PHYSICAL REVIEW E **73**, 065105(R) (2006)

Self-similar roughening of drying wet paper

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(Received 24 January 2006; published 9 June 2006)

We studied the kinetic roughening dynamics of drying wet paper. The configurations of dry paper sheets are found to be self-similar, rater than self-affine. Accordingly, the paper roughening dynamics corresponds to the new class of anomalous kinetic roughening [J. J. Ramasco, J. M. López, and M. A. Rodríguez, Phys. Rev. Lett. **84**, 2199 (2000)], characterized by the equal local and global roughness exponents $\zeta=\alpha=1$ and the dynamic exponent $z=1.0\pm0.2$, whereas the spectral roughness exponent $\alpha_S>1$ is determined by the long-range correlations characterized by the fractal dimension D of crumpled sheet.

DOI: 10.1103/PhysRevE.73.065105 PACS number(s): 05.90.+m, 68.35.Ct, 68.35.Fx, 05.45.Df

The kinetic roughening of surfaces has played the role of a paradigm in nonequilibrium statistical physics for the past two decades, with applications ranging from growth problems to fracture phenomena [1]. Typically, an initially flat d-dimensional surface $z(t,\vec{x})$ roughens if it is driven by some external noise. The roughening dynamics is commonly characterized by the time dependent global width of interface $W(L,t)=\overline{\langle[z(\vec{x},t)-\bar{z}]^2\rangle}^{1/2}$, where the overbar denotes the average over all $\vec{x}$ in a system of size L and the brackets denote the average over different realizations [1,2].

It was found that in many cases the roughening dynamics exhibits scaling invariance, i.e., their behavior does not change under rescaling of space and time combined with an appropriate rescaling of the observables and the control parameters [1,2]. Specifically, in the absence of any characteristic length in the system except L, the roughening dynamics satisfies the celebrated Family-Vicsek scaling ansatz [3],

$$W(L,t) \propto t^{\beta} f[L/\xi(t)], \tag{1}$$

where $\xi \propto t^{\beta/\alpha}$ is the horizontal correlation length, β, α, and $z=\alpha/\beta$ are the so-called growth, global roughness, and dynamic exponents, respectively; and the scaling function $f(y)$ behaves as $f \propto y^{\alpha}$ if $y \ll 1$ and it is a constant when $y \gg 1$. The scaling behavior of the local fluctuations of the surface is, however, generally characterized by different scaling exponents. According to the generic dynamic scaling ansatz suggested in [4], the local surface width, $w(\Delta,t)=\langle\{[z(\vec{x},t)-\langle z\rangle_{\Delta}]^2\}_{\Delta}\rangle^{1/2}$, where $\{\ldots\}_{\Delta}$ denotes an average over x in windows of size Δ, scales as

$$w(\Delta,t) \propto t^{\beta} f_l[\Delta/\xi(t)], \tag{2}$$

where the scaling function behaves as $f \propto y^{\zeta}$ if $y \ll 1$ and it is a constant when $y \gg 1$. The local roughness exponent ζ is generally less or equal to α. Kinetic roughening characterized by $\zeta<\alpha$ was called anomalous roughening in the literature [1,4]. Furthermore, to classify the scaling dynamics within a framework of generic scaling theory, we need to know the scaling behavior of the structure factor or power spectrum of surface, $S(k)=\langle Z(k)Z(-k)\rangle$, where $Z(k)$ is the Fourier transform of $z(x)$. In saturation $(t\to\infty, \xi\to L)$ the structure factor of rough d-dimensional surface behaves as

$$S(L,k) \propto k^{-(2\alpha_S+d)} L^{2(\alpha-\alpha_S)}, \tag{3}$$

where α_S is the spectral roughness exponent [4]. Accordingly, the Family-Vicsek scaling is associated with $\alpha_S=\alpha=\zeta$. The super-rough surfaces are characterized by $\alpha_S=\alpha>\zeta=1$, whereas the intrinsically anomalous roughness is characterized by $\alpha_S=\zeta<\alpha$ [4,5]. Furthermore, Ramasco *et al.* [4] has predicted the existence of a new class of anomalous dynamic scaling characterized by $\alpha_S>\alpha\geqslant\zeta=1$, which was observed in numerical simulations of one-dimensional Sneppen model of self-organized depinning [6]. However, as far as we know, this type of kinetic roughening was not observed in experiments [7].

