Relationship between the flow rate and the packing fraction in the choke area of the two-dimensional granular flow

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Two-dimensional granular flow in a channel with small exit is studied by both experiments and simulations. We first observe the time variation of the transition from dilute flow state to dense flow state by both experiments and simulations. Then we obtain a relationship between the local flow rate and the local packing fraction in the choke area by use of molecular dynamics simulations. The relationship is a continuous function rather than a discontinuous one. The flow rate has a maximum at a moderate packing fraction and the packing fraction is terminated at high value with negative slope. According to the relationship, four flow states—i.e., stable dilute flow state, metastable dilute flow state, unstable dense flow state, and stable dense flow state—are defined for fixed inflow rate. The discontinuities and the complex time variation behavior occurring in the transition between dilute and dense flow states can be attributed to the abrupt variation through unstable flow state.

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I. INTRODUCTION

Granular flow extensively exists in nature. Differing from the flow of gas or liquid, it may show discontinuous properties, such as, collapse, crowds, jamming, etc., which occur frequently in our daily life. The importance of this system stems from both basic understanding in theories and applications in technology [1,2]. The limiting case of the granular flow at very low density (dilute flow) or very high density (dense flow) has been widely studied, and some kinetic theories have been established. However, less work has hitherto been done in this intermediate density regime as compared to the very dense or the very dilute regime, though a very nice work has been done recently [3].

The system of granules flowing in a channel with a small exit is a good candidate to study the granular flow from dilute to dense state [4,5], because the density in the channel can be easily controlled. This problem is connected with the traffic flow [6,7], which is an important subject in city planning. A common property in these systems is that the outflow rate may be discontinuous as the inflow rate or the size of the exit changes [4–7]. Generally, the discontinuity is used to distinguish the flow states. For granular flow, two flow states are defined, i.e., dilute flow and dense flow, and for the traffic flow, three states are defined, i.e., free traffic flow, synchronized traffic flow, traffic jams. Under this definition, some physical quantities are discontinuous when transition happens. The discontinuity occurring here is apparently different from that of phase transitions, in that it is governed by bulk properties and that the discontinuity results from the breakdown of symmetry in an infinitely large system. In the problem of granular flow, the system is limited in size and its properties are seriously affected by the boundary conditions, which have some analogies with a dynamical system of liquid. However, such kinetic theory could not explain the discontinuity, so we have to find a new mechanism for the transitions. Besides the discontinuity, experiments in these systems also show very complex time-space behavior, which is also an open problem as of now.

The classical research for a material usually aims to establish a set of equations by use of the conservation laws and constitutive relations. For granular systems, only few kinds of situations can be studied by such kinetic theory. One example is the so-called granular gas system corresponding to the very dilute and high-speed case, which can be described by a kinetic theory similar to that of ordinary gas [8,9]. Another example corresponds to the very dense and low-speed case, in which a modified plastic model is adapted [10,11]. In establishing the kinetic theory, the constitutive equation is the key point, which is generally a relationship between physical quantities in differential volume elements. However, such a relationship is yet to be defined for general granular systems at present. Instead of a differential relationship, we could find a relationship in a larger local area, for example, in the area near the choke. This relationship includes some local information and can be considered as the first step in establishing a constitutive equation. The relationship can also be used to explain physical phenomena occurring in the system.

In this paper we investigate the two-dimensional (2D) granular flow in a channel with a small exit by both experiments and simulations. By experiment, we show some basic properties of the granular flow and the transitions between flow states. We suggest that the transition of granular flow between each flow state is a stochastic process, and discontinuity occurring in the transition is attributed to an abrupt variation through an unstable flow state. To show these in more detail, especially to reveal how the local choke area can affect the whole system, we also carry out molecular dynamics (MD) simulations to establish a relationship between the flow rate and the packing fraction in the choke area. By use of the relationship, we explain the time evolution of the system in different cases.

