Spectral form factor near the Ehrenfest time

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We calculate the Ehrenfest-time dependence of the leading quantum correction to the spectral form factor of a ballistic chaotic cavity using periodic orbit theory. For the case of broken time-reversal symmetry, when the quantum correction to the form factor involves two small-angle encounters of classical trajectories, our result differs from that previously obtained using field-theoretic methods [Tian and Larkin, Phys. Rev. B **70**, 035305 (2004)]. While we believe that the existing field-theoretic calculation is technically flawed, the question whether the field theoretic and periodic-orbit approaches agree when more than one small-angle encounter of classical orbits is involved remains unanswered.

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From a statistical point of view, the spectra of a quantum particle confined to a cavity with disorder or confined to a cavity with ballistic and chaotic classical dynamics are remarkably similar [1]. In both cases, the spectral statistics are independent of system specifics and follow the predictions of random matrix theory (RMT) on energy scales comparable to the mean spacing between energy levels Δ or, equivalently, on time scales comparable to the Heisenberg time $t_H = 2\pi\hbar/\Delta$ [2,3], provided that the latter is much larger than the time $\tau_{\rm erg}$ needed for ergodic exploration of the phase space. Calculations of the spectral statistics have been the theoretical vehicle through which the profound correspondence between RMT, impurity diagrammatic perturbation theory, periodic-orbit theory, and the field-theoretic zero-dimensional σ model has been demonstrated [2,4–7].

In the last decade, it has been understood that chaotic quantum systems are characterized by a time scale intermediate between $\tau_{\rm erg}$ and τ_H that does not have its counterpart in diffusive systems. This time scale is the "Ehrenfest time" τ_E [8–11]. The Ehrenfest time is the time required for two classical trajectories initially a quantum distance (wavelength) apart to diverge and reach a classical separation (system size). It is expressed in terms of the Lyapunov exponent λ of the corresponding classical dynamics as $\tau_E = \lambda^{-1}\ln(c^2/\hbar)$, where c^2 is a classical (action) scale. The existence of the Ehrenfest time does not affect the universality of the spectral statistics. However, it is responsible for differences between otherwise universal properties of chaotic and disordered quantum systems for energies $\sim\hbar/\tau_E$, which is well inside the universal range if $\hbar/c^2 \to 0$ [5,11–15]. In this article we address the spectral statistics at this energy scale.

Most of the literature on spectral statistics considers the spectral form factor $K(t)$, which is the Fourier transform of the two-point correlation function of the level density $\rho(\varepsilon)$,

$$K(t) = \hbar \left\langle \int d\omega e^{i\omega t} \rho(\varepsilon + \hbar\,\omega/2)\rho(\varepsilon - \hbar\,\omega/2) \right\rangle_c . \qquad (1)$$

Here the brackets $\langle\cdots\rangle_c$ denote the connected average obtained by varying the center energy ε and/or other parameters in the system. For $\tau_{\rm erg} \ll t < t_H$, the RMT predicts that $K(t)$ is dominated by a perturbative expansion in t/t_H [1,16,17]

$$K(t) = \frac{t}{\beta\pi\hbar} + \delta K_\beta(t), \qquad (2)$$

where $\beta=1$ (2) in the presence (absence) of time-reversal symmetry and

$$\delta K_1(t) = -\frac{t}{\pi\hbar}\left(\frac{t}{t_H} - \frac{t^2}{t_H^2} + \cdots\right),$$

$$\delta K_2(t) = 0. \qquad (3)$$

For times $t \gtrsim t_H$ the perturbative expansion in t/t_H breaks down and $K(t)$ is governed by nonperturbative contributions [16].

