Painlevé classification of a generalized coupled Hirota system

Gui-qiong Xu*

Department of Information Management, College of Business and Management, Shanghai University, Shanghai 201800, China

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By means of singularity analysis, the integrability of a system of generalized coupled Hirota equations with parameter coefficients is tested. It is proven that the system passes the Painlevé test for integrability only in two distinct cases.

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Many powerful methods to solve integrable models, such as the inverse scattering transformation (IST), Bäcklund and Darboux transformation, symmetry reduction, the bilinear approach, and Painlevé analysis, have been established by many famous mathematical physicists. The usual real physical nonlinear systems can be treated as perturbations of the related integrable models [1]. So to find as many as possible completely integrable models plays an important role in nonlinear science [2].

When one says that a model is integrable, one should point out under what special meaning(s) this is true. For instance, we say a model is P integrable (Painlevé integrable) if it possesses the Painlevé property and a model is Lax or IST integrable if it has a Lax pair and then can be solved by the IST approach.

In recent years, the Painlevé analysis has been identified as a powerful tool in the search for new integrable systems [3,4]. The remarkable feature of this analysis, particularly for soliton equations, is that a natural connection exists in relation to the Lax pair, Bäcklund transformation, and Hirota bilinear forms. Therefore, investigating the underlying integrable models of a general form of soliton equations by means of this analysis is quite interesting [5–8].

We consider a generalized coupled Hirota (GCH) system in the form [11,12]

$$
\begin{aligned}
&iq_{1t}+c_1q_{1zz}+2(\alpha|q_1|^2+\beta|q_2|^2)q_1-i\epsilon[q_{1zzz}+(2\mu_1|q_1|^2\\
&\qquad+\nu_1|q_2|^2)q_{1z}+\nu_1q_1q_2^{\star}q_{2z}]=0,\\
&iq_{2t}+c_2q_{2zz}+2(\beta|q_1|^2+\gamma|q_2|^2)q_2-i\epsilon[q_{2zzz}+(\nu_2|q_1|^2\\
&\qquad+2\mu_2|q_2|^2)q_{2z}+\nu_2q_1q_2^{\star}q_{1z}]=0,
\end{aligned}
\tag{1}
$$

which explains the simultaneous propagation of two fields in a nonlinear optical fiber with the inclusion of higher-order linear and self-steepening effects. In (1), q_j is the complex amplitude of the pulse envelope and z and t represent the spatial and temporal coordinates. c_1, c_2, α, β, γ, ϵ, μ_1, μ_2, ν_1, and ν_2 are real parameters.

Integrable cases of the GCH system attract both theoretical and experimental interest because they support a variety of exact solutions and the initial-value problem is solvable. The system (1) is solvable via the IST method for the conditions $c_1=c_2=\alpha=\beta=\gamma$, $\mu_1=\nu_1=\mu_2=\nu_2=3$ [9]. The bilinear integrability condition of (1) has been studied in Ref. [10]. The Painlevé analysis of the GCH system, which has been carried out by Bindu *et al.* [11] and Porsezian [12], indicates that the system (1) is integrable only in the following three cases:

$$c_1=c_2,\quad \alpha=\beta=\gamma,\quad \mu_1=\nu_1=\mu_2=\nu_2=3, \tag{2}$$

$$c_1=c_2=-1,\quad \alpha=\beta=\gamma,\quad \mu_1=\nu_1=\mu_2=\nu_2=-3, \tag{3}$$

$$c_1=-c_2,\quad \alpha=-\beta=\gamma,\quad \mu_1=-\mu_2=-\nu_1=\nu_2=3. \tag{4}$$

In this Brief Report, we analyze the GCH system again, and find that (1) is not P integrable for the conditions (2)–(4). In addition, we obtain two new sets of P-integrable parametric conditions for the GCH system. In order to apply the Painlevé analysis, we define $q_1=p$, $q_1^*=q$, $q_2=r$, $q_2^*=s$, and rewrite (1) and its complex conjugate as

$$
\begin{aligned}
&ip_t+c_1p_{zz}+2(\alpha pq+\beta rs)p\\
&\qquad-i\epsilon[p_{zzz}+(2\mu_1pq+\nu_1rs)p_z+\nu_1psr_z]=0,\\
&-iq_t+c_1q_{zz}+2(\alpha pq+\beta rs)q\\
&\qquad+i\epsilon[q_{zzz}+(2\mu_1pq+\nu_1rs)q_z+\nu_1qrs_z]=0,\\
&ir_t+c_2r_{zz}+2(\beta pq+\gamma rs)r\\
&\qquad-i\epsilon[r_{zzz}+(\nu_2pq+2\mu_2rs)r_z+\nu_2qrp_z]=0,\\
&-is_t+c_2s_{zz}+2(\beta pq+\gamma rs)s\\
&\qquad+i\epsilon[s_{zzz}+(\nu_2pq+2\mu_2rs)s_z+\nu_2psq_z]=0.
\end{aligned}
\tag{5}
$$

