

Stopping and time reversal of light in dynamic photonic structures via Bloch oscillations

Stefano Longhi

Dipartimento di Fisica and Istituto di Fotonica e Nanotecnologie del CNR, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milan, Italy

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It is theoretically shown that storage and time reversal of light pulses can be achieved in a coupled-resonator optical waveguide by dynamic tuning of the cavity resonances without maintaining the translational invariance of the system. The control exploits the Bloch oscillation motion of a light pulse in the presence of a refractive index ramp, and it is therefore rather different from the mechanism of adiabatic band compression and reversal proposed by Yanik and Fan in recent works [Phys. Rev. Lett., **92**, 083901 (2004); **93**, 173903 (2004)].

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The possibility of dynamically controlling the resonant properties of microresonator systems via small refractive index modulation represents a promising and powerful approach for an all-optical coherent control of light in nanophotonic structures [1–5]. Recently, several theoretical papers have shown that a temporal modulation of the refractive index in photonic crystals (PC's) and coupled-resonator optical waveguides (CROW's) can be exploited to coherently and reversibly control the spectrum of light, with important applications such as all-optical storage of light pulses [1,3], time reversal [2], and wavelength conversion [5,6]. The existence of a frequency shift in the spectrum of a light pulse reflected by a shock-wave front traveling in a PC was pointed out by Reed *et al.* [7,8], and adiabatic wavelength conversion by simple dynamic refractive index tuning of a high- Q microcavity in a PC has been numerically demonstrated in Ref. [5]. In a series of recent papers, Yanik and Fan showed that an adiabatic and translationally invariant tuning of the refractive index in a waveguide-resonator system can be exploited to stop, store, and time-reverse light pulses [1–3,9]. The general conditions requested to coherently stop or reverse light pulses have been stated in Refs. [4,9], and the possibility of overcoming the fundamental bandwidth-delay constraint of static resonator structures has been pointed out. The basic idea of these previous papers is that the band structure of a translationally invariant waveguide-resonator system can be dynamically modified by a proper tuning of the refractive index *without breaking the translational invariance of the system*. For instance, stopping a light pulse corresponds to an adiabatic band compression process: an initial state of the system, having a relatively wide band to accommodate the incoming pulse, adiabatically evolves toward a final state in which the bandwidth shrinks to zero [4]. In practice, the adiabatic evolution is attained by a slow change of the refractive index of certain cavities forming the photonic structure [1–3]. The condition that the dynamic refractive index change not break the translational invariance of the system is important because it ensures that (i) the system can be described in terms of a band diagram with a dispersion relation $\omega = \omega(k)$ relating the frequency ω and the wave vector k of its eigenmodes and (ii) different wave vector components of the pulse are not mixed, so that all the coherent information encoded in the original pulse is maintained while its spectrum is adiabatically changed [4,9].

In this work it is shown that a coherent and reversible control of light in a photonic structure by dynamic refractive index change does not necessarily require one to maintain the translational invariance of the system. We illustrate this by demonstrating the possibility of stopping and time-reversing light pulses in a CROW [10–12] with a dynamic refractive index gradient. In this system, light stopping and reversal is not due to adiabatic shrinking and reversal of the waveguide band structure, as in Refs. [1,2], but it is a consequence of the coherent Bloch oscillation (BO) motion of the light pulse induced by the index gradient. It is remarkable that, though temporal [13–15] and spatial [16,17] BO's and related phenomena have been studied to a great extent in several linear optical systems, they have not yet been proposed as an all-optical means to stop or time-reverse light pulses.

We consider a CROW made of a periodic array of identical coupled optical cavities and indicate by $\omega_n = \omega_0 + \delta\omega_n(t)$ the resonance frequency of the n th cavity in the array, where $\delta\omega_n(t)$ is a small frequency shift from the common frequency ω_0 which can be dynamically and externally changed by, e.g., local refractive index control, as discussed in previous works [1,5]. Practical implementations of CROW structures have been demonstrated in photonic crystals with coupled defect cavities [12,18] or in a chain of coupled microrings [19]. In most cases, coupled-mode theory [1,11,20] can be used to describe the evolution of the field amplitudes a_n in the cavities and therefore the process of coherent light control; the results obtained from coupled-mode theory have been shown in fact to be in excellent agreement with full numerical simulations using finite-difference time-domain methods (see, for instance, [1,2]). For our system, the coupled-mode equations read

$$i \frac{da_n}{dt} = -\kappa(a_{n-1} + a_{n+1}) - \delta\omega_n(t)a_n, \quad (1)$$

where κ is the hopping amplitude between two adjacent cavities, which defines the bandwidth (4κ) of the CROW. Note that cavity losses are not included in Eq. (1); however a nonvanishing loss rate would just introduce a uniform exponential decay in time of a_n which would set a maximum limit to the achievable delay time, as discussed in Ref. [1]. As in Refs. [1–4], field propagation is considered at a classical