Many studies of roughening dynamics were performed with the use of a paper as a model random medium (see, for example, Refs. [2,8], and references therein). An interesting example of kinetic roughening is a crumpling of a paper sheet when wet paper dries [9]. As moisture is removed, paper tends to shrink [10]. Dried paper does not reveal any characteristic length scale; rather smaller "mountains" appear inside larger ones up to the fiber scale. The mechanisms of paper roughening due to drying are discussed in details in Refs. [8–11]. The only known to us model of kinetic roughening of drying wet paper suggested in Ref. [9] predicts a trivial result $\alpha_S=\zeta=\alpha=1$, which was not supported experimentally. In this work we performed an experimental study of roughening dynamics of drying wet paper.

We used the square sheets of different sizes L of three kinds of Filtro paper with open, medium, and closed porosity [12]. The main properties of these papers are given in Table I. The sheet size was varied from $L_0=2$ cm to $L_{\max}=50$ cm with the relation $L=\lambda L_0$ for scaling factors $\lambda=2, 2.5, 3, 5, 10, 15, 20, 25$.

All samples were placed in a bath of distilled water for 10 min. After that, each sheet was removed from the bath and placed on a blotter to remove dripping water and then positioned on the stainless steel screen and allowed to free drain until no changes in the sheet shape were observed. Thirty samples of each size of each paper were tested [13].

TABLE I. Physical and mechanical properties of papers (ρ_A, E, ν, and σ_y are the area density, Young modulus, Poisson ratio, and yield stress, respectively; subscripts denote the direction of tension test: L, in the machine direction of paper, and T, across the machine direction) and characteristic parameters of kinetic roughening of drying wet papers and 1D surface formed in the Sneppen model of self-organized depinning.

	Filtro paper			1D Sneppen model[a]	
	Open porosity	Medium porosity	Closed porosity	Random segments	Identical segments
Thickness (mm)	0.32±0.06	0.25±0.05	0.21±0.04		
ρ_A (g/m^2)	128.4±3.3	103.4±3.1	102.4±2.8		
Porosity (%)[b]	73.2	72.4	67.5		
E_L (MPa)	1115±60	1570±170	2020±280		
E_T (MPa)	606±50	740±40	750±90		
ν_L	0.45±0.07	0.63±0.05	0.75±0.06		
ν_T	0.25±0.05	0.23±0.05	0.19±0.06		
σ_{yL} (MPa)	12.1±0.9	16±1	22±5		
σ_{yT} (Mpa)	6.6±0.7	7.8±0.7	8±1		
h (mm)	1.3±0.2	2.2±0.2	2.2±0.2		
k	0.65±0.1	0.95±0.1	1.3±0.2		
β	1.00±0.06	1.00±0.05	1.00±0.04	1[c]	1[c]
α	1.00±002	1.00±0.02	1.00±0.02	1[c]	1[c]
ζ	0.98±005	0.99±0.06	0.99±0.04	1[c]	1[c]
α_S	1.25±0.06	1.15±0.05	1.09±0.04	1.35±0.03[c]	1.5[c]
D_D	1.24±0.02	1.13±0.02	1.09±0.01	1.35±0.02	1.5±0.01
D_B	1.25±0.02	1.14±0.02	1.08±0.01	1.35±0.02	1.5±0.01
D_{PA}	1.24±0.02	1.12±0.02	1.1±0.01	—	—
D_S	2.24	2.13	2.09	—	—

[a]Reference [4].
[b]Reference [12].
[c]Reference [3].