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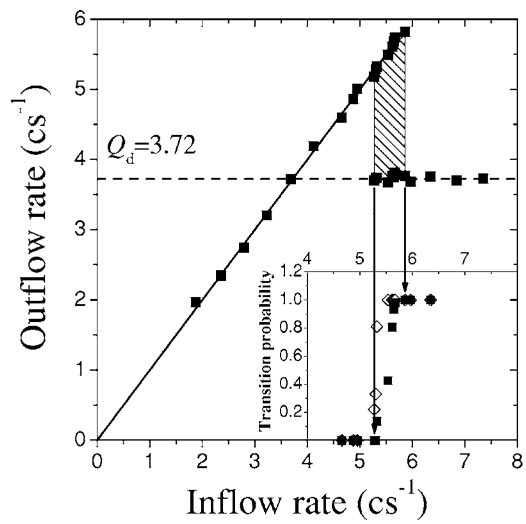

FIG. 1. Experimental results for outflow rate Q (solid squares) as a function of inflow rate Q_0. The solid line is $Q=Q_0$ and the dashed line is $Q=Q_d$, where Q_d is the outflow rate at the dense flow state. Both lines fit experimental data well in the corresponding range. The inset shows the transition probability from dilute flow at initial to dense flow in 5 seconds (solid squares) and 10 seconds (open diamonds), respectively. The abscissas for both the inset and the main graph are set to be the same for easier comparison.

II. EXPERIMENTS AND OBSERVATION

Our experiments are performed by falling steel spheres under gravity in a 2D channel with an inclination angle of 20°. The 2D channel is made by two glass plates separated by specially shaped metal spacers. The gap between the two glass plates (2.2 mm) is kept slightly larger than the diameter of the steel spheres $d_0=2.0\pm0.01$ mm to ensure single-layer flow. The channel is confined in a range with $200d_0=40.0$ cm long and $30d_0=6.0$ cm width. At the top of the channel, some thin plates are distributed, which control the inflow rate Q_0 and ensure the uniformity of the granular flow. The exit is located at the center of the bottom of the channel with a fixed width $d=7d_0$, and the outflow rate Q is obtained by the time differential of the total mass out of the exit, which is measured by an electronic balance per 0.1 second under the exit. In the experiments of this paper, the only adjustable parameter is the inflow rate, which is controlled by the thin plates at the top of the channel, and its value is measured by opening the exit fully. It is found that the particles behave much differently when they are magnetized, so we have taken special care to keep the particles far from magnetic field. The static charges on the particles can also affect the behavior of the granules in many systems, but it was avoided in our experiments by the main spacers and the frame of the apparatus, which are made by metal and are connected to the ground.

The experiment was started by drawing out a baffle plate at the entrance of the channel. The outflow rate will quickly tend to a steady value in most cases, after a very short turbulent initial state. Figure 1 shows the outflow rate at 5 seconds from the starting time as a function of Q_0. From Fig. 1, we can find that Q is almost equal to Q_0 for smaller inflow rate. In this case, the granules do not pile up in the channel and the granular flow is in the dilute flow state. For larger inflow rate, Q becomes a constant smaller than Q_0. In this case, the granules pile up in the channel and the granular flow is in the dense flow state. In between the two ranges, a discontinuity can be found, at where the boundary of dilute and dense flow states is defined as usual. At the larger (smaller) side of the inflow rate far from this boundary, the final flow state is dense (dilute) flow state decisively. However, with a medial inflow rate we find that the flow state observed at a certain finite time is not decisive, with some probability of being in either the dilute flow state or the dense flow state. The inset in Fig. 1 shows the probability of the final flow state being dense flow state at 5 and 10 seconds, respectively, which is described by the percentage of the final flow state being dense by observing 200 ensembles of experiments at corresponding time. It shows clearly that the probability is almost zero at smaller inflow rates and increases sharply to one at larger inflow rates. Comparing the probability at 5 and 10 seconds, we find that the curve of the probability at 10 seconds shifts slightly left of that at 5 seconds.

All these properties imply that the transition from dilute flow state to dense flow state is a stochastic process. Let us emphasize that the experiment starts with an empty channel, so the initial flow state should be the dilute flow state. For smaller inflow rates, the dilute flow state remains all the time, so it is a stable state. However, the initial dilute flow state becomes unstable or metastable when the inflow rate exceeds some value, so it transforms to the dense flow state at certain stochastic time. The transition also has the characteristic that the larger the inflow rate, the faster the transition occurs. When the inflow rate is very large, the transition usually happens very quickly and we always observe the dense flow state in the observation time. However, at a medial inflow rate, where the observation time is comparable to the survival time of the initial dilute state, the transition happens for only parts of ensembles. In this case, we can observe the probability of the dense flow state between 0 and 1 at the observation time. Under this scheme, the probability of the dense flow state at longer times should shift left slightly, relative to that for shorter times, which is just the experimental result for different time scale as we show in the inset of Fig. 1.