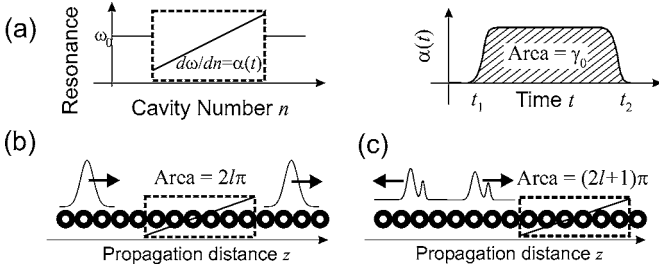


FIG. 1. Schematic of a dynamic CROW with a linear gradient of resonances. (a) Distribution of cavity resonances (left) and temporal behavior of gradient amplitude $\alpha(t)$ (right). (b) Process of pulse storage. (c) Process of time reversal.

level; a full quantum treatment, which would require the introduction of noise sources in Eq. (1) to account for quantum noise, is not necessary for the present analysis which deals with passive CROW structures. Contrary to Refs. [1,2], we assume that the modulation of cavity resonances used for coherent light control is not translational invariant; i.e., $\delta\omega_n$ depends on n . Precisely, we assume that a ramp with a time-varying slope $\alpha(t)$ is imposed onto the resonances of N adjacent cavities in the CROW, leading to a site-dependent frequency shift $\delta\omega_n(t) = n\alpha(t)$ for $1 \leq n \leq N$ and $\delta\omega_n(t) = 0$ for $n > N$ and for $n < 1$. Note that the dynamic part of the CROW which realizes stopping or time reversal of light is confined in the region $1 < n < N$, which is indicated by a rectangular dotted box in Fig. 1. The total length of the system realizing stopping or time reversal of light is therefore $L = Nd$, where d is the distance between two adjacent cavities. The modulation $\alpha(t)$ is assumed to vanish for $t < t_1$ and $t > t_2$ [see Fig. 1(a)], $[t_1, t_2]$ being the time interval needed to stop or time-reverse an incoming pulse. The switch-on time t_1 is chosen just after the pulse, propagating along the CROW and coming from $n = -\infty$, is fully entered in the dynamic part of the CROW, whereas the length L is chosen long enough to ensure that the pulse remains fully confined in the cavities $1 < n < N$ for the whole time interval $[t_1, t_2]$. Note that as for $t < t_1$ and $t > t_2$ the pulse propagates in the CROW at a constant group velocity, during the time interval $[t_1, t_2]$ the pulse motion as ruled by Eq. (1) is more involved and turns out to be fully analogous to the motion of a Bloch particle, within a tight-binding model, subjected to a time-dependent field $\alpha(t)$ (see, e.g., [21–23]). It is indeed such a Bloch motion that can be properly exploited to stop or time-reverse an incoming pulse. In fact, let us suppose that the incoming light pulse, propagating in the forward direction of the waveguide and coming from $n = -\infty$, has a carrier frequency tuned at the middle of the CROW transmission band and its spectral extension is smaller than the CROW bandwidth 4κ . For $t < t_1$ one can then write (see the Appendix for technical details)

$$a_n(t) = \int_{-\pi}^{\pi} dQ F(Q) \exp(iQn + 2i\kappa t \cos Q), \quad (2)$$

where the spectrum $F(Q)$ is nonvanishing in a small region at around $Q = Q_0 = \pi/2$. The shape of $F(Q)$ is determined