The drying paper sheets were observed to pass through a series of discrete visual stages. When first removed from the bath, the sample is flooded (stage 1). When the sample is placed on the drying screen, the surface appears wet. At the second stage ($t_1 \leqslant t \leqslant t_2$), the overall surface moisture decreases fairly quickly as the flooded water evaporates from the surface and the surface fibers begin to shrink up. At the third stage ($t_2 \leqslant t < t_S$), the sheet of paper roughens continuously until the roughness saturates, and at the final stage ($t_S \leqslant t \leqslant t_F$) only small changes in paper roughness are observed as the sheet achieves its final shape (see top insert of Fig. 1(a). For times $t \geqslant t_2$, we measured the maximal height of crumpling sheets, $H(L,t) = \max_{\vec{x}\in L\times L} z(t,x) - \min_{\vec{x}\in L\times L} z(t,x)$, which is assumed to satisfy the Family-Vicsek ansatz (1). Accordingly, we find that at the third stage. $t_2 \leqslant t < t_S$, the sheet height increases with time, such that $\Delta H(L,\tau) = H(L,\tau) - H(L,0)$ behaves as

$$\Delta H(L,\tau) \propto \tau f(L/\tau), \tag{4}$$

where $\tau = t - t_2$ [14]. The data collapse for sheets of different with are shown in Fig. 2(a). We also found that the heights of the final sheet shapes at $\tau_F = t_F - t_2$ scale with L as

$$H^* = H(L,\tau_F) - h = kL, \tag{5}$$

for all papers tested [see Fig. 2(b)], where h and k are constants (see Table I). This means that the roughness of all crumpled dry papers is characterized by $\alpha = 1.00 \pm 0.02$, $\beta = 1.00 \pm 0.05$, and so $z = \alpha/\beta = 1$.

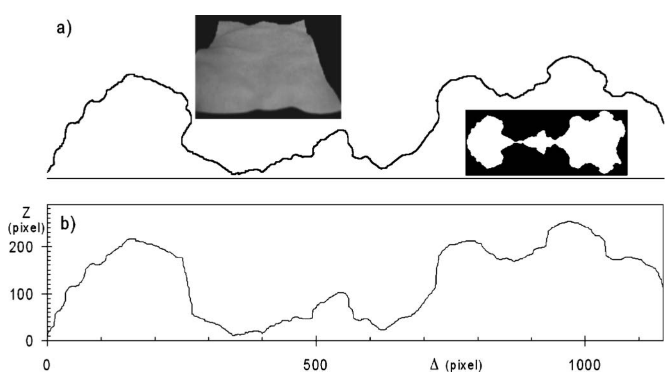

FIG. 1. (a) Profile of crumpled Filtro paper with closed porosity (inserts show photograph of crumpled sheet (top) and the closed contour profile presentation (bottom) used for perimeter-area measurements) and (b) its digitized single-valued presentation.

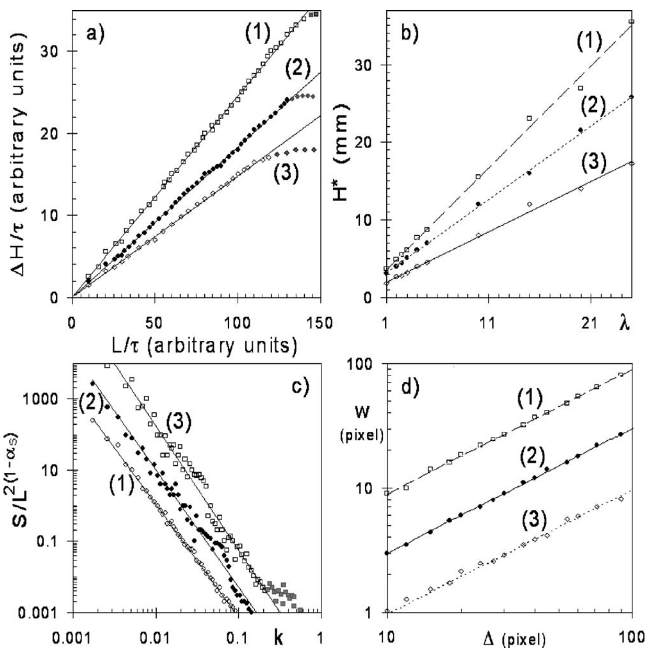

FIG. 2. (a) Data collapse for sheet heights $\Delta H(L,\tau)$ and (b) graphs of the sheet roughness amplitude, H^{*}, vs scaling factor of sheet size λ; (c) Data collapse for size dependent structure factor (3) of sheet profiles (in arbitrary units) and (d) Log-log graphs of local profile roughness $w(\Delta)$ vs window size Δ for Filtro papers with closed (1), medium (2), and open (3) porosity.

