

Minority games with score-dependent and agent-dependent payoffs

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Score-dependent and agent-dependent payoffs of the strategies are introduced into the standard minority game. The intrinsic periodicity is consequently removed, and the stylized facts arise, such as long-range volatility correlations and “fat tails” in the distribution of the returns. The agent dependence of the payoffs is essential in producing the long-range volatility correlations. The new payoffs lead to a better performance in the dynamic behavior nonlocal in time, and can coexist with the inactive strategy. We also observe that the standard deviation σ^2/N is significantly reduced, thus the efficiency of the system is distinctly improved. Based on this observation, we give a qualitative explanation for the long-range volatility correlations.

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I. INTRODUCTION

In past years a great deal of attention from physicists has been drawn to complex socioeconomic systems. Methods and concepts in statistical physics have been applied in understanding markets and economies. On the one hand, the phenomenological analysis of the financial data has led to the characterization of certain empirical statistical regularities, known as the “stylized facts,” such as long-range volatility correlations and “fat tails” in the distribution of the returns [1–12]. On the other hand, different microscopic models have been built for describing the financial markets [13–24]. Among them, the minority game (MG) is one of the important examples.

The standard MG [16,25] was initially designed as a simplification of Arthur’s famous El Farol’s Bar problem [26]. It describes a system in which many heterogeneous agents adaptively compete for a scarce resource, and it captures some key features of a generic market mechanism and the basic interaction between the agents and public information. However, it is a highly simplified model. To make it more realistic in comparison with the real markets, the microeconomic behavior of the agents should be taken into account.

Consequently, different variations [20,27–31] of the standard MG have been proposed. For example, the *inactive strategy* is introduced, which grants the agents with the possibility of not trading in the market, to mimic the real markets. In this case, the number of agents actively trading at each time step varies throughout the game. This type of extension is called the grand canonical MG. The inactive strategy in the grand canonical MG is considered to be the key ingredient leading to the stylized facts.

However, in the standard MG and its many variations, the double periodicity is quite annoying. Although the inactive strategy eliminates the periodicity from certain observables, the grand canonical MG still suffers from an intrinsic annoyance of the periodic dynamics [23,32]. For example, it fails in correctly characterizing the dynamic behavior nonlocal in time [32].

In most MGs, payoffs are generally referred to as rewards to the agents: each agent in the minority group is awarded one point, while each agent in the majority group is awarded nothing. In a recent variation of the MG, payoffs depending

on the size of the minority group have been introduced to quantify the performance of the strategies [33,34]. Thus the payoffs are generalized to be the rewards to the strategies. The results in Refs. [33,34] indicate that the main features of the standard deviation σ^2/N , which characterizes the fluctuation of the active agents, are not modified by the new forms of the payoffs.

Looking for more reasonable payoffs, which could mimic the financial markets, is also of great interest. There may be thousands of forms of the payoffs for the strategies. However, we may assume that the payoff of each strategy is mainly determined by two factors: the global information of the outcome after all the agents make their own decisions, i.e., the number of the agents in the minority group; and the cumulative performance of the individual strategy, i.e., the score of the strategy itself. Since the resource in the market is limited, the payoffs increase with a decreasing size of the minority group [33]. On the other hand, we assume that the strategies with higher scores should be rewarded or punished more than those with lower scores, since people would be more attentive to the outstanding strategies. More importantly, even for a same strategy, each agent may independently evaluate the performance of the strategies, and this gives rise to the *agent-dependent payoffs*.

In this paper, we propose a type of payoff that can be score-dependent and/or agent-dependent. Interestingly, the double periodicity is essentially removed. The stylized facts, especially the long-range volatility correlations and dynamic property nonlocal in time, can be reproduced. The efficiency of the system is also enhanced.

In Sec. II, the payoffs both score-dependent and agent-dependent are introduced to the standard MG. The autocorrelation function of the volatilities and the probability distribution of the returns are studied, and the agent-dependent essence of the payoffs will be revealed. In Sec. III, the new type of payoffs is introduced to the grand canonical MG, and the persistence probability is investigated for different variations of the MG. In Sec. IV, the market efficiency is concerned, and a qualitative explanation is given for the long-range volatility correlations. Section V contains the conclusion.

II. STANDARD MG WITH SCORE-DEPENDENT AND AGENT-DEPENDENT PAYOFFS

A. Model

The MG takes the form of a repeated game with an odd number of agents N who must choose an action independently whether to buy or sell. Those agents in the minority group are winners. Each agent chooses his or her decision at a given time step according to the prediction of a strategy based on the history, namely, the m most recent outcomes of the winner side. Since there are a total of 2^m possible history bit strings of the winner side, and there are two possible options (buy or sell) for each history bit string, there are a total of 2^{2^m} strategies.