from the excitation condition of the CROW at $n \rightarrow -\infty$ or, equivalently, from the field distribution $a_n(t_0)$ along the CROW at a given initial time $t = t_0 < t_1$. For instance, assuming without loss of generality $t_0 = 0$, the latter condition yields for the spectrum $F(Q) = 1/(2\pi) \sum_n a_n(0) \exp(-iQn)$ (see the Appendix). For $t_0 < t < t_1$, Eq. (2) describes a pulse which propagates along the waveguide with a group velocity $v_g = 2d\kappa \sin Q_0 = 2d\kappa$. At time $t = t_1$, we assume that the pulse is fully entered in the box system of Fig. 1, and the modulation of cavity resonances is then switched on. The *exact* solution to Eq. (1), which is the continuation of Eq. (2) at times $t > t_1$, can be calculated in a closed form and reads (see the Appendix)

$$a_n(t) = \exp[i\gamma(t)n] \int_{-\pi}^{\pi} dQ F(Q) \exp[iQn + i\theta(Q, t)], \quad (3)$$

where we have set $\gamma(t) = \int_{t_1}^t dt' \alpha(t')$ and $\theta(Q, t) = 2\kappa \int_{t_1}^t dt' \cos[Q + \gamma(t')] + 2\kappa t_1 \cos(Q)$. At time $t_2 = t_1 + \tau$, the modulation is switched off, and for $t > t_2$ one then has

$$a_n(t) = \exp(i\gamma_0 n) \int_{-\pi}^{\pi} dQ F(Q) \exp[i\phi(Q) + i\theta(Q)] \times \exp[iQn + 2i\kappa t \cos(Q + \gamma_0)], \quad (4)$$

where we have set $\gamma_0 = \int_{t_1}^{t_2} dt \alpha(t)$, $\phi(Q) = 2\kappa t_1 \cos Q - 2\kappa t_2 \cos(Q + \gamma_0)$, and $\theta(Q) = \theta(Q, t_2) = 2\kappa \int_{t_1}^{t_2} dt \cos[Q + \gamma(t)]$. The modulation parameters are chosen to either store or time-reverse the incoming pulse (see Fig. 1). In both cases, we assume that the length L of the system is large enough to entirely contain the pulse in the whole interval $[t_1, t_2]$. In case of pulse storage, after the modulation is switched off the pulse escapes from the system in the forward direction with the same group velocity $v_g = 2d\kappa$ as that of the incoming pulse, but it is delayed by a time $\sim \tau$ [see Fig. 1(b)]. In the case of time reversal [Fig. 1(c)], the incoming pulse is reflected from the system, which thus acts as a phase-conjugation mirror.

Consider first the process of *pulse storage*. To this aim, let us assume that the *area* γ_0 is an integer multiple of 2π . In this case, for $t > t_2$ from Eq. (4) one obtains

$$a_n(t) = \int_{-\pi}^{\pi} dQ F(Q) \exp[i\phi(Q)] \exp[iQn + 2i\kappa(t - \tau) \cos Q]. \quad (5)$$

A comparison of Eqs. (2) and (5) clearly shows that, if $\phi = 0$, the effect of the modulation is that of storing the pulse for a time $\tau = t_2 - t_1$ *without introducing any distortion*: in fact, one has $a_n(t_2) = a_n(t_1)$. If the area is an integer multiple of 2π but $\phi \neq 0$, the additional phase $\phi(Q)$ may introduce a non-negligible pulse distortion. The distortionless condition $\phi = 0$ is exactly satisfied in two important cases: a stepwise modulation $\alpha(t) = \alpha_0$ (with $\alpha_0 \tau = 2\pi l$, l an integer) and a sinusoidal modulation $\alpha(t) = \alpha_0 \cos(\Omega t)$, with $\tau \Omega = 2\pi l$ and $J_0(\alpha_0/\Omega) = 0$. These two cases realize the well-known dc or ac BO motion [21–23] of the light pulse in the interval $[t_1, t_2]$: pulse storage is therefore due to the periodic motion

