Dual synchronization of chaos in Mackey-Glass electronic circuits with time-delayed feedback

Satoshi Sano,[1] Atsushi Uchida,[1,2] Shigeru Yoshimori,[1] and Rajarshi Roy[2,3,4]
[1]*Department of Electronics and Computer Systems, Takushoku University, 815-1 Tatemachi, Hachioji, Tokyo 193-0985, Japan*
[2]*IREAP, University of Maryland, College Park, Maryland 20742, USA*
[3]*IPST, University of Maryland, College Park, Maryland 20742, USA*
[4]*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
(Received 18 August 2006; published 12 January 2007)

We experimentally and numerically demonstrate the dual synchronization of chaos in two pairs of one-way-coupled Mackey-Glass electronic circuits with time-delayed feedback. The outputs of the two drive circuits are mixed and used both for the feedback signal to the two drive circuits and for the transmission signal to the two response circuits. We investigate the regions for achieving dual synchronization of chaos when the delay time is mismatched between the drive and response circuits.

DOI: 10.1103/PhysRevE.75.016207 PACS number(s): 05.45.Xt

I. INTRODUCTION

Chaos synchronization has attracted increased interest for the applications of secure communications [1,2] and spread-spectrum communications [3]. Chaos synchronization can be used for sharing identical chaos in a transmitter and a receiver as a cryptographical code. Many studies on synchronization of chaos have been reported in one-way-coupled chaotic systems [1,2]. However, since the configuration of synchronization is limited to a single pair of one-way-coupled oscillators, this method cannot be applied for multiuser communication systems [4].

The technique of multiplexing is a very important issue for high-capacity communications [4]. Multiplexing chaos using synchronization has been reported in a simple map and electronic circuit model [5]. Dual synchronization of chaos for synchronizing two different pairs of chaotic maps and delay-differential equations has also been investigated [6]. Dual synchronization of chaos is a method to separate two mixed chaotic signals by using synchronization [6–8]. To synchronize each pair of chaotic systems, all the parameter settings must be identical between the transmitter and the receiver, whereas they must be slightly shifted between different pairs of chaotic systems. We have experimentally and numerically demonstrated dual synchronization of chaos in both microchip solid-state lasers and Colpitts electronic oscillators [7,8]. A similar scheme has been demonstrated in Chua circuits and Kennedy oscillators [9]. In the scheme of Ref. [8] the signals from both of the two response circuits are required to separate one of the drive signals from the other drive signal. All the response systems need to be synchronized to achieve dual synchronization [8].

In this paper, we propose a scheme for dual synchronization by introducing a time-delayed coupling between two drive systems, instead of a coupling between two response systems as seen in Ref. [8]. We can separate two mixed drive signals by using one response system in this coupling scheme. Our scheme is similar to the method for synchronization of globally coupled systems, as numerically demonstrated in Ref. [4]. However, we present experimental demonstration of dual synchronization of chaos to separate mixed chaotic signals in drive-coupled systems. We also introduce time-delay systems to increase the number of mixed chaotic attractors for multiplexing, since we can obtain a variety of chaotic attractors by changing delay time. We use Mackey-Glass electronic circuits with time-delayed feedback [10–12] and demonstrate the dual synchronization of chaos experimentally and numerically. The degree of synchronization is quantitatively evaluated by using cross correlation. We investigate the parameter regions for achieving dual synchronization when the delay time is mismatched between the drive and response circuits.

II. EXPERIMENT

A. Experimental setup

Figure 1 shows the diagram of our Mackey-Glass electronic circuit [11,12]. The circuit consists of a nonlinear part, amplifiers, a *RC* filter, and a delay line. A pair of *p*-channel and *n*-channel field-effect transistors (2SJ103 and 2SK30ATM, Toshiba) are used for generating the nonlinear function. Three operational amplifiers (LM741/CN, National Semiconductor) are used as electronic amplifiers. Time delay