At the beginning of the game, each agent i randomly picks S strategies from the full strategy space and keeps track of the cumulative performance of his or her trading strategy s , $s=1, \dots, S$ by assigning a score $U_{i,s}$ to it. The initial scores of the strategies are set to be zero. At each time step, each agent i adopts the strategy $s_i(t')$ with the highest score, and the action will then be $a_i(t') = \sigma_{i,s_i(t')}^{\mu(t')}$, according to the history $\mu(t')$, which is the common information shared by all the agents. $\sigma_{i,s_i(t')}^{\mu(t')} = +1$, or -1 denotes buying or selling. When a tie results from the strategies with the highest score, one of them is picked randomly. The *excess demand* is then defined as $A(t') = \sum_{i=1}^N a_i(t')$. We assume that the payoffs $g_{i,s}(t')$ generally depend on the scores of the strategies, then

$$U_{i,s}(t'+1) = U_{i,s}(t') + g_{i,s}(t'), \quad (1)$$

$$g_{i,s}(t') = -\sigma_{i,s}^{\mu(t')} A(t') \frac{a[U_{i,s}(t') - U_0] + b}{\sum_{s'=1}^S \{a[U_{i,s'}(t') - U_0] + b\}}. \quad (2)$$

Here U_0 is the lowest score of the strategies of the agent i at time t' . To ensure a positive weight $\{a[U_{i,s}(t') - U_0] + b\} / \sum_{s'=1}^S \{a[U_{i,s'}(t') - U_0] + b\}$, the parameters a and b are taken to be real and positive. Thus the strategies with higher scores will be rewarded or punished more than those with lower scores. Actually, the parameters a and b are *not independent*. If both the numerator and denominator in the weight are multiplied by a factor $1/b$, only the ratio a/b remains. Therefore, we simply put $b=1.0$. For the parameter $a=0$, the standard MG is recovered. The payoff is given by $g_{i,s}(t') \sim -\sigma_{i,s}^{\mu(t')} A(t')$. Those strategies giving the prediction consequently proved to be in the minority are rewarded with the same payoffs regardless of their scores, so the payoffs and the scores of the strategies are naturally not agent dependent.

The payoffs in Eq. (2) are *not only score-dependent, but also agent-dependent*, since the weight $\{a[U_{i,s}(t') - U_0] + b\} / \sum_{s'=1}^S \{a[U_{i,s'}(t') - U_0] + b\}$ is agent dependent. The score dependence of the payoffs is based on the assumption or plausible observation that people may be more attentive to the outstanding strategies. Once a high-score strategy gives a prediction, which is subsequently proved to be in the major-

ity, it will be restrained from the market for a longer time. When it gives a prediction which is subsequently proved to be in the minority, it will likely be adopted in the future for making more profits. For a low-score strategy, it is not so much affected by the outcome of the game. The agent dependence of the payoffs is assumed, since the evaluation of the strategies should vary with agents. In this sense, the agents in our model are heterogeneous. An agent evaluates his strategies according to their relative performances.

Following Refs. [20,35–37], a simple price dynamics of the returns $r(t')$ in terms of the excess demand $A(t')$ is defined as

$$r(t') = A(t'). \quad (3)$$

B. Autocorrelation function and cumulative distribution function

MGs behave differently in different regimes of the parameter space. In literature it is reported that the stylized facts of the financial market emerge in the region $2^m/N \leq \alpha_c$ with $\alpha_c \sim 0.32$ [38–40]. For a bigger $2^m/N$, the dynamics tends to be a random walk. Therefore we mainly present our results for a small m and a big N . Meanwhile, we take S much smaller than 2^m , the total number of the strategies, otherwise for a big S every agent may have almost the same strategies.

For the standard MG with a small m and a big N , however, a crowd of agents choose a particular strategy and thus induce the so-called double periodicity [41,42]. Assuming that the game visits each possible history bit string with an equal probability, a particular history bit string will be encountered about every 2^m time steps. If the odd occurrences of a particular history result in the increase of the scores of certain strategies, the even occurrences will lead to the deduction of the scores of the same strategies due to the overcrowded use of these strategies, which predict correctly in the previous occurrence of the history. This is a kind of periodic behavior with a period of 2×2^m time steps.

After introducing the new payoffs in Eq. (2), the dynamic evolution of the standard MG is then modified. For a same strategy, it may be the best strategy for one agent, but it can be a worse strategy for another agent. Thus the overcrowded use of certain strategies are avoided and the double periodicity is eliminated.

The long-range temporal correlations of the volatilities is a well-known stylized fact. The autocorrelation function of the volatilities decays by a power law

$$c(t) \sim t^{-\lambda}, \quad (4)$$

and the exponent λ is estimated to be about 0.3 in real financial markets [43,44]. In this paper, we define the volatility as $|A|$, and then the autocorrelation function

$$C(t) \equiv \frac{\langle |A(t')||A(t'+t)| \rangle - (\langle |A(t')| \rangle)^2}{\langle |A(t')|^2 \rangle - (\langle |A(t')| \rangle)^2}, \quad (5)$$

where $\langle \dots \rangle$ represents the average over the time t' . We have performed extensive numerical simulations of our improved MG. Here we present the results for $N=5001$, $S=2$, and m