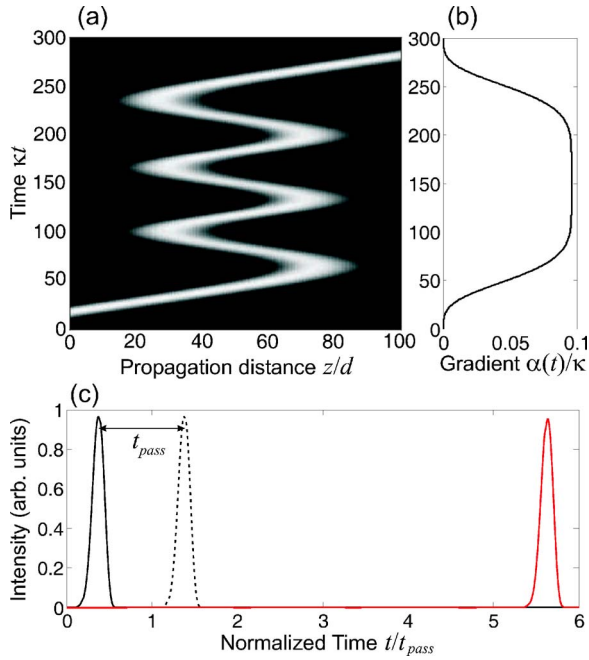


FIG. 2. (Color online) Storage of a Gaussian pulse in a coupled resonator waveguide consisting of $N=100$ cavities. (a) Gray-scale plot showing the space-time pulse intensity evolution (note the characteristic BO motion). (b) Profile of the applied modulation $\alpha(t)=\alpha_0 \exp\{-(\kappa t-150)/\tau_0\}^6$ with $\alpha_0/\kappa=0.0958$ and $\tau_0=106$, corresponding to an area $\gamma_0=6\pi$. (c) Process of pulse storage: the solid black curve is the intensity profile of the incoming pulse as recorded in the first cavity of the waveguide ($z=0$), whereas the solid red curve and dashed black curve are the intensity profiles of the outgoing pulse as recorded in the last cavity of the waveguide ($z=L=100d$) in the presence and in the absence of the modulation, respectively. In (c) time is normalized to the transit time $t_{pass}=N/(2\kappa)=50/\kappa$ of the pulse in the system.

of the light pulse which returns to its initial position after each BO (or ac field) period. It is worth commenting more deeply on the very different mechanisms underlying light stopping in the translationally invariant system of Ref. [1] with the one considered in the present work. In Ref. [1], the modulation of cavity resonances preserves the translational symmetry and, as a consequence, cross talk between different wave vector components of the pulse is prevented as the waveguide band accommodating the pulse shrinks to zero and the pulse group velocity adiabatically decreases. Additionally, the tuning process must be slow enough to ensure reversibility—i.e., to ensure the validity of the adiabatic theorem. In the present work, the tuning of the cavity resonances breaks the translational symmetry of the system and the different wave vector components Q of the pulse undergo a drift motion in the reciprocal space according to the well-known “acceleration theorem” of a Bloch particle studied in solid-state physics [22] (see the Appendix for more details). The motion of Q in the reciprocal space is accompanied by a shift of the pulse carrier frequency, which spans in a periodic fashion the full band of the waveguide, and by a periodic motion of the pulse in the “stopping box” of Fig. 1, which is hence trapped inside it [see Fig. 2(a), to be discussed later]. In particular, for a stepwise modulation $\alpha(t)$ which will be