III. SIMULATION METHOD AND THE SELECTED LOCAL AREA

To show that the transition from dilute flow state to dense flow state is a stochastic process, we also carry out MD simulations for the system. The main advantage of the simulation is that the information of each granule can be easily known, so we can treat the data in a more detailed way. We use a soft sphere approach in our MD simulations, because we have to treat the case of the dense flow state in which the multicollision of particles is not negligible. The simulation is carried out in a 2D system. The geometrical size in the simulation corresponds to that in our experiment, except the length of the channel, for which to decrease the simulation time we use a relatively shorter length $L=125d_0$ compensated by an increased muzzle velocity of the particle in the entrance to give equivalent effect of channel length. The shape of the grains is circular, and its rotation perpendicular to the 2D plane is considered. The effective gravity is set to be $g\sin 20°$ to simulate the inclined channel in experiment, where g is the gravity acceleration. The Kuwabara-Kono model is used as normal interactions between granules or side wall [12,13],

$$F_{ij}^{n} = -k_n \xi_{ij}^{3/2} - \eta_n \xi_{ij}^{1/2} V_{ij}^{n}, \tag{1}$$

and the tangential interactions are taken to the minor component in comparison to the viscous friction and dynamic friction,

$$F_{ij}^{\tau} = \min(\eta_\tau V_{ij}^{\tau}, \mu F_{ij}^{n}), \tag{2}$$

as is generally adopted in the literature. In Eqs. (1) and (2), $\xi_{ij}=\max(0,2d_0-|\mathbf{r}_i-\mathbf{r}_j|)$ is the overlap of particle i and j, $\mathbf{V}_{ij}=V_{ij}^{n}\mathbf{e}_n+V_{ij}^{\tau}\mathbf{e}_\tau$ is the relative velocity between particle i and j at contact point, and the superscript (subscript) n and τ express the normal and tangential components of a vector, respectively. The detail values of the elastic parameters are $k_n=5.0\times 10^9$ N/m$^{3/2}$, $\eta_n=300.0$ Ns/m$^{3/2}$, and $\eta_\tau=0.3$ Ns/m, and the coefficient of sliding friction is set to $\mu=0.2$. These parameters work out the normal restitution coefficient ranging from 0.7 to 0.9 as the impact velocity varies from $1000d_0$/s to $1d_0$/s. The 2D model meets the purpose that explains the basic physics in the granular flow of our experimental system, though it may be not quantitatively comparable with the experimental system due to sliding and rolling friction on the surface, the rotation of the particles on the other axes, etc.

It is logical that the outflow rate is seriously affected by the physical properties around the exits, i.e., the choke area. However, how to analyze the local properties is the key point of this research. The standard method is to establish a relationship between physical quantities in differential volume elements. But this may represent a constitutive equation, which has not been established at present for general granular flow. Instead of differential volume elements, we choose finite local areas (see Fig. 2) and try to establish a relationship between physical quantities in these areas. The most important quantities in the granular flow are the flow rate and the packing fraction, so we direct our attention on the relationship between them. For this purpose, the local packing fraction and local flow rate must be defined clearly in these areas. The local packing fraction of certain area is straightforwardly defined by the ratio of the volume occupied by the particles in the area and the whole volume of the area. For constant width σ of the area-(2 and 3), the local flow rate is easily defined as

$$Q = \frac{\sum_i q_i^{n}}{\sigma}, \tag{3}$$

where Σ_i represents the summation for all particles in the area and q_i^n is the flow rate of ith particle in the cross section. For area-1, the local flow rate is defined as