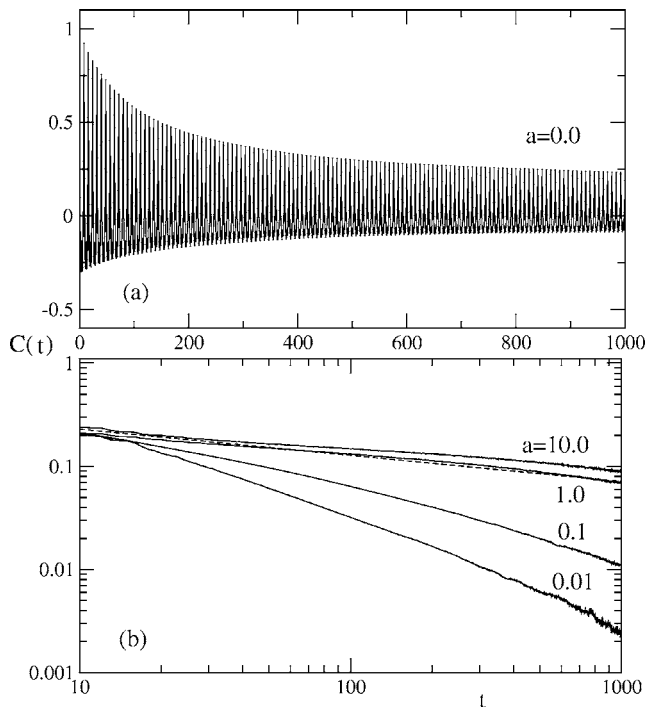


FIG. 1. Autocorrelation functions of the volatilities for different values of the parameter a : (a) $a=0.0$, (b) $a=0.01, 0.1, 1.0$, and 10.0 for the standard MG with the payoffs both score dependent and agent dependent with $N=5001$, $S=2$, and $m=2$. The dashed line has a slope of -0.25 .

$=2$. Obviously the parameter a plays a key role.

In Fig. 1, the autocorrelation function for different values of a are plotted. For each value of a , an average is taken over 100 runs with 10^5 iterations per run. Before collecting the data in each run, 5000 iterations have been performed for equilibration. The figure shows that as a changes from 0.0 to 10.0, a crossover behavior occurs. According to Eq. (2), the payoff for $a=0.0$ is equal to $-\frac{1}{2}\sigma_{i,s}^{\mu(t')}A(t')$ (here $S=2$). This is just the payoff of the standard MG except for a factor of $1/2$. As it is shown in Fig. 1(a), the curve for $a=0.0$ exhibits a strong periodic behavior with a period 2×2^2 , and the shape is the same as that of the standard MG.

In Fig. 1(b), the autocorrelation function for $a \neq 0.0$ are plotted on a log-log scale. For clarity, the curves for $a=10.0$ and $a=0.1, 0.01$ are shifted slightly upward and downward, respectively. We observe that as a increases, the autocorrelation function tends to show a power-law behavior, though for a relatively small $a < 0.001$ the periodic behavior still remains. The exponent λ changes from -1.05 to -0.21 as a changes from 0.01 to 10.0. The large a limit will be discussed in the next subsection.

In real markets, the cumulative probability distribution function (CDF) of the returns is known to have a fat tail such as

$$P(|A|) \sim |A|^{-\nu}, \quad (6)$$

with an exponent $\nu \sim 3.0$ on average [44]. The CDF of our model is also carefully investigated. Since the CDF for posi-

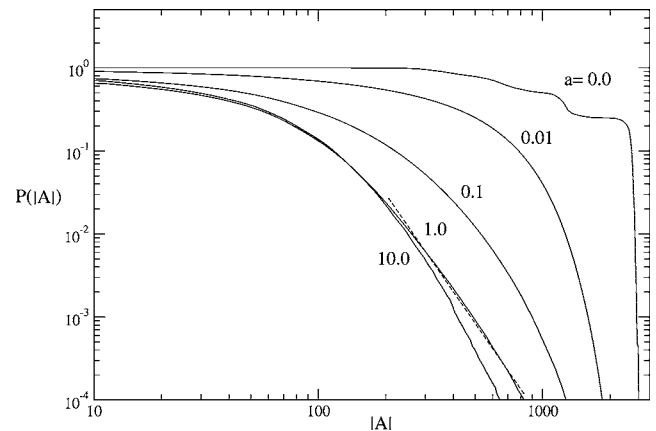


FIG. 2. Cumulant distribution functions of the volatilities for different values of the parameter a : (a) $a=0.0$, (b) $a=0.01, 0.1, 1.0$, and 10.0 for the standard MG with payoffs both score dependent and agent dependent with $N=5001$, $S=2$, and $m=2$. The dashed line has a slope of -3.9 .

tive and negative returns are symmetric, the CDF of the magnitude of the returns is computed.

The CDF also shows a crossover behavior similar to that of the autocorrelation function. In Fig. 2, the CDF for the parameter $a=0.0, 0.01, 0.1, 1.0$, and 10.0 are plotted on a log-log scale. For $a=0.0$, the CDF decays rapidly to zero, displaying a Gaussian-type shape. As the parameter a increases, a crossover behavior occurs. For a big value of a , a power-law tail of the CDF is observed. The exponent ν is equal to 3.9 for $a=1.0$.