mainly considered in this work [24], the temporal periodicity of the motion is $\tau_B=2\pi/\alpha_0$. This is the well-known periodic Bloch motion which is related to the existence for Eq. (1) of a discrete Wannier-Stark ladder spectrum instead of a continuous band spectrum (for more details see, for instance, [22]). Therefore, as in the translationally invariant waveguide system of Ref. [1] light stopping is achieved by adiabatically shrinking to zero the band of the pulse; in our system light stopping can be viewed as a trapping effect due to the appearance of the periodic Bloch motion in the dynamic part of the CROW structure. Note that, as opposed to the method of Ref. [1], in our case adiabaticity of the tuning process is not required; however, the stopping time τ is quantized since it must be an integer multiple of the Bloch period τ_B . Nevertheless, with a suitable choice of the gradient α_0 (and hence of τ_B), a target delay time τ can be achieved. If τ_p is the duration of the incoming pulse to be delayed (with $\tau_p < 1/\kappa$), we can estimate the minimum length L of the system as $L=L_p+L_b$, where $L_p \approx \tau_p v_g = 2d\kappa\tau_p$ is the spatial extension of the pulse in the waveguide in the absence of the modulation and $L_b \approx 4\kappa d/\alpha_0$ is the amplitude of the BO motion. Hence the minimum number of cavities of the system is given by $N=L/d \approx 2\kappa(\tau_p+2/\alpha_0)$. It should be noted that in practice a sharp stepwise modulation can never be realized and a finite rise time during switch on and off should be accounted for. Though $\phi(Q)$ does not exactly vanish in this case, dispersive effects can be kept, however, at a small level. As an example, Fig. 2 shows the process of light storage as obtained by a direct numerical simulation of Eq. (1) using a super-Gaussian profile for the gradient $\alpha(t)$. The area γ_0 is chosen to be 6π , so that pulse trapping corresponds to three BO periods, as clearly shown in the space-time plot of Fig. 2(a). Note that the system comprises $N=100$ cavities, and therefore the length L of the waveguide needed to perform light storage is $L=Nd=100d$. The intensity profile of the nearly Gaussian-shaped incoming pulse in the initial cavity ($z=0$) is indicated by the black solid line in Fig. 2(c). In the figure, the intensity profile of the outgoing pulse at the last cavity of the system ($z=100d$) is shown by the red solid line, whereas the dashed curve indicates the intensity profile of the output pulse at the last cavity in the absence of index gradient—i.e., when the pulse freely propagates along the system at the group velocity $v_g=2\kappa d$. In Figs. 2(a) and 2(b), time is normalized to $1/\kappa$, whereas in Fig. 2(c) time is normalized to the transit time $t_{pass}=L/v_g=N/(2\kappa)$ of the pulse in the system. Note that the maximum frequency shift of cavity resonance needed to achieve the process of pulse storage is $\delta\omega_{max}=\pm(N/2)\alpha_0 \sim \pm 5\kappa$. Assuming that a change δn of the refractive index n produces a change $\delta\omega \sim \omega_0(\delta n/n)$ of the cavity resonance ω_0 , the index ramp of Fig. 2(b) thus corresponds to a maximum refractive index change $\delta n/n \sim 5\kappa/\omega_0$. This value is comparable to the one requested for light stopping by means of adiabatic band compression in the translationally invariant case [1]. To get an idea of typical values in real physical units, let us assume, e.g., a carrier angular frequency $\omega_0 \approx 1.216 \times 10^{15}$ rad/s (corresponding to a wavelength $\lambda \approx 1.55 \mu\text{m}$) and a maximum index change $\delta n/n \sim 5 \times 10^{-4}$, which is comparable to the one used in previous studies (see, e.g., [1,5]). The bandwidth 2κ of the

waveguide and the transit time t_{pass} in the figure are then given by $2\kappa \sim 2 \times 10^{-4} \omega_0 \sim 2.43 \times 10^{11}$ rad/s (i.e., ~ 39 GHz) and $t_{pass} = N/(2\kappa) \approx 410$ ps, respectively. For such parameter values, Fig. 1 simulates the stopping of a ~ 68 -ps-long [full width at half maximum (FWHM)] Gaussian pulse with a storage time $\tau \sim 1.75$ ns.

The process of *time reversal* of a light pulse is simply achieved when the *area* γ_0 is equal to π , apart from integer multiples of 2π . In fact, in this case for $t > t_2$ from Eq. (4) one obtains

$$a_n(t) = (-1)^n \int_{-\pi}^{\pi} dQ F(Q) \exp[i\phi(Q)] \times \exp[iQn - 2i\kappa(t - t_1 - t_2) \cos Q]. \quad (6)$$

A comparison of Eqs. (2) and (6) clearly shows the sign reversal of the frequency $2\kappa \cos Q$ for any wave number Q in the integral term, which is the signature of time reversal of the pulse. Physically, time reversal is due to the fact that, for the π area, the spectrum of the wave packet in the reciprocal Q space (quasimomentum) is shifted in the Brillouin zone from $Q_0 = \pi/2$ to $Q_0 = -\pi/2$, thus producing spectral inversion. In addition, since the group velocity is correspondingly reversed, the pulse is reflected by the system and thus propagates backward. As in the previous case, the process of time reversal does not introduce pulse distortion provided that $\phi = 0$. For a stepwise modulation, contrary to the 2π area case $\phi(Q)$ does not vanish and one has $\phi(Q) = -(4\kappa/\alpha_0) \sin Q$. However, this additional phase term may be kept small by choosing, e.g., a sufficiently large value of α_0 , thus minimizing pulse distortion. An example of time reversal of an asymmetric pulse with minimal distortion, as obtained by a direct numerical simulation of Eq. (1) using a super-Gaussian profile for the gradient $\alpha(t)$, is shown in Fig. 3. Note that in this case the pulse undergoes a semi-integer number of BO periods.