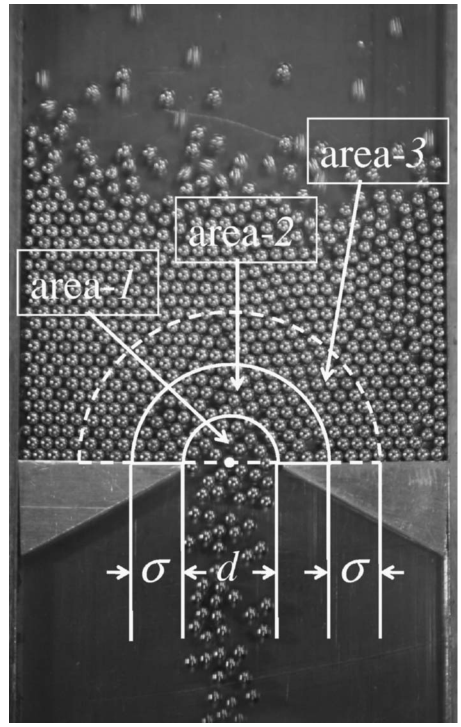

FIG. 2. A snapshot of the dense flow in the experiment. Three selected areas around the exit are sketched, including a half-circle (area-1), a small sector (area-2), and a large sector (area-3). The width of the cross section is $\sigma=4d_0$.

$$Q = \frac{\sum_i q_i^{y}}{A}, \tag{4}$$

where q_i^y is the flow rate of ith particle in vertical direction and A is a normalized constant. All the areas we have chosen have the properties that they separate the entrance and the exit completely. This is important because from the conservation law of mass, the local flow rate will be equivalent to the outflow rate at the exit for the stationary flow state, though it will show a phase difference for a time-dependent flow.

IV. SIMULATION RESULTS

Figure 3 shows the simulation results for the time evolution of the local flow rate in the three areas at three different inflow rates. Figure 3 shows that the flow rate is indeed equivalent for all three areas. We can also find that there is no abrupt variation in the flow rate for the smallest inflow rate (squares), which means that the dilute flow state remains throughout the whole simulation time. However, for the other two larger inflow rates (diamonds) and (triangles) an abrupt decrease of the flow rate at a certain time can be found, which corresponds to transition from the dilute flow state to the dense flow state. It also shows that the larger the inflow rate, the quicker the transition occurs. These properties are consistent with those observed in experiment.

Figure 4 shows the simulation results for time evolution of the local packing fraction in the three areas with the same

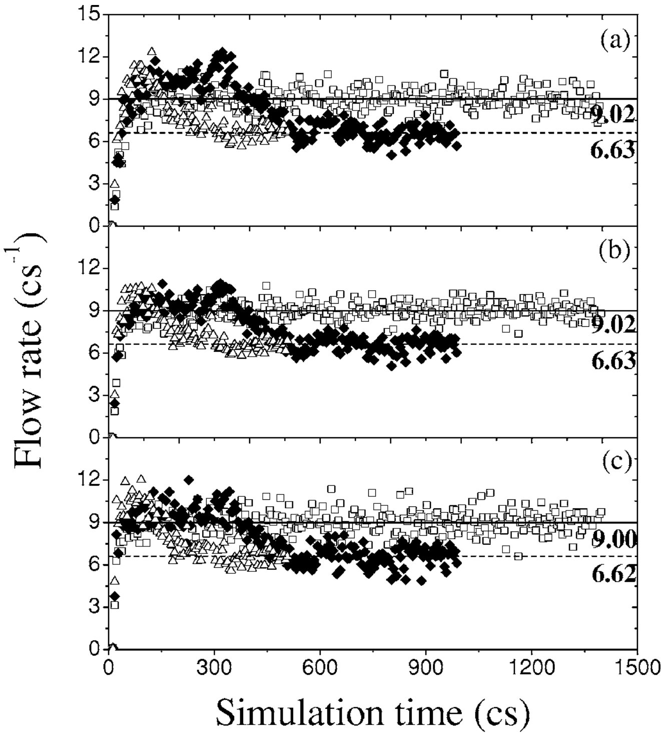

FIG. 3. Simulation results of local flow rate in (a) area-1, (b) area-2, and (c) area-3 as a function of time with several inflow rates. Open squares, closed diamonds, and open triangles are for Q_0 =9.0, 10.0, and 12.0 cs^{-1}, respectively, in all the cases.

inflow rate used in Fig. 3. Similar to Fig. 3, the abrupt changes in packing fraction means a transition from the dilute flow state to the dense flow state. Figure 4 also shows that the transition does not exist in the case of the smallest inflow rate, and it occurs for the other two cases. However, Fig. 4 shows different amounts of variation of the packing