In summary, the main features of our improved MG are dominated by the parameter a , which is a measure of the intensity of the reward and punishment. The larger a is, the more the high-score strategies are rewarded or punished, and the more heterogeneous the agents are. For $a=0.0$, the model behaves the same as the standard MG. As a increases, the dynamic behavior gradually stabilizes. For $a=1.0$, as is shown in Figs. 1 and 2, the autocorrelation function and CDF show a power-law behavior, and the exponents are estimated to be $\lambda=0.25$ and $\nu=3.9$, close to those of the real markets.

C. Agent-dependent payoffs

It is inspiring that the new payoffs in Eq. (2) make a distinct improvement on the dynamic behavior of the standard MG, and one may wonder what is the underlying mechanism responsible for it. To address the essence of the new payoffs, we consider the following two cases: the MG with the payoffs only score dependent and the MG with the payoffs only agent dependent.

For the MG with the payoffs only score dependent, we slightly modify the denominator in Eq. (2) such that

$$g_{i,s}(t') = -\sigma_{i,s}^{\mu(t')}A(t') \frac{a[U_{i,s}(t') - U_0] + 1}{M \sum_{s'=1} \{a[U_{i,s'}(t') - U_0] + 1\}}, \quad (7)$$

with $M=2^{2^m}$ being the total number of the strategies. Now U_0 is the lowest score of all the strategies at time t' . Since

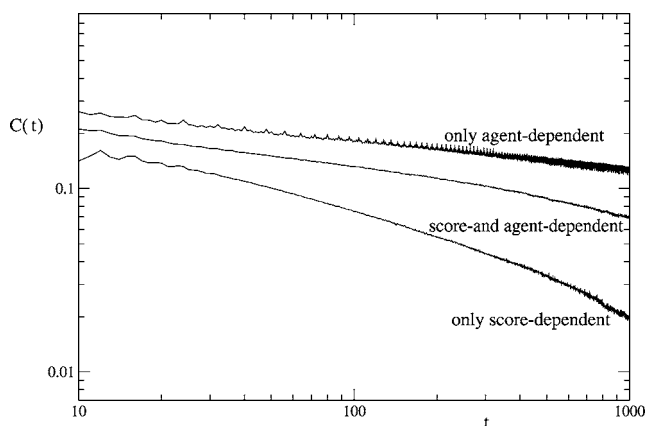


FIG. 3. Autocorrelation functions of the volatilities for the MGs with different types of payoffs for $N=5001$, $S=2$, and $m=2$: the payoffs only agent dependent, payoffs both score dependent and agent dependent, and payoffs only score dependent, respectively, denoted by solid lines from top to bottom.

the denominator sums over all the possible strategies, the payoffs are only score dependent, not agent dependent. In Fig. 3, the autocorrelation function of the MG with the payoffs only score dependent for $a=1.0$ is plotted. For a direct comparison, the autocorrelation function of the MG model with the payoffs both score dependent and agent dependent in Eq. (2) for $a=1.0$ is also plotted. Obviously, the payoffs only score dependent distinctly suppress the strong periodicity of the standard MG, but the autocorrelation function decays somewhat faster than a power law. For the MG with the payoffs both score dependent and agent dependent, the autocorrelation function exhibits a better power-law behavior, with an exponent close to that of the real markets.

For the MG with the payoffs only agent dependent, we consider the large a limit for the payoffs in Eq. (2). In this case, only the score of the strategy with the highest score of each agent at that moment is updated, but there may be more than one strategy with the highest score. Since the agent adopts one of the strategies with the highest score, let us further simplify the game and just update the strategy, which is actually adopted by the agent, and the others remain unchanged. Since the strategy with the highest score varies for different agents, the payoffs are agent dependent, not score dependent.

In Fig. 3, the autocorrelation function of the MG with the payoffs only agent dependent is also plotted. It follows a nice power-law behavior with an exponent slightly smaller than that of the MG with the payoffs both score dependent and agent dependent. But a weak periodic oscillation is still observed. We have also investigated other stylized facts of the MG with the payoffs only agent-dependent, e.g., the CDF, the persistence probability (see the next section), etc. The results indicate that the MG with the payoffs only agent dependent and the MG with the payoffs both score dependent and agent dependent behave not much differently.

In summary, the score dependence of the payoffs removes the strong periodic behavior of the standard MG, and the agent dependence of the payoffs brings a better power-law behavior of the autocorrelation function. The weak periodic

behavior of the MG with the payoffs only agent dependent may come from the oversimplified mechanism of scoring presented in this subsection. At this stage, the payoffs both score dependent and agent dependent are a better choice.

III. GRAND CANONICAL MG AND PERSISTENCE PROBABILITY

A. Grand canonical MG with payoffs score dependent and agent dependent

The grand canonical MG, in which an inactive strategy is included, has been proposed to mimic the real markets. In fact, the grand canonical MG reproduces most stylized facts [20,27–31], except for certain dynamic properties, such as the dynamic behavior nonlocal in time, etc. [32]. In the preceding section, we assume that the payoffs both score dependent and agent dependent is an essential ingredient of the MG, which also leads to the long-range temporal correlations. Theoretically, one expects that the score-dependent and agent-dependent payoffs could coexist with the inactive strategy in the MG. Therefore, we introduce the payoffs both score dependent and agent dependent to the grand canonical MG [20].