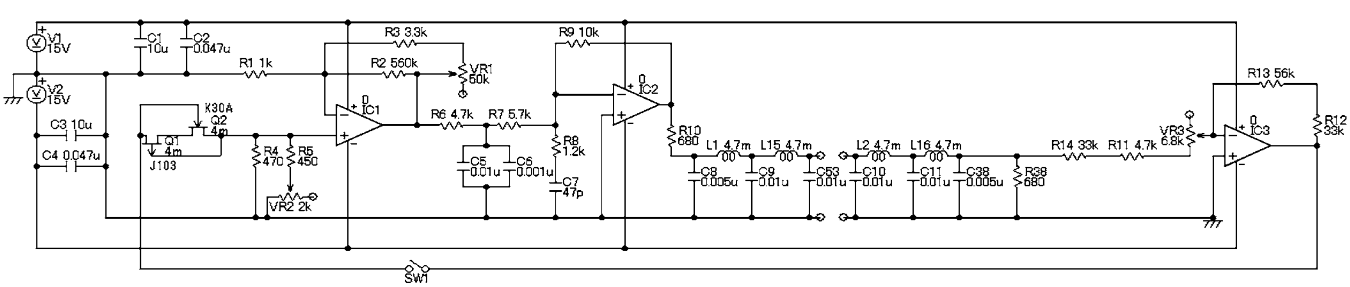

FIG. 1. Circuit diagram of the Mackey-Glass electronic circuit.

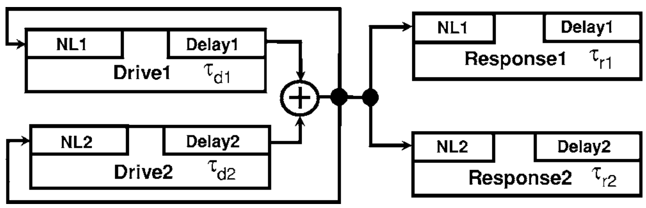

FIG. 2. Block diagram for dual synchronization of chaos in Mackey-Glass electronic circuits with time delay.

is introduced by using 50 pairs (not shown in Fig. 1) of capacitors of 10 nF (MMT50V103J, Nissei) and inductors of 4.7 mH (LHB0812-472K, OLE). One pair of the capacitor and inductor corresponds to the delay time of 6.0 μs. We can change the delay time by selecting the number of capacitor-inductor pairs. The time-delayed signal is fed back to the nonlinear part (transistor) to generate chaotic oscillations. We built four Mackey-Glass electronic circuits to test dual synchronization.

Figure 2 shows a block diagram for dual synchronization of chaos in Mackey-Glass electronic circuits with time delay. Two of the four electronic circuits are used as drive systems (drive 1 and drive 2). The other two circuits are used as response systems (response 1 and response 2). The voltages after the delay line in the two drive systems are added at an adder circuit and fed back to the nonlinear part (transistor) of each drive system with time delay to generate chaotic oscillations. This mixed feedback signal is also used as a transmission signal, and the signal is unidirectionally transmitted

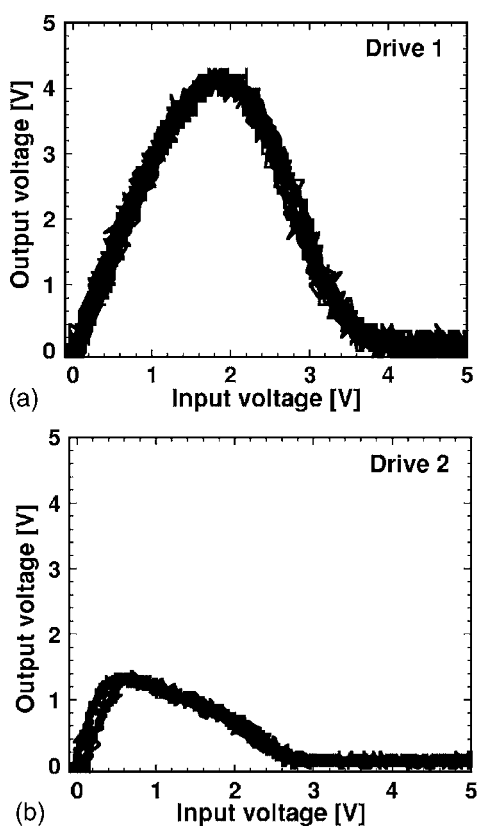

FIG. 3. Nonlinear functions for (a) drive 1 and (b) drive 2.