Though it is rather tedious to figure out the P-integrable models from the general model (5), to check the Painlevé property of (5) under parametric conditions (2)–(4) is quite easy by using some existing algebraic programs such as the MAPLE package WKPTEST developed by us [13,14]. Utilizing our package, the leading order and coefficients can be easily obtained; the resonances for the cases (2)–(4) appear at −1, 0, 0, 0, 1, 2, 2, 3, 4, 4, 4, and 5.

For the case (2), there are three arbitrary functions corresponding to the resonances at $j=0,0,0$. However, the resonance condition at $j=1$ is satisfied provided that $c_2=\gamma$. Under the new parametric constraints $c_1=c_2=\alpha=\beta=\gamma$, $\mu_1=\nu_1=\mu_2=\nu_2=3$, the system (5) admits sufficient number of arbitrary functions and so it is P integrable.

For the case (3), the resonances conditions hold at $j=0,0,0$, but the resonance condition at $j=1$ is satisfied along with additional parametric restriction $\gamma=1$. Under the

*Email address: xugq@staff.shu.edu.cn; xuguiqiong@yahoo.com

constraints $c_1=c_2=-1$, $\alpha=\beta=\gamma=1$, $\mu_1=\nu_1=\mu_2=\nu_2=-3$, the system (5) admits sufficient number of arbitrary functions and so it is P integrable too.

For the case (4), the resonances conditions hold at $j=0,0,0$, but the resonance condition at $j=1$ is satisfied only if $c_2=-\gamma$. Under the constraints $c_1=-c_2=\alpha=-\beta=\gamma$, $\mu_1=-\mu_2=-\nu_1=\nu_2=3$, the resonance condition at $j=2$ is satisfied only if $\gamma=0$. For the conditions $c_1=c_2=\alpha=\beta=\gamma=0$ and $\mu_1=-\mu_2=-\nu_1=\nu_2=3$, it is proved that the system (5) is P integrable. In Ref. [12], the author established the Painlevé property for the conditions $c_1=-c_2=\alpha=-\beta=\gamma=1$, $\mu_1=-\mu_2=-\nu_1=\nu_2=3$, in fact, (5) fails the Painlevé test in verifying the resonance condition at $j=2$.

The above analysis shows that cases (2)–(4) become P integrable only if replaced by

$$c_1=c_2=\alpha=\beta=\gamma, \quad \mu_1=\nu_1=\mu_2=\nu_2=3, \tag{6}$$

$$c_1=c_2=-1, \quad \alpha=\beta=\gamma=1, \quad \mu_1=\nu_1=\mu_2=\nu_2=-3, \tag{7}$$

$$c_1=c_2=\alpha=\beta=\gamma=0, \quad \mu_1=-\mu_2=-\nu_1=\nu_2=3. \tag{8}$$

The natural and important question is under what constraints on the parameters $\{\alpha,\beta,\gamma,c_1,c_2,\epsilon,\mu_1,\mu_2,\nu_1,\nu_2\}$ is the GCH system P-integrable. To answer this, we try to make a Painlevé classification for the system (5).

Now we describe how to obtain the possible Painlevé subcases for the GCH system. Following the standard Weiss-Tabor-Carnevale (WTC) approach [3], the system (5) is said to be P integrable if its solutions are "single valued" about arbitrary noncharacteristic, movable singularity manifolds. In other words, all solutions of (5) can be expressed as Laurent series,

$$p(z,t)=\sum_{j=0}^{\infty} p_j\phi(z,t)^{(j+\alpha_1)}, \quad q(z,t)=\sum_{j=0}^{\infty} q_j\phi(z,t)^{(j+\alpha_2)},$$

$$r(z,t)=\sum_{j=0}^{\infty} r_j\phi(z,t)^{(j+\alpha_3)}, \quad s(z,t)=\sum_{j=0}^{\infty} s_j\phi(z,t)^{(j+\alpha_4)} \tag{9}$$

with a sufficient number of arbitrary functions among p_j,q_j,r_j,s_j in addition to ϕ. Moreover, $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ should be negative integers. In order to simplify the calculations, we make use of the Kruskal ansatz $\phi(z,t)=z-\psi(t)$, where ψ is an arbitrary function of t. Then the coefficient functions p_j,q_j,r_j,s_j in Eqs. (9) will be functions of t alone.