The model consists of two types of agents: speculators and producers. Speculators are assigned a number $S+1$ of the strategies, among which S are active strategies, and the other is an inactive strategy with $\sigma_{i,0}^{\mu(t')}=0$. Each speculator keeps track of the cumulative performances of his or her strategies, and adopts the strategy with the highest score. The scores of the active strategies take the same updating form as in Eqs. (1) and (2). In the grand canonical MG, the score of the inactive strategy is usually set to be zero, since it does not predict any action. Sometimes such a scheme may be too simple. One may think that all strategies, including the inactive strategy, should be reasonably rewarded or punished.

Therefore, we assume that the payoffs of the inactive strategy could be evaluated according to the performance of the active strategy with the highest score,

$$g_{i,0}(t') = \sigma_{i,s_i(t')} A(t') \frac{a[U_{i,0}(t') - U_0] + 1}{\sum_{s'=0}^S \{a[U_{i,s'}(t') - U_0] + 1\}}, \quad (8)$$

where $s_i(t')$ is the active strategy with the highest score of the agent i . If the active strategy with the highest score makes a prediction, which is consequently proved to be in the minority group, the score of the inactive strategy should be deducted because one may lose the profit from the active strategy. If the active strategy with the highest score makes a prediction, which is consequently proved to be in the majority group, the score of the inactive strategy should be increased because it may help avoid the loss caused by the active strategy. With this updating form, the payoff of the inactive strategy is regulated to be both score dependent and agent dependent like other active strategies.

The other type of agents are producers who offer the information to feed the speculators [20]. The producers only have one active strategy randomly picked at the beginning of the game. Let N_s and N_p be the number of speculators and producers, both of them contribute to the outcome of the game, thus the excess demand is defined as $A(t') = \sum_{i=1}^{N_s+N_p} a_i(t')$.

In the literature, both the real history and random history have been adopted in the numerical simulations of the standard MG [45,46] and the grand canonical MG [15,20,28,31,38]. A random history bit string is drawn randomly and independently from the integers $1, \dots, 2^m$. Sometimes the random history is convenient for the analytical study. In Refs. [45,46], it has been argued that for some observables, the history of the standard MG is irrelevant. For the grand canonical MG, the real history and random history usually produce qualitatively similar results. Only in some cases the dynamic behavior of the grand canonical MG with the real history is somewhat uneven. For example, for a very small S and m such as $S=2$ and $m=2$, the periodic behavior still remains. The random history yields a relatively clean power-law behavior for the autocorrelation function, etc. In this subsection, we have also performed the numerical simulations with both the real history and random history. For a very small S and m such as $S=2$ and $m=2$ and with the real history, the grand canonical MG with the payoffs both score dependent and agent dependent also show certain irregular behavior. But as suggested above, this should not be the defect of the score-dependent and agent-dependent payoffs.

Here we report the simulations for $a=1.0$, and with a medium m and S , which are still small enough compared with N_s and N_p . In addition to the scoring of the inactive strategy in Eq. (8), we have also used an alternative scoring of the inactive strategy for comparison, which sets the score of the inactive strategy to be a random walk: $U_{i,0}(t'+1) = U_{i,0}(t') + \varepsilon$, with ε being a small random variable corresponding to the random noise of the social environment. Both types of scoring lead to qualitatively the same results.

In Fig. 4 we show the long-range temporal correlations of the volatilities in the grand canonical MG with the score-dependent and agent-dependent payoffs. The solid line is for $N_s=500$, $N_p=1001$, $S=3$, and $m=6$ with the random history, and the scoring in Eq. (8) for the inactive strategy. The slope of the curve in a time interval [100, 4000] is measured to be -0.14 . The lower dotted line is for $N_s=500$, $N_p=1001$, $S=4$, and $m=7$ with the random history, and the score of the inactive strategy is set to be the random walk. The curve obeys a nice power-law behavior with an exponent -0.43 . The upper dotted line is for $N_s=500$, $N_p=1001$, $S=4$, and $m=7$ with the real history, and the score of the inactive strategy is set to be the random walk. The slope of the curve in a time interval [100, 4000] is measured to be -0.35 , close to that of the real markets.

B. Persistence probability distribution

In the preceding sections and subsections, we have demonstrated that both the MG with the inactive strategy [20] and the MG with the score-dependent and agent-dependent

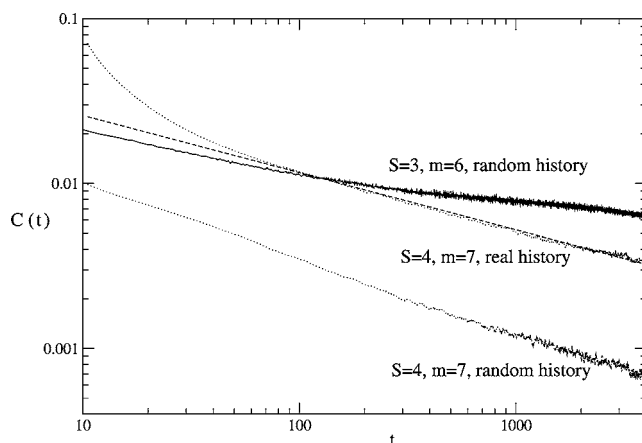


FIG. 4. Autocorrelation functions of the volatilities for the grand canonical MGs with the payoffs both score dependent and agent dependent for $N_s=500$ and $N_p=1001$. The solid line is obtained with the random history for $a=1.0$, $S=3$, and $m=6$, and the inactive strategy is evaluated by the highest score of the active strategy. Dotted lines are for $a=1.0$, $S=4$, and $m=7$, and the inactive strategy has the score of a random walk: the upper dotted line is obtained with the real history, the lower dotted line is with the random history. The dashed line has a slope of -0.35 .