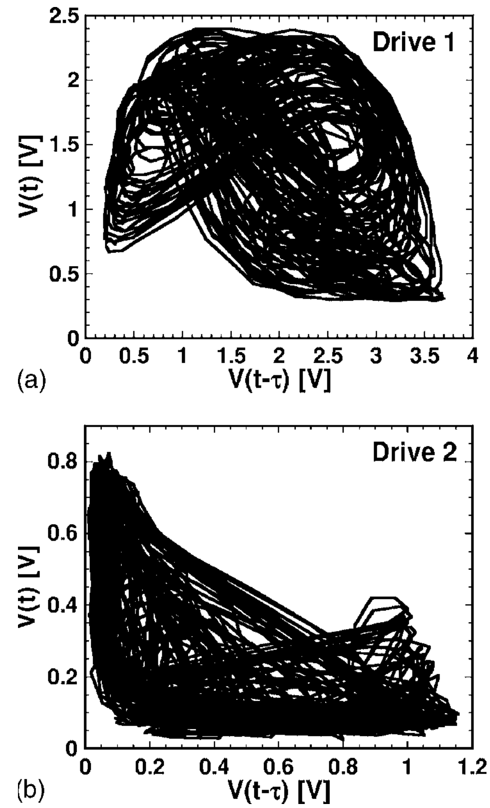

FIG. 4. Chaotic attractors generated in (a) drive 1 and (b) drive 2.

to the nonlinear parts and delay lines of the two response systems through an amplifier (gain of 1) for dual synchronization. No feedback signal is used for the two response systems to maintain the symmetry between the drive and response systems. All the parameters need to be matched between drive 1 and response 1, and between drive 2 and response 2, whereas some parameters for drive 1 and drive 2 are set to be different for achieving dual synchronization. When synchronization manifolds are stable between drive 1 and response 1, and between drive 2 and response 2, dual synchronization can be achieved.

B. Experimental results

We chose the delay time of 228 μs for drive 1 and response 1, and 307 μs for drive 2 and response 2. We also changed the shape of the nonlinear functions between drive 1 and drive 2. We selected the *p*-channel field-effect transistor of 2SJ103 (Toshiba) for drive 1 and response 1, and 2SJ104 (Toshiba) for drive 2 and response 2. The shapes of the nonlinear functions for drive 1 and drive 2 are different as shown in Fig. 3. Figure 4 shows chaotic attractors generated in drive 1 and drive 2. Two different chaotic attractors are observed for the two drive circuits with different delay times and different shapes of the nonlinear functions. The delay time corresponds to a half period of the chaotic wave form. When all the circuits are coupled as shown in Fig. 2, dual synchronization is observed. Figure 5 shows the temporal wave forms of the voltages for all four electronic circuits and the corre-