The presence of the Ehrenfest time does not affect the leading contribution to $K(t)$, but it does impact δK. The leading τ_E dependence of δK_1 in the perturbative regime, which already occurs to order $(t/t_H)^2$, was first considered by Aleiner and Larkin [12], using a field-theoretic approach. Recently, Tian and Larkin used the field-theoretic approach to calculate the leading τ_E dependence of δK_2 [5], which appears to order $(t/t_H)^3$ and causes the perturbative quantum correction δK_2 no longer to be strictly zero. Their result is

$$\delta K_1(t) = -\frac{t^2}{\pi\,\hbar\,t_H}\theta(t - 2\tau_E) + \cdots, \qquad (4)$$

$$\delta K_2(t) = -\frac{t^2}{2\pi\,\hbar\,t_H^2}[\Theta(t - 3\tau_E) - \Theta(t - 4\tau_E)] + \cdots, \qquad (5)$$

where $\theta(x)=1$ if $x>0$ and $\theta(x)=0$ otherwise, $\Theta(x) = \int_0^x dx'\,\theta(x') = x\theta(x)$, and the ellipses refer to terms of higher order in t/t_H that were not considered in the calculation. (Tian and Larkin also considered the τ_E dependence of nonperturbative contributions to the spectral form factor, but these will not be considered here.)

In a parallel development, the full perturbation expansion for $K(t)$ was derived from periodic-orbit theory [6,7,18]. The connection between $K(t)$, which is a quantum-mechanical object, and classical periodic orbits follows from Gutzwiller's trace formula, which expresses $K(t)$ as a double sum over periodic orbits γ and γ' [17,19],

$$K(t) = \left\langle \sum_{\gamma,\gamma'} A_\gamma A^*_{\gamma'} e^{i(S_\gamma - S_{\gamma'})/\hbar} \delta\left(t - \frac{T_\gamma + T_{\gamma'}}{2}\right) \right\rangle. \quad (6)$$

Here A_γ, S_γ, and T_γ are the stability amplitude, classical action, and period of γ, respectively. While the leading contribution to $K(t)$ comes from the diagonal terms $\gamma = \gamma'$ (up to time reversal, if $\beta = 1$) [17], off-diagonal contributions are responsible for $\delta K(t)$. Sieber and Richter [6] and Heusler *et al.* [7,18] succeeded in classifying the relevant off-diagonal orbit pairs and calculated their contribution to $\delta K(t)$ in the limit $\tau_E / t_H \to 0$.

Below, we show that periodic-orbit theory can also be used to calculate the Ehrenfest-time dependence of $\delta K(t)$. Interestingly, while we confirm the field-theoretic result for the $O(t^2)$ term in $\delta K_1(t)$ [5,20], our result for the leading $O(t^3)$ contribution to $\delta K_2(t)$ differs from that of Ref. [5],

$$\delta K_2(t) = \frac{3t^2}{2\pi\hbar\, t_H^2}[\Theta(t - 2\tau_E) - 2\Theta(t - 3\tau_E) + \Theta(t - 4\tau_E)]. \quad (7)$$

In particular, we find that the minimum duration of off-diagonal pairs of orbits that contribute to δK_2 is $2\tau_E$, not $3\tau_E$. We also find that there are no τ_E-dependent corrections for $t > 4\tau_E$, in contrast to Ref. [5], where the τ_E dependent corrections to δK_2 persist up to $t \sim t_H$. (However, our leading-order perturbative calculation does not answer the question whether such τ_E insensitivity at long times persist to the higher order terms in t/t_H.)

Instead of calculating $K(t)$ directly, it is more convenient to calculate the Laplace transform

$$K(\alpha) = \left\langle \sum_{\gamma,\gamma'} A_\gamma A^*_{\gamma'} e^{i(S_\gamma - S_{\gamma'})/\hbar - \alpha(T_\gamma + T_{\gamma'})/2} \right\rangle. \quad (8)$$

The leading diagonal contribution to K can be calculated using the sum rule of Hannay and Ozorio de Almeida [21]

$$\left\langle \sum_\gamma |A_\gamma|^2 e^{-\alpha T_\gamma} \right\rangle = \frac{1}{2\pi\hbar\,\alpha^2}, \quad (9)$$

so that

$$K(\alpha) = \frac{1}{\pi\hbar\,\alpha^2\beta} + \delta K(\alpha). \quad (10)$$

The inverse Laplace transform of the first term in Eq. (10) reproduces the leading term in Eq. (2) above.