payoffs are rather successful in producing the stylized facts of the financial markets. In this subsection, we show that the MG with the inactive strategy still suffers from certain characteristics of the periodic dynamics, especially with respect to the dynamic behavior nonlocal in time. On the other hand, the MG with the payoffs both score dependent and agent dependent correctly produce the dynamic behavior nonlocal in time.

In recent years, much attention in nonequilibrium dynamics has been drawn to the dynamic behavior nonlocal in time [47]. Such a concept has also been introduced to the financial dynamics. An example is the persistence probability distribution [22,32,48,49], which describes the dynamic behavior nonlocal in time, and also corresponds to the cumulative distribution function of the first passage time [50,51].

Starting from a time t' and $|A(t')|$, the persistence probability $P_-(t)$ [$P_+(t)$] is defined as the probability that $|A(t'+\tilde{t})|$ has always been below (above) $|A(t')|$ in a time t , i.e., $|A(t'+\tilde{t})| < |A(t')|$ [$|A(t'+\tilde{t})| > |A(t')|$] for all $\tilde{t} < t$. The average is taken over t' . In general, the persistence probability distribution provides additional information to the autocorrelation function.

In Fig. 5, we compare the persistence probability distribution $P_-(t)$ of the daily data of the German DAX with that of a random walk, the MG with the inactive strategy [20], the MGs with the payoffs only score dependent, only agent dependent, and both score dependent and agent dependent. For the daily data of the German DAX, $P_-(t)$ obeys a universal power law

$$P_-(t) \sim t^{-\theta}, \quad (9)$$

with an exponent θ estimated to be $0.90(2)$. The exponent $\theta < 1.0$ indicates a long-range correlation nonlocal in time.

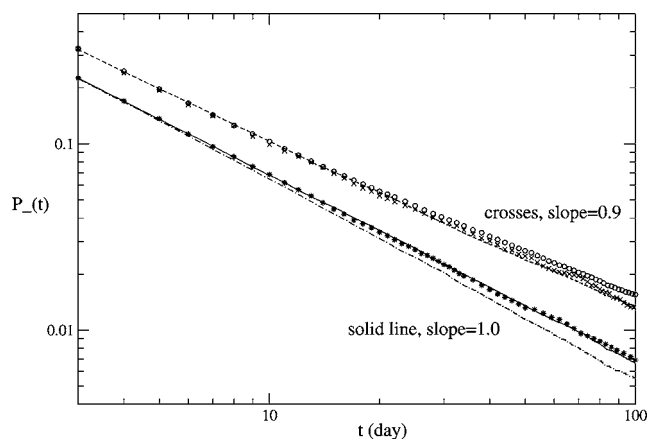


FIG. 5. The persistence probability distribution $P_-(t)$ of the daily data of German DAX is compared with those of a random walk and the MGs. Crosses denote $P_-(t)$ measured with the daily records of the German DAX from October 1959 to January 1997. The dashed line is $P_-(t)$ of the MG with the payoffs both score dependent and agent dependent at $a=1.0$, and circles denote that of the MG with the payoffs only agent dependent. The dotted-dashed line is $P_-(t)$ of the MG with the payoffs only score dependent. The solid line denotes $P_-(t)$ of the random walk and stars denote that of the MG with the inactive strategy.

$P_-(t)$ of the random walk and the MG with the inactive strategy obey a power-law behavior with the same exponent of 1.0, different from that of the German DAX. The MG with the inactive strategy fails to produce the long-range correlation nonlocal in time, and may still suffer from an implicit periodicity.

$P_-(t)$ of the MG with the payoffs both score dependent and agent dependent is consistent with that of the daily data of German DAX. $P_-(t)$ of the MG with the payoffs only agent dependent yields a curve with an exponent $\theta=0.82$, slightly smaller than that of the German DAX. The power-law behavior of $P_-(t)$ of the MG with the payoffs only score dependent is less clean, with an effective exponent bigger than 1.0. This further confirms our observation that the agent dependence of the payoffs enhances the temporal correlation and the score dependence of the payoffs helps mainly remove the intrinsic periodicity.