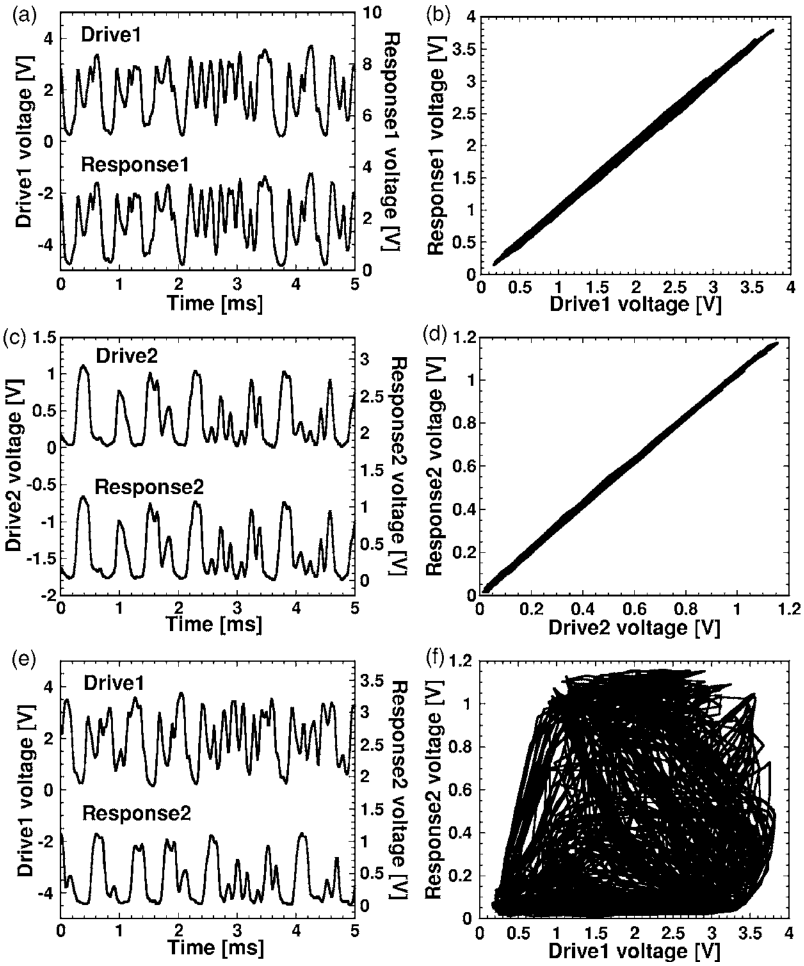

FIG. 5. Experimental results of temporal wave forms and their correlation plots. (a), (b) drive 1 and response 1, (c), (d) drive 2 and response 2, and (e), (f) drive 1 and response 2. Dual synchronization is achieved in the pair of drive 1–response 1 and drive 2–response 2.

lation plots of the pairs of drive 1–response 1, drive 2–response 2, and drive 1–response 2. Synchronization of chaotic oscillations is independently achieved for the pairs of drive 1–response 1 and drive 2–response 2, as shown in Figs. 5(a)–5(d). A good linear correlation is observed as shown in Figs. 5(b) and 5(d). Conversely, synchronization of chaos is not achieved for the pair of drive 1–response 2, as shown in Figs. 5(e) and 5(f).

C. Parameter dependence

To investigate the characteristics of dual synchronization, we introduce the cross correlation C of two temporal wave forms normalized by the product of their standard deviations, i.e.,

$$C = \frac{\langle (V_\mathrm{d} - \bar{V}_\mathrm{d})(V_\mathrm{r} - \bar{V}_\mathrm{r}) \rangle}{\sigma_\mathrm{d}\sigma_\mathrm{r}}, \quad (1)$$

where V_d and V_r are the voltages of the drive and response circuits, $\bar{V}_\mathrm{d}$ and $\bar{V}_\mathrm{r}$ are the mean values of the drive and response wave forms, and σ_d and σ_r are the standard deviations of the drive and response wave forms. The angular brackets denote time averaging.

We quantitatively investigate chaos-synchronization regions against parameter mismatch in the two pairs of Mackey-Glass electronic circuits. The delay times for drive 1, response 1, and drive 2 are fixed at 327, 327, and 398 μs, respectively. The delay time of response 2 is changed and the cross correlation between the temporal wave forms of drive 2 and response 2 is calculated. Figure 6 shows the cross correlation of the two temporal wave forms for the pair of drive 2 and response 2. We found that the best correlation is obtained at the matched delay time for the pair of drive 2 and response 2. This result demonstrates that mismatched delay times reduce the accuracy of synchronization. We also found that it is important to set parameters significantly different

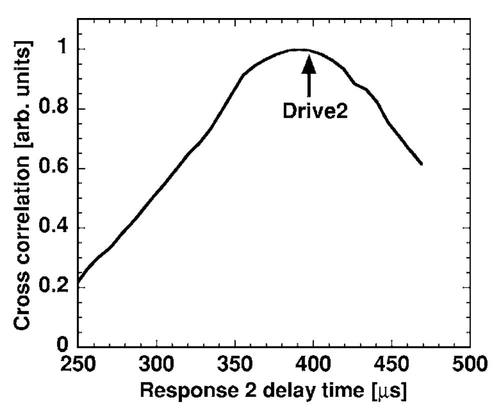

FIG. 6. Cross correlation between drive 2 and response 2 as a function of the delay time for response 2. The vertical arrows indicate the parameter matching condition of the delay time (398 μs).