First, find the leading order and coefficients. To reach this aim, we substitute

$$p(z,t)=p_0\phi^{\alpha_1}, \quad q(z,t)=q_0\phi^{\alpha_2}, \quad r(z,t)=r_0\phi^{\alpha_3},$$

$$s(z,t)=s_0\phi^{\alpha_4}$$

into (5). Upon balancing the dominant terms, we obtain

$$\alpha_1=\alpha_2=\alpha_3=\alpha_4=-1, \quad r_0=\frac{3(\nu_2-\mu_1)}{(\mu_1\mu_2-\nu_1\nu_2)s_0}, \tag{10}$$

$$p_0=\frac{3(\nu_1-\mu_2)}{(\mu_1\mu_2-\nu_1\nu_2)q_0},$$

where s_0 and q_0 are arbitrary functions.

Next, in order to find the resonances that are the powers at which the arbitrary coefficients enter into the Laurent series (9), we substitute

$$p(z,t)=p_0\phi^{-1}+p_j\phi^{j-1}, \quad q(z,t)=q_0\phi^{-1}+q_j\phi^{j-1},$$

$$r(z,t)=r_0\phi^{-1}+r_j\phi^{j-1}, \quad s(z,t)=s_0\phi^{-1}+s_j\phi^{j-1}$$

into (5). Detailed calculations give the following resonance equation for the exponent j:

$$\begin{aligned}&\epsilon^4q_0^4s_0^4(\mu_1\mu_2-\nu_1\nu_2)^4j^2(j+1)(j-1)(j-3)(j-5)(j-4)^4\\&\quad\times[(\mu_1\mu_2-\nu_1\nu_2)j^2-6(\mu_1\mu_2-\nu_1\nu_2)j+3(\mu_1\nu_1+\mu_2\nu_2)\\&\quad+5\mu_1\mu_2-11\nu_1\nu_2][(\mu_1\mu_2-\nu_1\nu_2)j^2\\&\quad-2(\mu_1\mu_2-\nu_1\nu_2)j-3(\mu_1\mu_2+\nu_1\nu_2)+3(\mu_1\nu_1+\mu_2\nu_2)]\\&\quad=0.\end{aligned} \tag{11}$$

Due to (11), eight resonances lie in the positions $j=-1,0,0,1,3,4,4,5$, and the other four resonances are denoted as j_1, j_2, j_3, j_4. From (11), we have the following three possibilities (the case $\epsilon=0$ is not considered here):

$$\nu_2=0, \quad \nu_1=\mu_2-\frac{\mu_2j_4^2}{3}+\frac{2\mu_2j_4}{3},$$

$$j_2=4-j_4, \quad j_1=2+j_4, \quad j_3=2-j_4, \tag{12a}$$

$$\mu_1=\frac{\nu_2(3\nu_1-3\mu_2+\nu_1j_4^2-2\nu_1j_4)}{3\nu_1-3\mu_2+\mu_2j_4^2-2\mu_2j_4},$$

$$j_2=4-j_4, \quad j_1=2+j_4, \quad j_3=2-j_4, \tag{12b}$$

$$\nu_1=\mu_2, \quad j_1=2, \quad j_2=4, \quad j_3=0, \quad j_4=2. \tag{12c}$$

We require that the considered branch be generic, i.e., 11 resonances lie in non-negative integer positions. Taking into account the admissible multiplicity of resonances, we find from (12) that the considered branch is generic only for the five specific choices of parameters shown in Table I.

For the parameter choices listed in Table I, the Painlevé test should be performed again from the first step. That is to say, the leading-order analysis, the resonance determination,

TABLE I. Parameter sets leading to a generic branch.

