, showing only a weak, short-range attraction, and at most forming linear chains. The interactions increase rapidly as the film thickness increases, with the drops eventually aggregating into 2D clusters in the usual way. In the case of deeper baths ($H\gtrsim 5$ mm), the drops clearly prefer to bounce near the center of the container, especially if the bath is of small diameter. The same positional tendency is observed when the driving frequency is raised or the number of drops in the cluster is increased.

We may ask whether these observations can be attributed to curvature of the oil surface. The positive meniscus at the container walls causes the surface of the oil bath to be slightly concave, which could account, in principle, for the movement of drops toward the center of the bath. However, the capillary length $\ell=(\sigma/g\rho)^{1/2}$, where σ and ρ are, respectively, the surface tension and density of the oil, is estimated to be only a few mm, so any residual surface curvature would be tiny near the center of the bath. Nevertheless, the apparent long-range centripetal force on drops is weaker in wider containers (consistent with smaller surface curvature), and on an essentially flat surface (such as the thin oil film on a microscope slide) the drops have no global positional preference at all.

Meniscus waves produced at the circular sidewalls of the bath may also contribute to repelling drops from the boundaries. The decay length of a surface wave with frequency ω and wave number k can be estimated [7] by

$$\ell_{\text{decay}} \approx \frac{(\omega/k)}{2(\eta/\rho)k^2} = \frac{\sigma}{2\eta\omega},$$

where η is the viscosity of the fluid. For a driving frequency of 60 Hz, the decay length of meniscus waves is approxi-

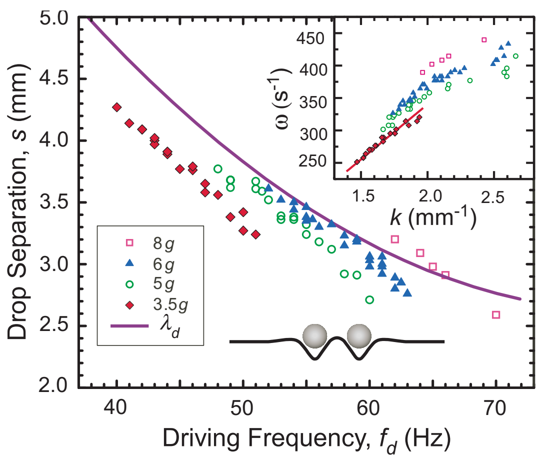

FIG. 3. (Color online) Drop separation vs driving frequency for a pair of drops (d=2.5 mm), bouncing on a bath of vegetable oil (H=2 cm, D=3 cm), vibrated with different amplitudes (shown in legend). The smooth curve shows the theoretical variation of the pulse train wavelength λ_d with driving frequency, assuming that the disturbances due to the drops travel at $c_{\text{min}}=19$ cm/s, the minimum speed of capillary-gravity waves when $\sigma=0.03$ mN/m and $\rho=920$ kg/m^3. The close agreement with experiment confirms that the drops are located at neigboring antinodes of their mutual standing wave. Inset: Dispersion relation $\omega=2\pi f_d$ vs $k=2\pi/s$. The average group velocity $d\omega/dk$ increases slightly with forcing amplitude, with a value of $c_{\text{min}}\sim 17$ cm/s for the fit shown, and increasing to about 19 cm/s for the largest forcing amplitude.

mately 0.5 mm. Even in our smallest diameter container (D=4 cm), the effects of any meniscus waves near the center of the bath should be negligible. In fact, no meniscus waves were ever observed in our experiments, even when the bath was driven above the Faraday instability.

As we saw in Fig. 1, the average spacing of drops in the clusters decreases with increasing frequency. We measured the separation s of a single pair of bouncing drops as a function of driving frequency f_d for several different forcing amplitudes (see Fig. 3). We will show below that the separation corresponds to the wavelength of a surface pulse train generated by the periodic impact of the falling drops and traveling approximately at c_{min}, the minimum speed given by the standard dispersion relation for capillary-gravity waves.

For impulsive waves created by small objects, the fluid surface naturally selects the components with minimum phase velocity [6,7]: waves of higher velocity are damped out quickly and may be ignored. The remaining disturbance is a nonlinear soliton which propagates at or near the minimum wave speed [12]. The speed c of a (sinusoidal) surface wave of wavelength λ is given by the usual dispersion relation

$$c=\sqrt{\left(\frac{g\lambda}{2\pi}+\frac{2\pi\sigma}{\rho\lambda}\right)\tanh\left[\frac{2\pi H}{\lambda}\right]}, \qquad (1)$$

where H is the depth of the fluid and g is the acceleration due to gravity [13]. When the bath depth $H \gtrsim 0.42\lambda$, then $\tanh(2\pi H/\lambda)\approx 1$ and the minimum wave speed derived from Eq. (1) is found to be

$$c_{\text{min}}=\left(\frac{4g\sigma}{\rho}\right)^{1/4}.$$

The wavelength of a periodic train of solitary waves generated by a drop bouncing at frequency f_d is then simply

$$\lambda_d=\frac{c_{\text{min}}}{f_d}.$$

The smooth curve in Fig. 3 shows the prediction of this model, assuming reasonable values for the density and surface tension of the oil. It is evident from the graph that the separation s between drops does indeed correspond to this forcing wavelength. The corresponding ω vs k plots in the inset of Fig. 3 show that the average group velocity varies slightly (by about 10%) with the forcing amplitude. This effect is obviously not accounted for in Eq. (1) but is broadly consistent with solitary wave behavior.