In conclusion, it has been theoretically shown that storage and time reversal of light can be realized by exploiting BO's in a dynamic coupled-resonator waveguide. The proposed scheme is rather distinct from the adiabatic band compression technique recently proposed in Refs. [1,2] and provides a noteworthy example of coherent light control in a system with broken translational invariance.

APPENDIX

In this appendix we provide a detailed derivation of the solution to the coupled-mode equation (1) at times $t < t_1$, $t_1 < t < t_2$, and $t > t_2$ presented in the text [Eqs. (2)–(4)]. To this aim, we follow a rather standard technique (see, for instance, [21]) and introduce the time-varying Fourier spectrum $G(Q, t)$ defined by the relation

$$G(Q, t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} a_n(t) \exp(-iQn). \quad (A1)$$

The amplitudes $a_n(t)$ can be derived from the spectrum $G(Q, t)$ after inversion according to the relation

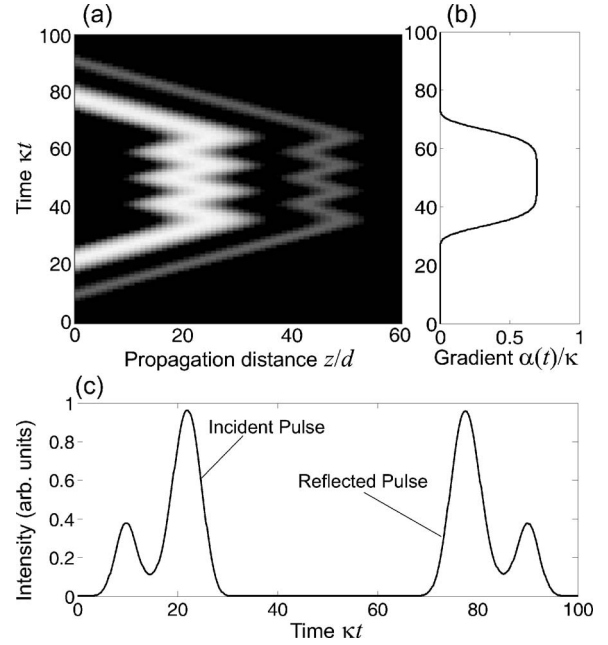


FIG. 3. Time reversal of a nonsymmetric double-peaked optical pulse in a coupled resonator optical waveguide consisting of $N = 60$ cavities. (a) Gray-scale plot showing the space-time pulse intensity evolution. (b) Profile of the applied modulation $\alpha(t) = \alpha_0 \exp\{-[(\kappa t - 50)/\tau_0]^6\}$ with $\alpha_0/\kappa = 0.7$ and $\tau_0 = 17$, corresponding to an area $\gamma_0 = 7\pi$. (c) Pulse intensity profile versus scaled time κt as recorded in the first cavity of the waveguide ($z = 0$).

$$a_n(t) = \int_{-\pi}^{\pi} dQ G(Q, t) \exp(iQn). \quad (A2)$$

Using the coupled-mode equation (1) with $\delta\omega_n(t) = \alpha(t)n$, the following differential equation for the spectrum G can be easily derived [25]:

$$\frac{\partial G}{\partial t} + \alpha \frac{\partial G}{\partial Q} = 2i\kappa G \cos Q. \quad (A3)$$

For $t < t_1$, we have $\alpha(t) = 0$, and therefore the solution to Eq. (A3) is simply given by

$$G(Q, t) = F(Q) \exp(2i\kappa t \cos Q) \quad (t < t_1), \quad (A4)$$

where the profile $F(Q)$ is determined by the spectrum G at an initial time $t = t_0$ by means of Eq. (A1) once the field distribution $a_n(t_0)$ is assigned. Note that substitution of Eq. (A4) into Eq. (A2) yields Eq. (2) given in the text.