Case	Parameter choices	Resonances (j)
I	$\nu_2=0, \quad \nu_1=\frac{4\mu_2}{3}$	$-1, 0, 0, 1, 1, 1, 3, 3, 3, 4, 4, 5$
II	$\nu_2=0, \quad \nu_1=\mu_2$	$-1, 0, 0, 0, 1, 2, 2, 3, 4, 4, 4, 5$
III	$\mu_1=\frac{\nu_2(2\nu_1-3\mu_2)}{3\nu_1-4\mu_2}$	$-1, 0, 0, 1, 1, 1, 3, 3, 3, 4, 4, 5$
IV	$\mu_1=\nu_2$	$-1, 0, 0, 0, 1, 2, 2, 3, 4, 4, 4, 5$
V	$\nu_1=\mu_2$	$-1, 0, 0, 0, 1, 2, 2, 3, 4, 4, 4, 5$

and the test of resonance conditions are required.

Considering case I, the leading-order analysis is also given by (10), there is only one generic branch, and the resonances are located at $-1,0,0,1,1,1,3,3,3,4,4,5$. Further analysis shows that the resonance conditions at $j=1,1,1$ reduce to

$$p_1=(i(27\gamma\mu_1^2q_0-3\mu_2^2\mu_1c_1q_0-3\mu_2^2\alpha q_0-9\mu_2\mu_1^2c_2q_0+2i\epsilon\mu_1\mu_2^3s_0q_0r_1-3i\epsilon\mu_1\mu_2^2q_1))/(3\epsilon\mu_1^2\mu_2^2q_0^2),$$
$$s_1=\frac{\mu_2s_0^2r_1}{3},\quad \beta=\frac{\mu_1(3\gamma-\mu_2c_2)}{\mu_2}.$$

It is obvious that there are only two arbitrary functions, r_1,q_1, so the system (5) is not P integrable in this case.

Considering case II, we have to set $\mu_1=0$ such that three coefficients among r_0, s_0, p_0, q_0 are arbitrary. Along with the additional parametric constraint $\mu_1=0$, the resonance condition turns out to be inconsistent at $j=1$, as in the previous case; thus the system (5) is not P integrable in this case.

Considering case III, the leading-order analysis and the resonance determination provide two possible generic branches.

Branch (i): $\alpha_i=-1$ $(i=1,\ldots,4)$, $r_0=-1/[(\nu_1-\mu_2)s_0]$, $q_0=-(3\nu_1-4\mu_2)/[\nu_2(\nu_1-\mu_2)p_0]$, and the resonances occur at $j=-1,0,0,1,1,1,3,3,3,4,4,5$.

Branch (ii): $\alpha_i=-1$ $(i=1,\ldots,4)$, $r_0=-(3+\nu_2p_0q_0)/(\nu_2s_0)$, $\nu_1=\mu_2$, and the resonances occur at $j=-1,0,0,0,1,2,2,3,4,4,4,5$.

In the case of branch (i), there are two arbitrary functions p_0,s_0 corresponding to the resonances $j=0,0$. However, the resonance conditions turn out to be inconsistent at $j=1,1,1$; thus this branch should be ignored. In the case of branch (ii), p_0,q_0,s_0 are arbitrary and correspond to the resonances $j=0,0,0$. At resonance $j=1$, p_1 is an arbitrary function; the remaining expansion coefficients q_1,r_1,s_1 are

$$q_1=-\frac{q_0p_1}{p_0},$$
$$s_1=\frac{-is_0(2c_2\mu_2-2\gamma p_0q_0\nu_2-i\epsilon\nu_2p_1q_0\mu_2+2\beta p_0q_0\mu_2-6\gamma)}{\epsilon\nu_2p_0q_0\mu_2},$$
$$r_1=\frac{-i(2c_2\mu_2-2\gamma p_0q_0\nu_2-i\epsilon\nu_2p_1q_0\mu_2+2\beta p_0q_0\mu_2-6\gamma)(p_0q_0\nu_2+3)}{s_0\epsilon\nu_2p_0q_0\mu_2^2},$$

where

$$c_1=[(\nu_2\beta\mu_2-\nu_2^2\gamma-\alpha\mu_2\nu_2+\beta\nu_2^2)q_0^2p_0^2+(3\beta\mu_2-6\gamma\nu_2+\nu_2c_2\mu_2+3\beta\nu_2)q_0p_0+3c_2\mu_2-9\gamma]/(\mu_2\nu_2p_0q_0). \tag{13}$$

p_0 and q_0 are arbitrary functions, so we find from (13) that

$$\alpha=\frac{c_1\nu_2}{3},\quad \gamma=\frac{\mu_2c_2}{3},\quad \beta=\frac{\mu_2\nu_2(c_1+c_2)}{3(\mu_2+\nu_2)}. \tag{14}$$