The leading $O(t^2)$ quantum correction for $K(\alpha)$ exists in the presence of time-reversal symmetry only. The relevant pairs of periodic orbits γ and γ' are shown in Fig. 1(a). The existence of such pairs was pointed out by Sieber and Richter [6]; an equivalent configuration of classical trajectories appears in the field-theoretic formulation [5,12,20] and in the diagrammatic calculation of the form factor for disordered cavities [4]. The periodic orbit γ in Fig. 1 has a small-angle self-intersection. There are two loops of duration T_1 and T_2 through which γ returns to the self-intersection. The trajectory γ' is equal to γ in one of these loops, whereas γ' is the time reversal of γ in the other loop. Following Refs. [7,18], we perform the sum over such periodic orbits with the help of a Poincaré surface of section taken at an arbitrary point during the self-intersection. The Poincaré surface of section is parametrized using stable and unstable phase space coordinates s and u, normalized such that $dsdu$ is the cross-sectional area element. Denoting the coordinate differences between the two points, where γ pierces the Poincaré surface of section by s and u, the action difference $S_\gamma - S_{\gamma'} = su$ [22,23]. The duration $t_{\rm enc}$ of the self-encounter is defined as the time during which the two stretches of γ are within a phase space distance c, where c is a classical scale below which the classical dynamics can be linearized. The periodic-orbit sum is then expressed in terms of an integral over s, u, T_1, and T_2 [7,18],

$$\delta K_1(\alpha) = \int dT_1 dT_2 \int_{-c}^{c} dsdu \frac{(T_1 + T_2 + 2t_{\rm enc})^2}{(2\pi\hbar)^2 t_{\rm H} t_{\rm enc}} \times \cos(su/\hbar) e^{-\alpha(T_1+T_2+2t_{\rm enc})}$$
$$= \frac{\partial^2}{\partial\alpha^2}\frac{1}{\alpha^2}\int_{-c}^{c} dsdu \frac{\cos(su/\hbar)e^{-2\alpha t_{\rm enc}}}{(2\pi\hbar)^2 t_{\rm H} t_{\rm enc}}, \quad (11)$$

where

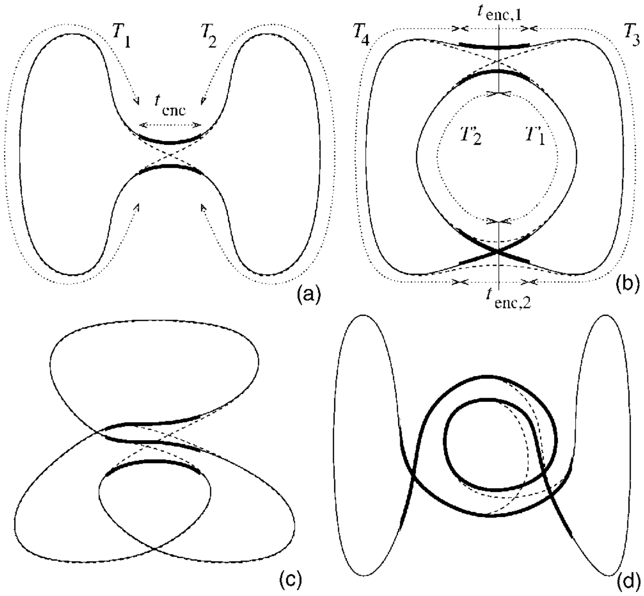

FIG. 1. Schematic drawing of a pair of orbits (shown solid and dashed) contributing to the leading interference correction to the spectral form factor in the presence of time-reversal symmetry (a) and in the absence of time-reversal symmetry (b), (c), (d). The true orbits are piecewise straight, with specular reflection off the cavity's boundaries. The small-angle self-encounters are shown thick. Panels (a) and (b) also show the definitions of various durations used in the text.