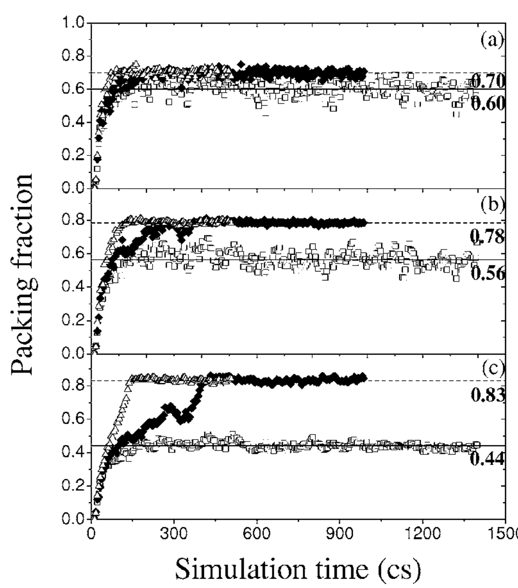

FIG. 4. Simulation results for local packing fraction in three areas as a function of time. The symbols are used in the same way as in Fig. 3.

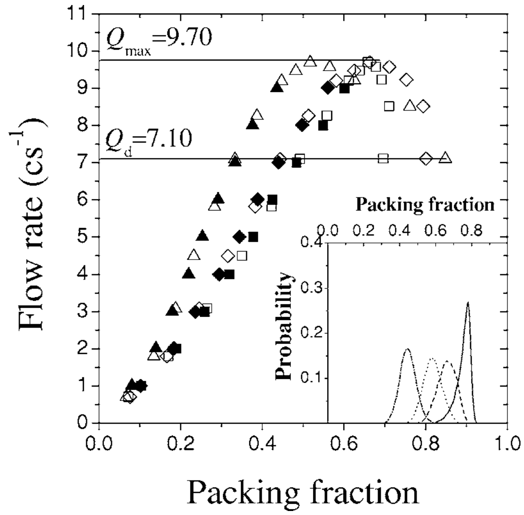

FIG. 5. Simulation results of the relationship between local flow rate and packing fraction. Squares, diamonds, and triangles are results for area-1, 2, and 3, respectively, and the open (closed) symbols are results obtained by fixed particle number (fixed inflow rate) simulation. $Q_{\max}$ and Q_d are the maximum flow rate and the flow rate of stable dense flow state, respectively. The inset is the histogram of the local packing fraction in area-2 for fixed particle number N=300 (dash-dotted line), 400 (dotted line), 450 (dashed line), and 500 (solid line), which is normalized for all the simulation steps.

fraction for different areas when transition occurs. We can see that the packing fraction increases to a high value near the random close packing fraction in Figs. 4(b) and 4(c), which is around 0.83 [14]. In contrast, only a small variation in the packing fraction is observed in Fig. 4(a). This suggests that the crowding first happens at area-2, and then extends to the area behind it—i.e., area-2 is the choke area in this system.

We have shown the local flow rate and packing fraction separately. It is interesting and important to see if they have any inherent relations. For this purpose, the average local flow rates as a function of average local packing fraction are plotted in Fig. 5. In these calculations, two kinds of simulation conditions are used to control the local packing fraction. One is the fixed inflow rate conditions, which is the same as that we used in the experiment. Under this condition, the results show that the packing fraction increases successively as the inflow rate increases. However, this method cannot be used for larger inflow rate, because the high inflow rate will cause particle piling-up in the channel and the long time averaging could not be accomplished. Another simulation condition is the fixed particle number condition, which can overcome the disadvantages occurring in the fixed inflow rate simulation. The fixed particle number condition is realized by creating an equal number of particles at the top of the channel when some particles flow out. We can find that the local packing fraction increases also successively as the particle number increases. In these simulation conditions, no particle accumulation occurs in the channel, even for the dense flow state, so we can obtain the relationship in the whole range.