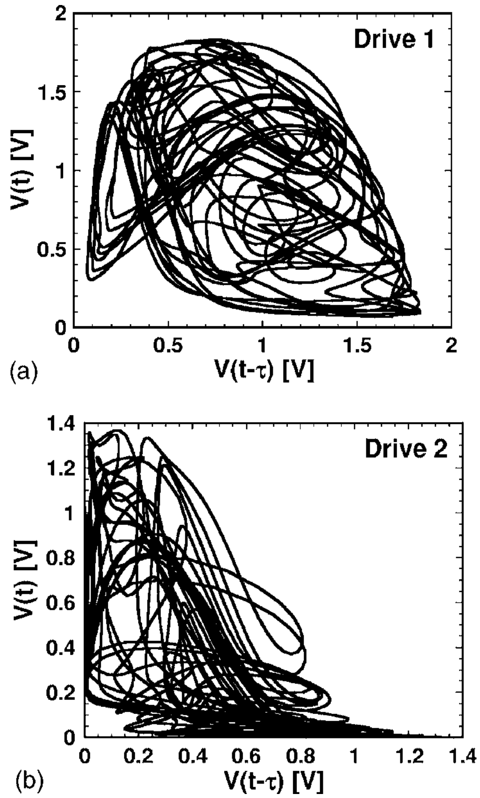

FIG. 7. (a) Chaotic attractors for (a) drive 1 and (b) drive 2 generated by numerical simulation.

between drive 1 and drive 2 for the achievement of dual synchronization. When similar chaotic attractors are mixed in the drive systems, the two response circuits cannot distinguish between the two chaotic attractors and can synchronize with one of the drive circuits (e.g., drive 1 and response 2 can be synchronized). Therefore, one cannot control the pair of synchronization under the mixing of similar chaotic attractors. Time-delayed chaotic systems are suitable for a dual synchronization scheme since chaotic attractors can be significantly changed at different delay times.

III. NUMERICAL SIMULATION

A. Model

To explain our experimental observation, we numerically calculate a model for coupled Mackey-Glass electronic circuits shown in Fig. 2. The model equations are as follows:

$$\frac{dV_{d1}(t)}{dt} = \frac{a_{d1}c_{d1}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}}{1+[c_{d1}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}]^{n_{d1}}} - b_{d1}V_{d1}(t), \quad (2)$$

$$\frac{dV_{d2}(t)}{dt} = \frac{a_{d2}c_{d2}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}}{1+[c_{d2}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}]^{n_{d2}}} - b_{d2}V_{d2}(t), \quad (3)$$

$$\frac{dV_{r1}(t)}{dt} = \frac{a_{r1}c_{r1}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}}{1+[c_{r1}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}]^{n_{r1}}} - b_{r1}V_{r1}(t), \quad (4)$$

$$\frac{dV_{r2}(t)}{dt} = \frac{a_{r2}c_{r2}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}}{1+[c_{r2}\{V_{d1}(t-\tau_{d1}) + V_{d2}(t-\tau_{d2})\}]^{n_{r2}}} - b_{r2}V_{r2}(t), \quad (5)$$

where $V(t)$ is the voltage of the Mackey-Glass electronic circuits. The subscripts d1, d2, r1, and r2 indicate drive 1, drive 2, response 1, and response 2, respectively. a is the height of the nonlinear function, b is the RC constant, τ is the delay time, n is the shape of the nonlinear function, and c is the feedback or coupling strength. All the parameter values are matched between drive 1 and response 1, and between drive 2 and response 2. However, some of the parameter values are different between drive 1 and drive 2. The parameter values are set as follows: for drive 1 and response 1, $a_{d1}=a_{r1}=3.2$, $n_{d1}=n_{r1}=4.5$, $\tau_{d1}=\tau_{r1}=8.2$, $b_{d1}=b_{r1}=1.0$, and $c_{d1}=c_{r1}=1.0$; for drive 2 and response 2, $a_{d2}=a_{r2}=2.0$, $n_{d2}=n_{r2}=10.0$, $\tau_{d2}=\tau_{r2}=11.0$, $b_{d2}=b_{r2}=1.0$, and $c_{d2}=c_{r2}=1.0$. We compared the time-delayed drive wave form $V_d(t-\tau_d)$ with the time-delayed response wave form $V_r(t-\tau_r)$ to evaluate the accuracy of synchronization. The matching of the delay time between the drive and response systems is required for high quality of synchronization.