$P_+(t)$ behaves much differently from $P_-(t)$ and decays faster than a power law. The behavior of $P_+(t)$ is not universal, for example, in the sense that it depends on the time scale [32,49]. In Fig. 6, $P_+(t)$ and $P_-(t)$ of the minute-to-minute data of the German DAX and the MG with the payoffs both score dependent and agent dependent are plotted. Within the errors the exponent $\theta=0.88(2)$ for $P_-(t)$ measured with the minute-to-minute data of the German DAX is consistent with $\theta=0.90(2)$ with the daily data. $P_+(t)$ decays faster than $P_-(t)$. We observe that $P_+(t)$ of the MG with the payoffs both score dependent and agent dependent behaves similarly to that of the minute-to-minute data of the German DAX. The grand canonical MG leads to qualitatively the same results, but the standard MG fails (not shown in the figure). As one gradually changes the time scale from a minute to a day, and to a week, etc., $P_+(t)$ tends to come

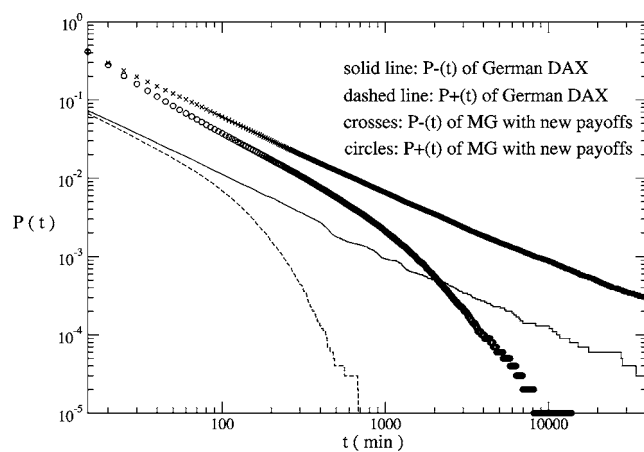


FIG. 6. Persistence probability distributions $P_-(t)$ and $P_+(t)$ of the minute-to-minute data of the German DAX are compared with those of the MG with the payoffs both score dependent and agent dependent. $P_-(t)$ and $P_+(t)$ measured with the minute-to-minute records of the German DAX from December 1993 to July 1997 are denoted, respectively, by the solid line and the dashed line. $P_-(t)$ and $P_+(t)$ of the MG with the payoffs both score dependent and agent dependent at $a=1.0$ are denoted, respectively, by crosses and circles.

closer to $P_-(t)$. None of the above models could exactly follow this crossover.

In the MGs, the total number of the strategies is usually rather limited, and every agent selects S strategies and keeps them unchanged during the dynamic evolution. Even though the inactive strategy can modulate the number of the agents who share a same strategy with the highest score, it is not sufficient to completely change the periodic nature of the dynamics. After introducing the score-dependent and agent-dependent payoffs, however, a same strategy is independently evaluated by different agents. It looks like many degrees of freedom are generated, and therefore, a correct long-range correlation nonlocal in time emerges and the double periodicity of the dynamics is essentially eliminated.

IV. MARKET EFFICIENCY AND LONG-RANGE VOLATILITY CORRELATIONS

To understand the efficiency of the system, let us define the standard deviation of the number of the agents in the buyer group,

$$\frac{\sigma^2}{N} = \frac{1}{N} \left\langle \left(N_b(t') - \frac{N}{2} \right)^2 \right\rangle, \quad (10)$$

$N_b(t')$ is the number of the agents in the buyer group at each time step, and $\langle \dots \rangle$ represents the average over the time t' . σ^2/N is a convenient reciprocal measure of how efficient the system is at distributing resources. The smaller it is, the smaller the magnitude of the excess demand $A(t')$ is. In Ref. [33] it has been suggested that the main features of the standard deviation do not depend on the choice of the payoffs. In this paper, different types of payoffs are introduced. Here we are interested in the effect of the new payoffs on the standard

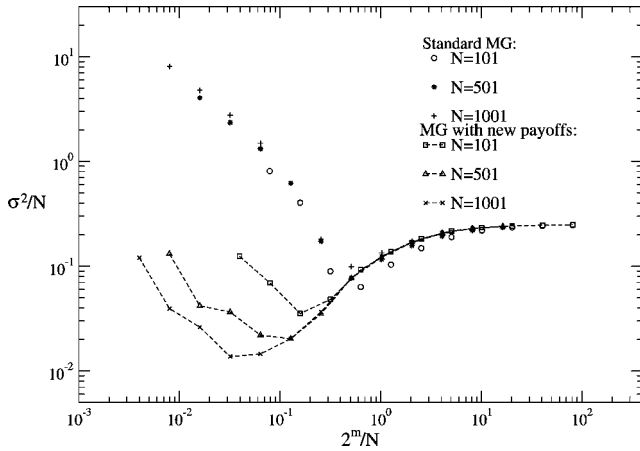


FIG. 7. σ^2/N as a function of $z=2^m/N$ for the standard MG with the payoffs both score dependent and agent dependent with $a=1.0$, $S=2$, and $N=101, 501, 1001$ denoted by squares, triangles, and crosses, respectively, in comparison with those for the standard MG denoted by circles, stars, and pluses.

deviation. We report the result of σ^2/N at $a=1.0$, since it provides a power-law behavior for the autocorrelation function and CDF with the exponents close to those of the real markets.

In Fig. 7, σ^2/N for the standard MG is shown to be a function only of $z=2^m/N$ [42]. In a certain sense, the system undergoes a phase transition: (1) for a small z , σ^2/N is very large, and decreases as z increases; (2) at the transition point $z=z_c$, it reaches a minimum; (3) as z goes beyond z_c , σ^2/N slowly increases and approaches the value for a random walk for a large z .