$$t_{\mathrm{enc}} = (1/\lambda)\ln(c^2/|su|), \tag{12}$$

λ being the Lyapunov exponent of the classical dynamics in the cavity. The factor t_{enc} in the denominator cancels an unwanted contribution from the freedom to choose the Poincaré surface of section anywhere inside the encounter region. Whereas Refs. [7,18] calculate the remaining integral in the limit $\alpha\tau_E \to 0$, we need to consider the effect of a finite Ehrenfest time. Such integrals have been considered in the context of quantum transport and we will be able to obtain the remaining integral in Eq. (11) as well as all other necessary integrals from the literature. The integral needed here has been calculated in Ref. [24] and gives

$$\delta K_1(\alpha) = -\frac{1}{\pi\hbar\, t_H}\frac{\partial^2}{\partial\alpha^2}\frac{e^{-2\alpha\tau_E}}{\alpha}, \tag{13}$$

where

$$\tau_E = \frac{1}{\lambda}\ln\frac{c^2}{\hbar}. \tag{14}$$

This result is in agreement with the τ_E dependence of δK_1 calculated by Tian and Larkin [5]. Its inverse Laplace transform is Eq. (4) above.

We now consider the Ehrenfest-time dependent correction $\delta K_2(t)$ to the spectral form factor in the absence of time-reversal symmetry. There are three classes of periodic orbits that give a contribution to $\delta K_2(t)$ to order t^3. These are shown in Figs. 1(b)–1(d). Now, the classical orbit γ has two small-angle self encounters. Its partner orbit γ' follows γ between the encounters, but connects the ends of the encounters in a different way. Figure 1(b) shows two separate encounters. Figure 1(c) shows a "three-encounter," which arises when the two small-angle encounters of Fig. 1(b) are merged along one of the connecting stretches (while keeping the duration of the other stretches finite). The orbit γ of Fig. 1(c) goes through three loops between returns to the encounter region. Finally, Fig. 1(d) shows a periodic orbit for which the self-encounter fully extends along one of these loops [25]. Note that the encounter region cannot simultaneously extend along two or more of the loops, because then these loops and, hence, the trajectories γ and γ' would be equal.

The configurations of Fig. 1(b) and 1(c) were also considered by Heusler *et al.* [7,18]. These two contributions cancel in the limit $\alpha\tau_E \to 0$, so that one finds $\delta K_2 = 0$ in that limit. The contribution of the trajectories shown Fig. 1(d) vanishes in the limit $\alpha\tau_E \to 0$, which is why it was not considered in Refs. [7,18]. However, as we will show below, it is needed to be taken into account when calculating τ_E-dependent corrections. The field-theoretic calculation of Ref. [5] also has two contributions to δK_2 only, but these cannot *a priori* be identified with any one of the three contributions shown in Figs. 1(b)–1(d).

The contribution δK_{2b} of the two separate encounters for the periodic-orbit pair in Fig. 1(b) factorizes. Taking a Poincaré surface of section at each of the encounters and proceeding as in the calculation of δK_1, one finds

$$\delta K_{2b}(\alpha) = \frac{1}{8\pi\hbar}\frac{\partial^2}{\partial\alpha^2}\frac{1}{\alpha^4}\left[\int_{-c}^{c} ds du \frac{\cos(su/\hbar)e^{-2\alpha t_{\mathrm{enc}}}}{2\pi\hbar\, t_H t_{\mathrm{enc}}}\right]^2 = \frac{1}{2\pi\hbar\, t_H^2}\frac{\partial^2}{\partial\alpha^2}\frac{e^{-4\alpha\tau_E}}{\alpha^2}, \tag{15}$$

where we included a combinatorial factor $1/4$ to account for permutations of the 2×2 passages of γ through the two encounters [7,18].