Further, the one-dimensional (1D) profiles [see Fig. 1(a)] of dry crumpled sheets of size $L \geqslant 150$ mm were obtained with help the one-dimensional laser profilometry. Only a small number of overhangs were observed in obtained profiles, which were neglected in order to represent the profiles by single-valued functions $z(x)$ [see Fig. 1(b)]. Accordingly, we found the spectral α_S and the local roughness ζ exponents from the scaling relations (3) and $w \propto \Delta^{\zeta}$, respectively [see Figs. 2(c) and 2(d)]. We note that $\zeta = 1.0 \pm 0.03$ for all papers, whereas the spectral exponents were found to be different for different papers (see Table I). So the saturated roughness of dry paper sheets is characterized by

$$\alpha_S > \alpha = \zeta = 1, \tag{6}$$

i.e., the sheet roughness is consistent with a new class of kinetic roughening suggested in Refs. [4,15]. This result differs from the prediction of the model of crumpling of dried paper suggested in Ref. [9], as well as from the results of experimental studies of the flattening of randomly folded sheets (see Refs. [16,17]). In experiments with mechanically crumpled papers, it was found that unfolded sheets are self-affine, i.e., the sheet roughness is invariant under affine transformation $\vec{x} \rightarrow \lambda x, z \rightarrow \lambda^{\zeta} z$ with $\zeta = 0.71 \pm 0.01$ (Ref. [16]) and $\zeta = 0.88 \pm 0.03$ (Ref. [17]). In contrast to this, we found that crumpled dry sheets are invariant under the similarity transformation

$$\vec{x} \rightarrow \lambda x, z \rightarrow \lambda z, \tag{7}$$

and the roughening dynamics (4) is self-similar, i.e., $t \rightarrow \lambda t$). It is interesting to note that the wrinkling of plastically deformed sheets is also self-similar [18].