In Fig. 7, we also plot σ^2/N as a function of z for $S=2$ and $N=101, 501, 1001$ for the MG with the payoffs both score dependent and agent dependent in comparison with the standard MG. We find that the main phase structure of the standard MG remains, and for a large z , σ^2/N also approaches the value for a random walk. Different from that of the standard MG, however, σ^2/N is a function not only of $z=2^m/N$ when z is small. Different N lead to different curves, although the curves look similar in shape. This phenomenon can be qualitatively understood. We note that $2^m \sim \ln M$ and $M=2^{2^m}$ is the total number of the strategies. Since now the agents independently evaluate their strategies, the *effective number of the strategies* increases with the number N of the agents. Therefore, σ^2 does not increase in proportion to N , and σ^2/N as well as the critical value z_c decreases as N increases. This strongly indicates that the MG with the payoffs both score dependent and agent dependent makes a distinct improvement in the utilization of the overall resource at the small value of z . It confirms also our conjecture that the new payoffs could efficiently remove the crowd effect of the standard MG.

If one can calculate the effective number M_{eff} of the strategies, in principle, σ^2/N might be a function only of $\ln M_{eff}/N$. But we have not been able to find M_{eff} , and therefore this remains as an interesting problem.

To further understand the payoffs both score dependent and agent dependent, we investigate the average frequency

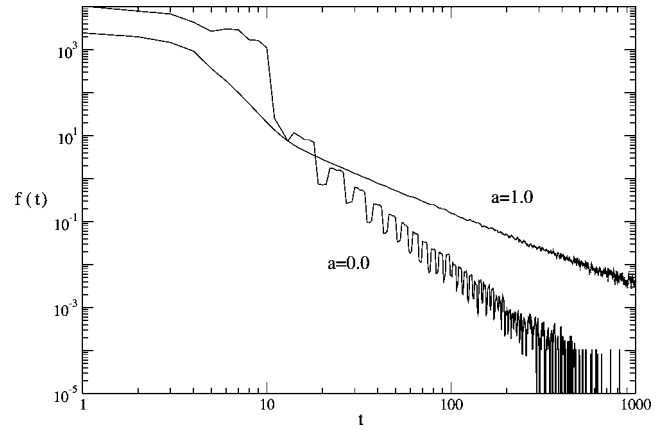


FIG. 8. The average frequency $f(t)$ of the strategies cited continuously in time t for each agent. The results are for $S=2$ and $m=2$, and averaged over 100 runs. The upper solid line is for $a=1.0$ and the lower solid line is for $a=0.0$.

$f(t)$ of the strategies cited continuously in time t for each agent. The results for $a=0.0$ and 1.0 are averaged over 100 runs and are shown in Fig. 8. The curves for a single run look similar but with larger fluctuations.

For $a=0.0$, $f(t)$ displays a periodicity of 2×2^m , and it consists with that of the standard MG [41]. For $a=1.0$, $f(t)$ shows a power-law behavior and no periodic behavior is observed. Since the payoffs both score dependent and agent dependent could efficiently remove the crowd effect, the payoffs of the strategies for the even occurrences of a particular history may be positive, thus the strategies could be cited continuously in a long period of time, and the periodicity is consequently eliminated. This has also been confirmed in the calculations of the autocorrelation function.

When we look more carefully at the curves of $f(t)$ in Fig. 8, we observe (i) at the early stage of the time evolution, for example, $t \in [1, 16]$, $f(t)$ for $a=0.0$ decays almost 4 orders of magnitude, while the frequency for $a=1.0$ decays only 3 orders of magnitude; (ii) for $t > 500$, $f(t)$ for $a=0.0$ drops rapidly to zero, while $f(t)$ for $a=1.0$ obeys a power-law behavior. Therefore, for $a=1.0$ the agents continuously cite certain strategies for quite a long time until the performances of his or her strategies are reversed. It seems that the agents are more cautious of their choices after introducing the payoffs both score dependent and agent dependent. This may lead to the long-range volatility correlations.

In summary, we testify that the payoffs both score dependent and agent dependent efficiently remove the crowd effect of the standard MG. The periodicity of the standard MG is consequently eliminated and the efficiency is essentially increased. On the other hand, since the agents could continuously cite the strategies for a long time, a nonperiodic and long-range time correlation is observed.

V. CONCLUSIONS

In summary, we introduce the payoffs both score dependent and agent dependent into the standard minority game (MG), and testify that this kind of payoff of the strategies

may serve as an alternative dynamic mechanism of reproducing the stylized facts of the real markets, and can coexist with the inactive strategy. The agent dependence of the new payoffs is essential in producing the long-range volatility correlations. Compared with the MG with the inactive strategy and other variations of the MG, the MG with the payoffs both score dependent and agent dependent makes a distinct improvement in removing the intrinsic periodicity, and especially in the dynamic behavior nonlocal in time. In addition, the efficiency of the system is significantly enhanced at the

small value of z by avoiding the overcrowded use of certain strategies. Based on this observation, an explanation is given for the long-range volatility correlations.

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