For the calculation of the contribution of the trajectory of Fig. 1(c) one needs only one Poincaré surface of section, taken at a point where all three stretches of γ are within a phase space distance c. Labeling the phase space coordinates of the three piercings of γ through the surface of section as (s_i, u_i), $i=1,2,3$, the action difference is [7,18]

$$S_\gamma - S_{\gamma'} = su + s'u', \tag{16}$$

where $s = s_1 - s_3$, $s' = s_1 - s_5$, $u = u_1 - u_3$, and $u' = u_1 - u_5$. Integrating over the durations of the three stretches of γ that connect the three-encounter to itself, we find

$$\delta K_{2c}(\alpha) = \frac{1}{3}\frac{\partial^2}{\partial\alpha^2}\frac{1}{\alpha^2}\int ds ds' du du' \times \frac{\cos[(su+s'u')/\hbar]e^{-\alpha(2t_{\mathrm{enc}}+t'_{\mathrm{enc}})}}{(2\pi\hbar)^3 t_H^2 t'_{\mathrm{enc}}}, \tag{17}$$

where t_{enc} is the duration of the encounter

$$t_{\mathrm{enc}} = \frac{1}{\lambda}\ln\frac{c^2}{\min(|s|,|s'|,|s-s'|)\min(|u|,|u'|,|u+u'|)}$$

and t'_{enc} is the time that all three trajectories involved in the encounter are within a phase space distance c,

$$t'_{\mathrm{enc}} = \frac{1}{\lambda}\ln\frac{c^2}{\max(|s|,|s'|,|s-s'|)\max(|u|,|u'|,|u+u'|)}.$$

The prefactor $1/3$ in Eq. (17) accounts for permutations of the three passages of γ through the three encounter. The integration domain in Eq. (17) is $\max(|u|,|u'|,|u+u'|)$, $\max(|s|,|s'|,|s-s'|) < c$. The exponential factor contains the total time $2t_{\mathrm{enc}} + t'_{\mathrm{enc}}$ that the orbit γ spends in the encounter region. Taking the remaining integral over s, s', u, and u' from Ref. [26], we find

$$\delta K_{2c}(\alpha) = \frac{1}{2\pi\hbar\, t_H^2}\frac{\partial^2}{\partial\alpha^2}\frac{1}{\alpha^2}[3e^{-3\alpha\tau_E} - 4e^{-4\alpha\tau_E}]. \tag{18}$$

Finally, we have to calculate the contribution from trajectories with a three-encounter, where the encounter region fully wraps around one of the loops. In order to make optimal use of the literature on the Ehrenfest-time dependence of quantum transport, we calculate this contribution in an indirect way: We again consider the case of two two-encounters, but now allow the encounters to approach each other and overlap along two preassigned stretches of γ. This situation is shown in Fig. 1(b) (again), and we allow the encounters to approach each other along the central loop in the figure. We take a Poincaré surface of section at each encounter, and measure the durations between the two surfaces of section along the central loop by T_1' and T_2'. Along the remaining stretches [the outer loop in Fig. 1(b)], we still require nonoverlapping encounters in order to enforce $\gamma \neq \gamma'$. Hence, we parametrize their duration using times T_3 and T_4 measured between the ends of the encounters. Finally, $t_{\mathrm{enc},1}$ and $t_{\mathrm{enc},2}$ denote time that the inner and outer loops are within a phase space distance c, and t_s and t_u are the durations of eventual stretches that the two segments of the outer loop are within a phase space distance c from each other but not from the inner loop [24]. (The times t_s and t_u are zero except in the case of overlapping encounters.) With this parametrization, the total duration of γ is

$$T_\gamma = T_1' + T_2' + T_3 + T_4 + t_{\mathrm{enc},1} + t_{\mathrm{enc},2} + 2t_s + 2t_u. \tag{19}$$