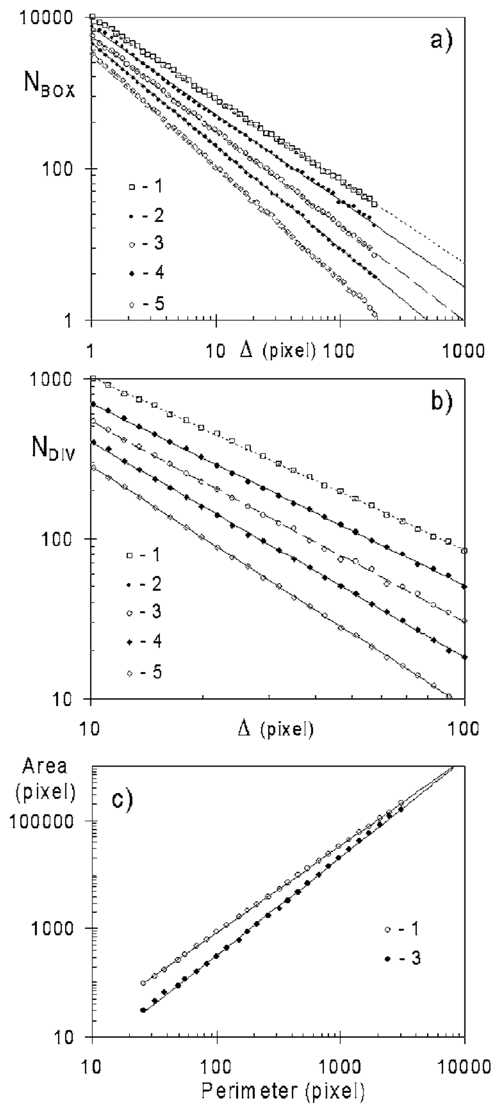

FIG. 3. Log-log graphs of (a) number of boxes covered sheet profile $[N_{BOX}(\Delta)]$ vs box size (Δ); (b) number of dividers along profile $[N_{DIV}(\Delta)]$ vs divider size; and (c) area (S) versus perimeter (P) of closed contours composed from crumpled sheet profiles [see inset in Fig. 1(b)]. Points are experimental data, and straight lines are data fitting with the scaling relations $N_{BOX} \propto \Delta^{-D_B}$, $N_{DIV} \propto \Delta^{-D_D}$, and $S \propto P^{2/D_{PA}}$ for graphs in panels (a), (b), and (c) respectively. The numbers in all panel correspond to sheets of Filtro papers (1–3) with open (1), medium (2), and closed (3) porosity and to interfaces formed in one-dimensional Sneppen model of self-organized depinning (4) and (5) in the cases of interfaces formed by facets with constant slope ±1 (4) and by a finite number of identical segments (5). Notice that all graphs are shifted along ordinate for clarity.

The nontrivial values of $\alpha_S > 1$ indicate the long-range correlations in the paper roughness [19]. Generally, self-similar shapes are characterized by the nontrivial fractal dimension D, rater than by the roughness exponent [20]. Accordingly, we determined the fractal dimension of each original profile by the box-counting [Fig. 3(a), divider [Fig. 3(b)], and perimeter-area [Fig. 3(c) (Ref. [21])] methods with the help of commercial BENOIT 1.3 software [22]]. We found that three methods give an almost the same value of fractal dimension $D=D_B=D_D=D_{PA}$ (see Table I; notice that the fractal dimensions D_B, D_D, and D_{PA} are reoffered dimensions obtained by the box-counting and divider methods and via the perimeter-area relation, respectively), as is expected for self-similar fractals [23]. Hence, the surface fractal dimension is assumed to be $D_S=D+1$ (see Ref. [24] and Table I). Notice that D_S is found to be different for different kinds of paper (see also Refs. [12,25]).

We also note the numerical coincidence between the fractal dimension D and the spectral roughness exponent α_S of self-similar profiles of crumpled sheets (see Table I). To verify the physical significance of this coincidence, we performed numerical study of the one-dimensional Sneppen model of self-organized depinning (model A). Ramasco *et al.* [4] have found that this model exhibits a type of anomalous roughening, characterized by $\alpha_S > \alpha = \zeta = \beta = z = 1$, where $\alpha_S = 1.5$ in the case of a faceted interface formed by a finite number of identical segments, and $\alpha_S = 1.35$ if the interface is formed by facets with constant slope ±1 (Ref. [4]). In this work we determine the fractal dimension for these two types of faceted interfaces [see Fig. 3(a) and 3(b), and Table I). Accordingly, we found again that numerically $\alpha_S = D$ (Ref. [26]).

In summary, we present an experimental observation of anomalous kinetic roughening of the class suggested theoretically in Ref. [4]. Moreover, we have show that in reported experiments, as well as in numerical simulations with the Sneppen model, this class of anomalous roughening is associated with self-similar scaling dynamics. The open questions are (1) Is the equality $\alpha_S = D$ valid for all self-similar interfaces? And if so, why? (2) Is it possible the scaling characterized by $\alpha_S > \alpha > \zeta = 1$ and/or $z \neq 1$, or this later type of kinetic roughening is always self-similar?

The authors would like to thank Miguel Angel Rodriguez and Rodolfo Cuerno for useful discussions. This work has been supported by CONACyT of the Mexican government under Project No. 44722.

[1] A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, England, 1995); P. Meakin, *Fractals, Scaling and Growth Far from Equilibrium* (Cambridge University Press, New York, 1998).

[2] M. Alava, M. Dubé, and M. Rost, Adv. Phys. **53**, 83 (2004).

[3] F. Family and T. Vicsek, J. Phys. A **18**, L75 (1985).