For $t > t_1$, $\alpha(t)$ is nonvanishing and the solution to Eq. (A3), which is a continuation of Eq. (A4) for times $t > t_1$, can be easily obtained after the change of variables $\eta = t$ and $\xi = Q - \gamma(t)$, where we have set

$$\gamma(t) = \int_{t_1}^t dt' \alpha(t'). \quad (A5)$$

With these new variables, Eq. (A3) is transformed into the equation

$$\frac{\partial G(\xi, \eta)}{\partial \eta} = 2i\kappa G(\xi, \eta) \cos[\xi + \gamma(\eta)], \quad (\text{A6})$$

which can be easily integrated with the initial condition $G(\xi, \eta=t_1) = F(\xi) \exp(2i\kappa t_1 \cos \xi)$. Upon reintroducing the old variables Q and t , one then obtains

$$G(Q, t) = F(Q - \gamma(t)) \exp\{2i\kappa t_1 \cos[Q - \gamma(t)]\} \\ \times \exp\left\{2i\kappa \int_{t_1}^t dt' \cos[Q + \gamma(t') - \gamma(t)]\right\} \quad (t > t_1). \quad (\text{A7})$$

Substituting Eq. (A7) into Eq. (A2), after the change of integration variable $Q' = Q - \gamma(t)$ and taking into account the 2π periodicity of the spectrum $G(Q, t)$ with respect to the variable Q , one then readily obtains Eq. (3) given in the text. Note that $|G(Q, t)|^2 = |F(Q - \gamma(t))|^2$; i.e., the role of the gradient $\alpha(t)$ is to induce a rigid drift of the initial spectrum, a

result which is known as the ‘‘acceleration theorem’’ in the solid-state physics context [22]. In particular, for a constant gradient $\alpha(t) = \alpha_0$, the drift of the spectrum is uniform in time. In this case, from Eq. (A5) it follows that, after a time $\tau_B = 2\pi/\alpha_0$ from the initial time $t = t_1$, one has $\gamma(t_1 + \tau_B) = 2\pi$ and, from Eq. (A7), $G(Q, t_1 + \tau_B) = G(Q, t_1)$; i.e., the initial field distribution in the CROW structure is retrieved: τ_B plays the role of the BO period which is determined by the gradient α_0 .

For $t > t_2$, one has $\alpha(t) = 0$ and the spectrum $G(Q, t)$ is still given by Eq. (A7), where according to Eq. (A5) one has $\gamma(t) = \int_{t_1}^t dt' \alpha(t') \equiv \gamma_0$ for $t > t_2$. Note that for $t > t_2$ Eq. (A7) can be cast in the following form:

$$G(Q, t) = G(Q, t_2) \exp[2i\kappa(t - t_2) \cos Q] \quad (t > t_2). \quad (\text{A8})$$

Substitution of Eq. (A8) into Eq. (A2), with $G(Q, t_2)$ given by Eq. (A7) with $t = t_2$, and after the change of variable $Q' = Q - \gamma_0$ in the integral of Eq. (A2), one finally obtains Eq. (4) given in the text for the solution at times $t > t_2$.

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 [24] For a sinusoidal modulation, Eq. (1) admits of a quasienergy band and the condition $J_0(\alpha_0/\Omega) = 0$ corresponds to the collapse of the quasienergy band [M. Holthaus, Phys. Rev. Lett. **69**, 351 (1992)]. More generally, for a fast sinusoidal modulation $\alpha(t) = \alpha_0(t) \sin(\Omega t)$ of frequency Ω much larger than κ and amplitude α_0 which is slowly varying in time, by a multiple-scale analysis one can remove the rapidly varying terms in Eq. (1) and obtain the reduced equations $ida_n/dt = -\kappa_e(t)(a_{n+1} + a_{n-1})$ with an effective slowly varying hopping amplitude $\kappa_e = \kappa J_0(\alpha_0/\Omega)$ [see, e. g., S. Longhi, Phys. Rev. B **73**, 193305 (2006)]. A slow change of α_0 leads to an adiabatic change of the bandwidth of the waveguide. Light slowing down and stopping in this case is thus analogous to adiabatic band compression of Ref. [1]. The requested modulation frequency Ω , however, turns out to be too high (larger than κ) to be reasonably achieved in practice.
 [25] In deriving Eq. (A3) we assumed that the relation $\omega_n = n\alpha(t)$ holds even for $n < 1$ and $n > N$, though in practice the dynamic tuning of cavity resonances is applied solely for the cavities with index n in the range $1 < n < N$ (see Fig. 1). However, taking into account that $\alpha(t)$ vanishes outside the interval $[t_1, t_2]$ and that the size N of the stopping box is large enough to fully contain the pulse during the delay interval—i.e., $a_n(t) = 0$ for $n < 1$, $n > N$ in the time interval $t_1 < t < t_2$ —the previous assumption does not change the solution of the problem.