We now consider the integral

$$I = \int dT_1' dT_2' dT_3 dT_4 \int_{-c}^{c} ds_1 du_1 ds_2 du_2 \times \frac{T_\gamma^2 \cos[(u_1 s_1 + u_2 s_2)/\hbar] e^{-\alpha T_\gamma}}{(2\pi\hbar)^3 t_H^2 t_{\mathrm{enc},1} t_{\mathrm{enc},2}}, \tag{20}$$

where s_i and u_i are phase space coordinates at the two Poincaré surfaces of section, $i=1,2$. This integral contains both the case that the two encounters are separate and the case that the two encounters overlap. If the two encounters are separate, which requires $T_1' > \lambda^{-1}\ln(|u_1 s_2|/c^2)$ and $T_2' > \lambda^{-1}\ln(|u_2 s_1|/c^2)$, the two stretches in the outer loop are never close to each other, hence $t_s = t_u = 0$. One then recovers the expression for δK_{2b}, multiplied by 4 because of the combinatorial factor $1/4$ which is present in Eq. (15) but not in Eq. (20). If the two encounters overlap at one end but not at the other end, one recovers the scenario for δK_{2c}, multiplied by three because of the combinatorial factor $1/3$ which is present in Eq. (17) but not in Eq. (20). (Note that in this case the times t_s and t_u need not be zero.) If the encounters overlap at two ends, they span the reference loop. This scenario is neither contained in δK_{2b} nor in δK_{2c}. Since there is no combinatorial factor in this case, this is precisely the contribution δK_{2d} corresponding to the trajectories of the type shown in Fig. 1(d). Hence

$$\delta K_{2d}(\alpha) = I - 4\,\delta K_{2b}(\alpha) - 3\,\delta K_{2c}(\alpha). \tag{21}$$

From Sec. IV of Ref. [24], where an integral similar to I was calculated, we find

$$I = \frac{1}{2\pi\hbar\, t_H^2} \frac{\partial^2}{\partial\alpha^2} \frac{1}{\alpha^2} [3e^{-2\alpha\tau_E} - 2e^{-4\alpha\tau_E}]. \tag{22}$$

Combining everything, we arrive at

$$\delta K_2(\alpha) = \delta K_{2b}(\alpha) + \delta K_{2c}(\alpha) + \delta K_{2d}(\alpha) = \frac{3}{2\pi\hbar\, t_H^2} \frac{\partial^2}{\partial\alpha^2} \frac{e^{-2\alpha\tau_E}}{\alpha^2} [1 - e^{-\alpha\tau_E}]^2. \tag{23}$$

The inverse Laplace transform of this result is Eq. (7) above.

The main difference between our result and that of Ref. [5] is that, in contrast to Ref. [5], in Eq. (7) universal quantum corrections already appear after a time $2\tau_E$ [27]. This shortest-duration contribution to δK_2 (which is labeled δK_{2d} in our calculation) stems from periodic orbit pairs which (for a part of their duration) wind around another, shorter, periodic orbit. Such orbits explained [24] the numerically observed τ_E independence of conductance fluctuations in chaotic cavities [28,29]. However, even the remaining contribution $\delta K_{2b} + \delta K_{2c}$ differs from the field-theoretic calculation of Ref. [5]. The orbit configurations contributing to $\delta K_{2b} + \delta K_{2c}$ are essential for the validity of the "effective random matrix theory" [15] of the Ehrenfest-time dependence of the spectral gap in a chaotic cavity coupled to a superconductor [26].

Previously, a difference between periodic-orbit theory and the field theoretic approach appeared concerning the role of repetitions of periodic orbits [30], which were not treated correctly in the ballistic nonlinear sigma model of Refs. [31,32]. In that case, an amended field theory was eventually reported [33], the result of which agrees with periodic-orbit theory. In our case, the difference between the two calculations should have a different origin. This follows, e.g., because the disagreement between the two calculations persists even without inclusion of the contribution δK_{2d}. Instead, we attribute this discrepancy to the incorrect handling of multiple encounter regions in Ref. [5], an issue that is closely related to the implementation of the short-time regularization procedure in the field theory. (Note that, in all cases where calculations were reported for both approaches, the two approaches agree if only one small-angle encounter is involved.) While, at least, the technical evaluation of products of classical propagators connecting two encounter regions in Ref. [5] is flawed [34], the question of whether or not the field theory—properly evaluated with the original regularization of Ref. [12] or in later formulations that use a weaker regularization [20,33]—and the periodic-orbit theory will eventually agree must remain unanswered here.

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