[4] J. J. Ramasco, J. M. López, and M. A. Rodríguez, Phys. Rev. Lett. **84**, 2199 (2000).

[5] A. S. Balankin *et al.*, Phys. Rev. E **72**, 065101(R) (2005).

[6] K. Sneppen, Phys. Rev. Lett. **69**, 3539 (1992).

[7] Another type of anomalous multiscaling, characterized by $\alpha_S < \zeta_2 = \zeta < \alpha \leqslant 1$, was observed in paper burning experiments [A. S. Balankin and D. M. Matamoros, Phys. Rev. E **71**, 056102 (2005)].

[8] M. Alava and K. Niskanen, Rep. Prog. Phys. **53**, 669 (2006).

[9] F. Tzschichholz, A. Hansen, and S. Roux, e-print cond-mat/9507056.

[10] G. M. Laudone, G. P. Matthews, and P. A. C. Gane, Ind. Eng. Chem. Res. **43**, 712 (2004); G. M. Laudone *et al.*, Am. Inst. Chem. J. **50**, 2721 (2004).

[11] H. L. Noboa and J. Seyed-Yagoobi, in *Proceedings of the 1999 ASME International Mechanical Engineering Congress and Exposition*, edited by John W. Sutherland (ASME, Nashville, Tennessee, 1999); D. J. Gates and M. Westcott, ANZIAM J. **44**, E269 (2003).

[12] Early these kinds of paper were early used for paper imbibition, fracturing, and burning experiments [see A. S. Balankin, O. Susarrey, and J. M. Gonzáles, Phys. Rev. Lett. **90**, 096101 (2003); A. S. Balankin and D. Morales, Phys. Lett. A **399**, 23 (2005); A. S. Balankin *et al.*, Phys. Rev. Lett. **96**, 056101 (2006)].

[13] In this work, the temperature and air humidity were not controlled, because we found that the evaporation rate during paper drying does not affect the measured scaling exponents; nevertheless the drying kinetics is controlled by the evaporation rate.

[14] The time duration of each stage strongly depends on the evaporation rate controlled by the air humidity and temperature (see Ref. [13]).

[15] Notice that the author of Ref. [4] does not restrict the class by the equality of local and global roughness exponents.

[16] D. L. Blair and A. Kudrolli, Phys. Rev. Lett. **94**, 166107 (2005).

[17] F. Plouraboue and S. Roux, Physica A **227**, 173 (1996).

[18] E. Sharon *et al.*, Nature **419**, 579 (2002).

[19] C. Tricot, *Curves and Fractal Dimension* (Springer-Verlag, New York, 1995).

[20] For self-affine fractals $D=d+1-\zeta$, whereas for self-similar fractals $\zeta=1$, nevertheless $D>1$ (Ref. [19]).

[21] To make use the perimeter-area relation, the profiles were converted to contours [see the bottom insert in Fig. 1(a)].

[22] http://www.trusoft-international.com (BENOIT 1.3, 1999); W. Seffens, Science **285**, 1228 (1999).

[23] For self-affine curves $D_D=1/\zeta>D_B=2-\zeta$ (Ref. [24]).

[24] K. Falconer, *Fractal Geometry: Mathematical Foundations and Applications* (Wiley, New York, 1990).

[25] A. S. Balankin *et al.*, Proc. R. Soc. London, Ser. A **455**, 2565 (1999); A. S. Balankin and O. Susarrey, Philos. Mag. Lett. **79**, 629 (1999); A. S. Balankin, O. Susarrey, and A. Bravo, Phys. Rev. E **64**, 066131 (2001); I. L. Menezes-Sobrinho, M. S. Couto, and I. R. B. Ribeiro, *ibid.* **71**, 066121 (2005).

[26] It should be emphasized that we studied the Steppen model as the only known example of anomalous kinetic roughening of this class to support our experimental finding that $\alpha_S=D$, and not as the model of kinetic roughening of drying wet paper. The physical mechanisms at work in the drying-induced roughening process are discussed in Refs. [8–11].