Couder *et al.* have suggested that the equilibrium spacing of separated drops is generally determined by damped standing surface waves produced by the bouncing drops themselves [4,5]. In our experiments, the drop pairs always impact the surface of the bath in phase. We propose that since two drops located at adjacent antinodes of their mutual standing wave (one wavelength apart) can continue to bounce in phase and with maximum amplitude, this will be their favored separation. We do not observe any other stable separations in our experiments with two drops (see Fig. 3), presumably because the standing wave is heavily damped at distances of much more than one wavelength.

Finally, we have observed that ordered droplet clusters spin spontaneously when driven above a minimum forcing amplitude and within a well-defined frequency range, indicated for a pair of drops in Fig. 4. In the spinning regime, the angular velocity (spinning speed) α can be controlled by varying either the driving frequency f_d or the forcing amplitude of the bath. Increasing the forcing amplitude always results in an increase in the spinning speed, at least until the Faraday instability sets in, after which the drops move around rather chaotically and soon disappear, making any meaningful measurements in this regime difficult. At fixed amplitude, a decrease in driving frequency below the upper threshold f_{c1} initially causes an increase in α, until a maximum spinning speed is reached, as shown in Fig. 5. Lowering the frequency further results in a steady reduction of the spinning speed, until finally the cluster returns to a stationary state at f_{c2} (unless this is preempted, for example, in a deeper bath by the onset of the Faraday instability, as is the case in Fig. 4). The spinning speed varies as the square root of the reduced driving frequency, i.e., $\alpha \propto |f_d-f_c|^{1/2}$, where f_c represents f_{c1} or f_{c2}. Analogous behavior has been reported in several condensed systems that exhibit similar parity-breaking bifurcations, for example, in rotating surface patterns of a driven air-fluid interface [8], the drifting of oil fingers between two rotating cylinders [14], the rotation of cellular flames [15], and rotating bound states in gas discharge systems [16]. In the experiments on fluids, the drift velocity is also reported to increase as the square root of a control parameter related to the forcing signal, and a rotational bifurcation has recently been predicted theoretically in

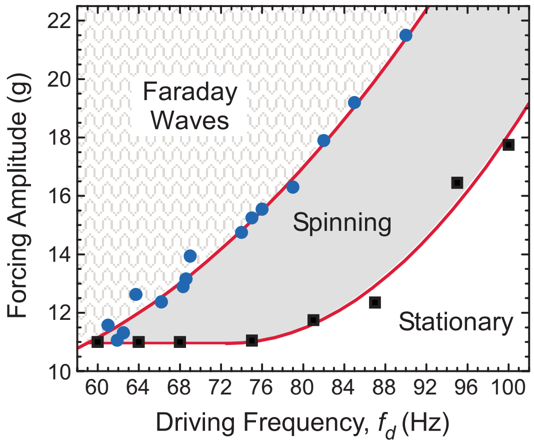

FIG. 4. (Color online) Spinning instability of drop pairs. Clusters of bouncing oil drops are unstable to collective spinning within a well-defined forcing frequency and amplitude regime, depicted here for a pair of drops ($N=2$, $d=2.5$ mm). The clusters can be made to stop and start spinning by changing the driving frequency, rotating fastest near the middle of the spinning regime. The direction of spin can be reversed by carefully directing short puffs of air at the cluster. In this case (with a bath $D=2.5$ cm in diameter and $H=2$ cm deep), as the driving frequency is lowered, spinning is eventually perturbed by the onset of the Faraday instability but in shallower baths the spinning regime is bounded on the low frequency side also by a distinct stationary region (shown, for example, in Fig. 5). Regardless of driving frequency, drop pairs could not be made to spin if the bath was driven at less than about 11 g.

computer simulations of reaction-diffusion systems [17]. Interestingly, Couder and co-workers have recently reported a drift bifurcation in the translational motion of "walking" drops [5].

We note that no collision or previous motion of drop pairs or clusters was necessary for rotation to occur in our experiments (in contrast to Couder's early observations of the orbiting behavior of walking drops [18]): stationary clusters start or stop spinning spontaneously when the driving frequency is changed. The initial direction of spin is arbitrary and judiciously directed puffs of air from a pipette can be

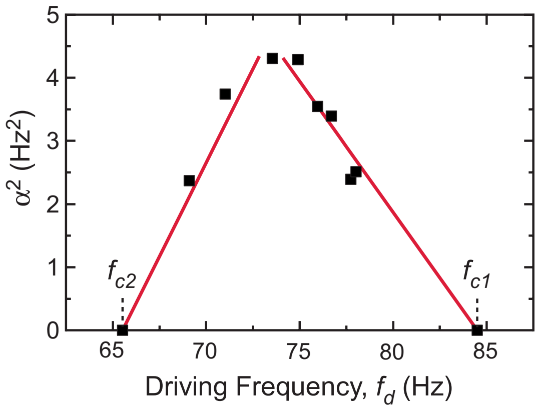

FIG. 5. (Color online) The angular velocity α of a spinning pair of drops depends on reduced driving frequency as $\alpha \propto |f_d - f_c|^{1/2}$, where f_c marks the onset of spinning. These experimental data correspond to a pair of drops ($d=2.5$ mm, $N=2$), bouncing on a bath of vegetable oil ($H=1.5$ cm, $D=3$ cm), driven at 23 g.

used to reverse the rotation direction of an already spinning cluster.

In summary, oil drops bouncing on a vibrating bath form stable, ordered structures in two dimensions. The appearance of these droplet clusters can be controlled by varying the driving frequency and amplitude. The separation of a pair of drops corresponds approximately to the wavelength of a periodic train of surface disturbances traveling with the minimum speed predicted by the standard dispersion relation for capillary-gravity waves. We propose that each falling drop causes a local disturbance of the oil bath that propagates as a heavily damped solitary wave. The droplets are preferentially located at antinodes of the standing wave caused by all of the drops. Under certain driving conditions a parity-breaking spinning instability occurs, in which clusters of bouncing drops start rotating spontaneously.

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