B. Numerical results

Figure 7 shows chaotic attractors generated in drive 1 and drive 2. These attractors are qualitatively similar to those observed in out experiment (see Fig. 4). Figure 8 shows the numerical results of temporal wave forms and their correlation plots for dual synchronization. Chaos synchronization is achieved between drive 1 and response 1, and between drive 2 and response 2 as shown in Figs. 8(a)–8(d). Conversely, no synchronization is observed between drive 1 and response 2 [Figs. 8(e) and 8(f)]. These numerical results agree well with our experimental observation shown in Fig. 5.

C. Parameter dependence

We changed the delay time of response 2 and investigated the degree of synchronization between drive 2 and response 2. Figure 9 shows the cross correlation of the two temporal wave forms between drive 2 and response 2 as a function of the delay time for response 2. We found that synchronization can be achieved when the delay time for response 2 is set to be equal to the delay time for drive 2. This result demonstrates that mismatched delay times reduce the accuracy of synchronization. The result of Fig. 9 agrees well with our experimental observation shown in Fig. 6.

Our dual-synchronization scheme can be extended for many pairs of drive-response systems (i.e., multiple synchronization). The crucial point is the coupling between the drive systems, because this coupling makes the drive systems synchronized. We carefully need to choose different parameter values for the drive systems to avoid synchronization among the drive systems. The increase of the pairs of drive-response systems is currently under investigation for multiple synchronization.

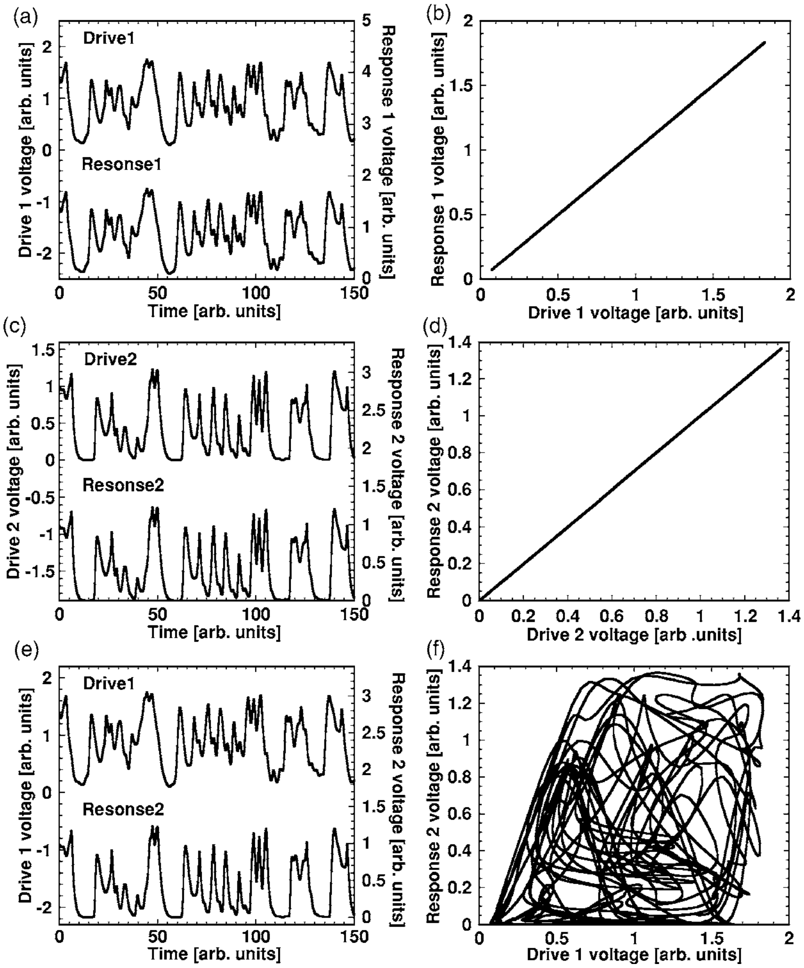

FIG. 8. Numerical results of temporal wave forms and their correlation plots. (a), (b) drive 1 and response 1, (c), (d) drive 2 and response 2, and (e), (f) drive 1 and response 2. Dual synchronization is achieved in the pair of drive 1–response 1 and drive 2–response 2.