Together with the parameter choice (14) in addition to $\mu_1=\nu_2(2\nu_1-3\mu_2)/(3\nu_1-4\mu_2)$, $\nu_1=\mu_2$, we perform the Painlevé test of (5) again, and know that there is only one branch: the leading orders for p,q,r,s are -1, $p_0=-(3+\mu_2r_0s_0)/(\nu_2q_0)$, and r_0,s_0,q_0 are arbitrary functions. The resonances occur at $j=-1,0,0,0,1,2,2,3,4,4,4,5$. Now substituting the expansion (9) into (5) and collecting the coefficients of $(\phi^{-3},\phi^{-3},\phi^{-3},\phi^{-3})$, the determining equations of the coefficients p_1,q_1,r_1,s_1 are obtained. Solving them we get

$$p_1=\frac{(3+\mu_2r_0s_0)q_1}{\nu_2q_0^2},$$
$$r_1=-\frac{r_0(3\epsilon q_1\nu_2+3\epsilon q_1\mu_2+2ic_2q_0\nu_2-2i\mu_2c_1q_0)}{3\epsilon q_0(\nu_2+\mu_2)},$$
$$s_1=\frac{s_0(3\epsilon q_1\nu_2+3\epsilon q_1\mu_2+2ic_2q_0\nu_2-2i\mu_2c_1q_0)}{3\epsilon q_0(\nu_2+\mu_2)},$$

where q_1 is an arbitrary function corresponding to the resonance $j=1$.

In a similar manner, collecting the coefficients of $(\phi^{-2},\phi^{-2},\phi^{-2},\phi^{-2})$, the determining equations of the coefficients p_2,q_2,r_2,s_2 are obtained. From the obtained equations we see that two coefficients among r_2,s_2,p_2,q_2 cannot become arbitrary. However, we have the following three possibilities for specific choices of c_1, c_2, μ_2, ν_2.

Case IIIa: when $c_1=c_2,\nu_2=\mu_2$,

$$s_2=[-i(i\nu_2^2r_0s_0\mu_2q_1^2\epsilon-3i\nu_2^2q_1^2\epsilon-2q_1\mu_2q_0c_2\nu_2+2i\epsilon q_2\mu_2^3r_0s_0q_0-i\psi_tq_0^2\mu_2^2-3i\epsilon q_1^2\mu_2^2+6i\epsilon q_2q_0\mu_2^2-ir_0s_0q_1^2\epsilon\mu_2^3)]/(2\epsilon\mu_2^3r_0q_0^2),$$

$$p_2=[-i(i\nu_2^2r_0s_0\mu_2q_1^2\epsilon-3i\nu_2^2q_1^2\epsilon-2q_1\mu_2q_0c_2\nu_2-3i\epsilon q_1^2\mu_2^2-2i\epsilon\mu_2^3q_0^2r_2s_0-i\psi_tq_0^2\mu_2^2-ir_0s_0q_1^2\epsilon\mu_2^3)]/(2\epsilon\nu_2q_0^3\mu_2^2),$$

where r_2,q_2 are arbitrary functions.

Case IIIb: when $c_1=-c_2,\nu_2\neq 0$,

$$p_2=[i(18i\epsilon^2\mu_2 q_0^2 r_2 s_0+4i\mu_2 r_0 s_0 c_2^2 q_0^2+12\mu_2 r_0 s_0\epsilon q_1 c_2 q_0 +9i\psi_t q_0^2\epsilon-18c_2 q_1\epsilon q_0 +54i\epsilon^2 q_1^2)]/(18q_0^3\epsilon^2\nu_2),$$

$$s_2=[-i(18i\epsilon^2 q_2\mu_2 r_0 s_0 q_0-12\mu_2 r_0 s_0\epsilon q_1 c_2 q_0-54i\epsilon^2 q_1^2 +54i\epsilon^2 q_2 q_0-4i\mu_2 r_0 s_0 c_2^2 q_0^2 -9i\psi_t q_0^2\epsilon+18c_2 q_1\epsilon q_0)]/(18q_0^2\epsilon^2\mu_2 r_0),$$

where r_2, q_2 are arbitrary functions.