Figure 5 gives the relationship between the local flow rate and packing fraction for all three areas. First, we compare the results for two simulation conditions, and it is easy to find that they agree very well in dilute flow. So it is reasonable to consider the relationship being inherent and robust. To exclude that the relationship is obtained from mixed states, we have checked the histogram of the local packing fraction. The inset in Fig. 5 shows the histogram of the packing fraction in area-2 for several particle numbers, which correspond to different flow states. It is clear that the histogram has only one peak for all the cases, and the position of the peak moves smoothly as the particle number changes. The histogram of the local flow rate shows similar properties. In Fig. 5, the most important characteristics in the relationship are that the flow rate has a maximum $Q_{\max}$ at a moderate packing fraction and the packing fraction is terminated at a high value with a negative slope for the flow rate. For all three areas, these characteristics are similar, but the curve is wider for the area further from the exit.

The relationship is useful in explanation of the flow state and the transition between them when the inflow rate is fixed. First, we can find that the end point at highest packing fraction corresponds to a dense flow state, which is crowded with particles and the flow rate Q_d is less than the maximum flow rate. It is stable for inflow rates higher than this point, because the outflow rate is smaller than the inflow rate, so the packing fraction would remain at this high value. Thus, we define this point as stable dense flow state. We shall show that this is the only stable state when the inflow rate is higher than Q_d. In the range of smaller packing fraction, a fixed inflow rate less than Q_d will give a unique small packing fraction, which corresponds to the dilute flow state. In this region, the flow rate increases with the increase of the packing fraction. The positive slope of the relation between the flow rate and packing fraction results in the flow state being stable for the fluctuation of the inflow rate. The mechanism is like the following. For an instantaneous increase (decrease) of the inflow rate, the packing fraction will increase (decrease) first. The increase of the packing fraction will increase (decrease) the flow rate if it is in the range of the positive slope. Then in turn, the increase (decrease) of the flow rate will then decrease (increase) the packing fraction. This feedback mechanism could keep on the packing fraction at its stable value. Thus, the flow state in this range is defined as stable dilute flow state. Similar to the previous consideration, we can find that the flow state with negative slope is unstable in fixed inflow rate condition, because the larger packing fraction will decrease the flow rate in this case, which will cause further crowding, so the flow state will depart from its original state further and move to a stable dense flow point finally. The process going to stable dense flow point is very fast, and the surviving time of the state is almost neglected in experiments. Considering both discussions on the stable and unstable states, the flow state with positive slope but having higher flow rate than Q_d should be metastable, because the flow state in this range is unstable in large fluctuations, though it is stable in small fluctuation. When the fluctuation exceeds the certain allowed range, the high packing fraction causes the local flow rate to be less than the inflow rate, and then the area will begin to accumulate particles and further decrease the local flow rate, and so on. In this metastable state, the dilute flow state can survive for some time, but finally transforms to the stable dense flow point, and the surviving time will be longer if the inflow rate is far from the maximum flow rate, because it will take some time for large fluctuations to occur. The boundary between the metastable and unstable states is at the maximum point, which closes to the boundary of the dilute flow state and dense flow state defined by the discontinuity of the flow rate or packing fraction if the observation time is short. Near this point, the abrupt change to the stable dense flow point is usually considered as discontinuity. To be consistent with preexisting works, we suggest to call the metastable state as metastable dilute flow state and the unstable state as unstable dense flow state.

Our simulations show very similar characteristic with that of the backward propagating shockwaves [15,16], for example, the piling-up from area-2 and then spreading to the area behind it. We know that the effective sound speed of granular system is very slow because of the larger mass of the particles, so it is natural to see some properties of the shockwaves. It would be interesting to check if an acoustic equation can give a similar relationship between the inflow rate and the packing fraction as ours, we will do this in near future.

V. CONCLUSIONS

In conclusion, we have first observed the time variation of the transition from dilute flow state to dense flow state by both experiments and simulations. Then we have shown that both the flow rate and the packing fraction are continuous in the fixed particle number condition. The relationship between the flow rate and the packing fraction has a maximum at a moderate packing fraction and is terminated at high density with a negative slope. By discussing these characteristics in detail, we have defined four flow states clearly for the fixed inflow rate condition and show that the transition between dilute flow state and dense flow state is a stochastic process. The discontinuities and the complex time variation behavior occurring in the transition have been explained by the abrupt variation through an unstable flow state. All these descriptions are consistent with those we observed in both experiments and simulations.

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