IV. CONCLUSION

We have demonstrated dual synchronization of chaos in two pairs of one-way-coupled Mackey-Glass electronic circuits. The outputs of the two drive circuits are added and used both for the feedback signal to the two drive circuits and for the transmission signal to the two response circuits. The delay time and the shape of nonlinear function are set to be different between drive 1 and drive 2, whereas all the parameters are set to be identical between drive 1 and response 1, and between drive 2 and response 2. We have observed dual synchronization of chaos in both experiment and numerical calculation. We have also investigated the regions for achieving dual synchronization of chaos when the delay time is mismatched between the drive and response circuits. We found that synchronization can be achieved when the delay time for the response circuit is set to be equal to the delay time for the drive circuit. The technique of dual synchronization could be useful for multiplexing communications using chaos and for the identification of chaos from mixed chaotic wave forms.

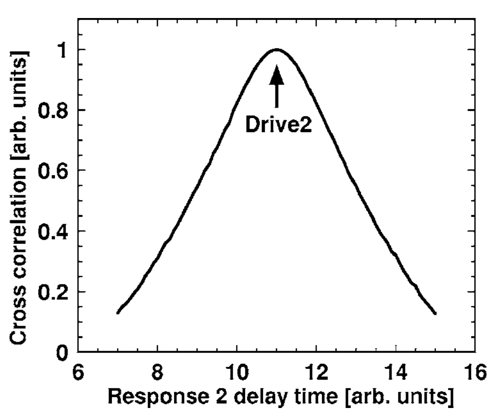

FIG. 9. Numerical result of cross correlation between drive 2 and response 2 as a function of the delay time for response 2. The vertical arrows indicate the parameter matching condition of the delay time ($\tau_{d2}=\tau_{r2}=11.0$).

ACKNOWLEDGMENTS

The authors thank Arturo Buscarino, Ned J. Corron, Peter Davis, Vasily Dronov, Min-Young Kim, Louis M. Pecora, Dave T. Sodaitis, Christopher Sramek, Ken Umeno, and Kazuyuki Yoshimura for helpful discussions. We gratefully acknowledge support from the Support Center for Advanced Telecommunications Technology Research and Grants-in-Aid for Scientific Research from the Japan Society for the Promotion of Science. R.R. thanks the Physics Division of the Office of Naval Research for support.

[1] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[2] K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993).
[3] M. P. Kennedy, R. Rovatti, and G. Setti, *Chaotic Electronics in Telecommunications* (CRC Press, Boca Raton, 2000).
[4] K. Yoshimura, Phys. Rev. E **60**, 1648 (1999).
[5] L. S. Tsimring and M. M. Sushchik, Phys. Lett. A **213**, 155 (1996).
[6] Y. Liu and P. Davis, Phys. Rev. E **61**, R2176 (2000).
[7] A. Uchida, S. Kinugawa, T. Matsuura, and S. Yoshimori, Phys. Rev. E **67**, 026220 (2003).
[8] A. Uchida, M. Kawano, and S. Yoshimori, Phys. Rev. E **68**, 056207 (2003).
[9] P. Arena, A. Buscarino, L. Fortuna, and M. Frasca, Phys. Rev. E **74**, 026212 (2006).
[10] M. C. Mackey and L. Glass, Science **197**, 287 (1977).
[11] A. Namajunas, K. Pyragas, and A. Tamasevicius, Phys. Lett. A **201**, 42 (1995).
[12] M.-Y. Kim, C. Sramek, A. Uchida, and R. Roy, Phys. Rev. E **74**, 016211 (2006).