Case IIIc: when $c_1=-c_2, \nu_2=0$,

$$r_2=\frac{\psi_t r_0^2+6\epsilon r_1^2+2ic_2 r_1 r_0}{6\epsilon r_0},\quad s_2=\frac{i(i\psi_t r_0^2-2c_2 r_1 r_0+6i\epsilon r_1^2)}{2\epsilon\mu_2 r_0^3},$$

where p_2, q_2 are arbitrary functions.

In this way by proceeding further and collecting the coefficients of $(\phi^{-1},\phi^{-1},\phi^{-1},\phi^{-1})$, $(\phi^0,\phi^0,\phi^0,\phi^0)$ and $(\phi^1,\phi^1,\phi^1,\phi^1)$, we establish the required number of arbitrary functions corresponding to $j=3$, $j=4,4,4$, and $j=5$ without any additional restrictions on the parameters. Thus we conclude that in case III the Laurent expansions of (5) contain a sufficient number of arbitrary functions for the three parametric restrictions:

$$\alpha=\beta=\gamma=\frac{c_2\mu_2}{3},\quad c_1=c_2,\quad \mu_1=\nu_2=\nu_1=\mu_2, \quad (15)$$

$$\alpha=-\frac{c_2\nu_2}{3},\quad \beta=0,\quad \gamma=\frac{c_2\mu_2}{3},\quad c_1=-c_2, \quad \nu_1=\mu_2,\quad \mu_1=\nu_2, \quad (16)$$

$$\alpha=\beta=0,\quad \gamma=\frac{\mu_2 c_2}{3},\quad \mu_1=\nu_2=0,\quad \nu_1=\mu_2,\quad c_1=-c_2. \quad (17)$$

It is obvious that Eqs. (15) and (16) are more general than the conditions (6)–(8). When $\mu_2=3$, the case (15) becomes the condition (6), when $\mu_2=-3$ and $c_2=-1$, the case (15) becomes the condition (7), and when $c_2=0$, $\nu_2=3$, $\mu_2=-3$, the case (16) becomes the condition (8).

In case III, we arrive at three P-integrable subcases of Eqs. (5). The first one is

$$iq_{1t}+c_2 q_{1zz}+\frac{2c_2\mu_2}{3}(|q_1|^2+|q_2|^2)q_1 -i\epsilon[q_{1zzz}+\mu_2(2|q_1|^2+|q_2|^2)q_{1z}+\mu_2 q_1 q_2^\star q_{2z}]=0,$$
$$iq_{2t}+c_2 q_{2zz}+\frac{2c_2\mu_2}{3}(|q_1|^2+|q_2|^2)q_2 -i\epsilon[q_{2zzz}+\mu_2(|q_1|^2+2|q_2|^2)q_{2z}+\mu_2 q_1 q_2^\star q_{1z}]=0, \quad (18)$$

where c_2, μ_2, ϵ are arbitrary constants, and the second one is

$$iq_{1t}-c_2 q_{1zz}-\frac{2c_2\nu_2}{3}|q_1|^2 q_1 -i\epsilon[q_{1zzz}+(2\nu_2|q_1|^2+\mu_2|q_2|^2)q_{1z}+\mu_2 q_1 q_2^\star q_{2z}]=0,$$
$$iq_{2t}+c_2 q_{2zz}+\frac{2c_2\mu_2}{3}|q_2|^2 q_2-i\epsilon[q_{2zzz} +(\nu_2|q_1|^2+2\mu_2|q_2|^2)q_{2z}+\nu_2 q_1 q_2^\star q_{1z}]=0, \quad (19)$$

where $c_2, \mu_2, \nu_2, \epsilon$ are arbitrary constants. The models (18) and (19) are, as far as we know, two types of subcase of P-integrable models of (1) first reported here.

From the parametric restriction (17), we get the third P-integrable model. However, the obtained model becomes decoupled and so it is omitted here.

After finishing a similar analysis for cases IV and V, we obtain a new parametric restriction in addition to (15)–(17),

$$\alpha=-\frac{c_2\nu_2}{3},\quad \beta=\gamma=0,\quad \mu_2=\nu_1=0,\quad \mu_1=\nu_2,\quad c_1=-c_2, \quad (20)$$

under the above parametric constraints, the system (1) becomes decoupled and so it is also omitted here.

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