PHYSICAL REVIEW E **73**, 060901(R) (2006)

Suppression of Winfree turbulence under weak spatiotemporal perturbation

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(Received 26 January 2006; published 16 June 2006)

Winfree turbulence is a chaotic wave pattern developing through negative-tension instability of scroll wave filaments in three-dimensional weak excitable media. Here, we investigate the response of Winfree turbulence to a spatiotemporal forcing in the form of a traveling-wave modulation of the medium excitability. It is shown that turbulent waves can be suppressed much more rapidly by this method, in comparison with the space-uniform modulation of the medium excitability. Since the occurrence of Winfree turbulence is currently regarded as one of the possible mechanisms underlying cardiac fibrillation, this method turns out to be suggestive of a possible low-amplitude defibrillation approach.

DOI: 10.1103/PhysRevE.73.060901 PACS number(s): 87.19.Hh, 82.40.Ck, 87.10.+e

Excitable media, which range from chemical systems to biological cells and tissues, usually show pattern formation in the form of spiral waves in two dimensions or more complex structures such as scroll waves in three dimensions [1–6]. The tip of spiral waves rotates rigidly around a circle or meanders about in the medium in an even more complicated manner, while the scroll waves rotate around a one-dimensional curve known as the filament that is an extension of a spiral tip into three dimensions. This filament may be straight or curved; it may also form a closed ring that, depending on the condition of the medium, may either contract or expand, or even form more complicated entangled loops. In particular, under low-excitability conditions, the filament may be unstable and lead to highly disordered wave patterns.

Winfree turbulence of scroll waves [7–12] is a special kind of spatiotemporal chaos that exists exclusively in three-dimensional excitable media and is currently considered one of the possible mechanisms of cardiac fibrillation. It is a chaotic wave pattern developing through the negative-tension instability of vortex filaments, which tend to spontaneously stretch, bend, loop, and produce an expanding tangle that fills up the volume. As to its relevance for cardiac fibrillation, how to control this form of spatiotemporal chaos with as small as possible damage to the cardiac tissues becomes a challenging problem. In Ref. [10], Alonso *et al.* demonstrate that such turbulence can be controlled by space-uniform modulation of the medium excitability. Recently, we developed a local stimulation method that suppresses Winfree turbulence by generating target waves [12].

However, both of the controlling methods, as well as some other works, focus only on purely temporal or purely spatial modulation of some control parameters. Some recent studies of spiral waves and spiral turbulence in two-dimensional excitable media are performed under spatiotemporal forcing in the form of a traveling-wave modulation of a control parameter [13,14]. Since Winfree turbulence is a kind of spatiotemporal pattern, it will be more interesting to study its response to a spatiotemporal forcing. In this Rapid Communication, we will show that turbulent waves can be suppressed by a traveling-wave forcing imposed on a control parameter of the system and a surprisingly high efficiency to control Winfree turbulence is achieved. Our numerical results will be reported to feature the processes of suppression of Winfree turbulence as well as the relationship between controlling speed and three parameters of the external forcing: the forcing amplitude b_f, the forcing frequency ω_f, and the forcing wave number k_f.

The excitable medium is described by the Barkley model [15]

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon} u(1-u)[u-(v+b)/a] + \nabla^2 u,$$

$$\frac{\partial v}{\partial t} = u - v. \tag{1}$$

Here, u and v are the activator and the inhibitor variables, respectively; ε is the ratio of their temporal scales; the dimensionless parameters a and b denote the activator kinetics with b effectively controlling the excitation threshold of the system. The effect of spatiotemporal forcing is introduced through the parameter b, since this parameter is relevant in determining the excitability of the medium. The contribution of forcing can be decomposed into a mean value b_0 and a modulation part in the form of a traveling wave:

$$b = b_0 + b_f \cos(\vec{k}_f \cdot \vec{r} - \omega_f t). \tag{2}$$

For a nonforcing system $b_f=0$, Eqs. (1) may admit an unstable scroll ring that contracts or expands [10]: the radius of the ring develops at a rate proportional to the filament tension coefficient α and inversely proportional to the radius of the ring R: $\dot{R}=-\alpha/R$. The negative-tension instability will

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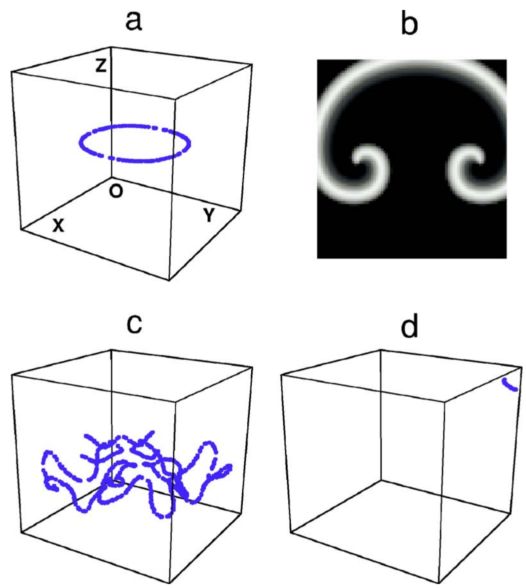

FIG. 1. (Color online). Winfree turbulence of scroll waves and its suppression by a weak spatiotemporal forcing in the form of traveling-wave modulation $b=b_0+b_f\cos(k_f y-\omega_f t)$. (a), (b) $t=0$; initial scroll ring and two-dimensional spiral wave pattern on its cross-section plane. (c) $t=224$; the filament is expanding and gives rise to a turbulence state. Here, we switch on the weak spatiotemporal forcing, $b_f=0.03$, $k_f=k_0$, $\omega_f=1.2$. (d) $t=400$; turbulent waves will disappear after 33.7 T_0 ($t=402$).

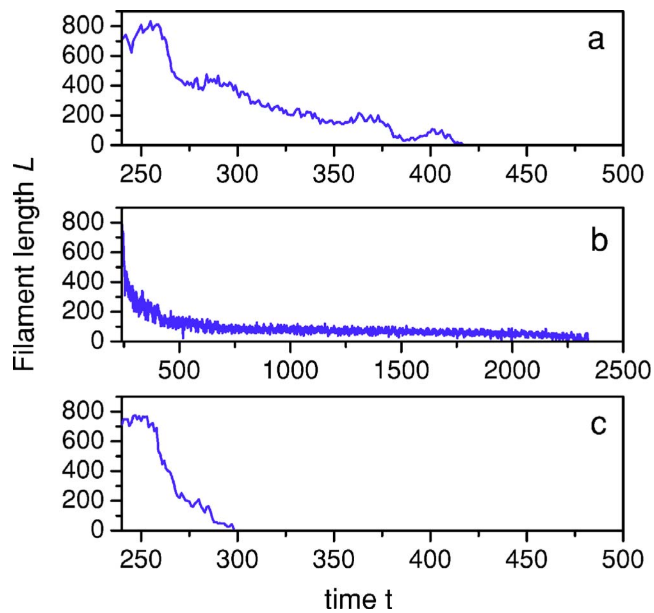

FIG. 2. (Color online) The variation of the vortex filament L versus time t under weak forcing. (a) Traveling-wave modulation: $b=b_0+b_f\cos(k_f y-\omega_f t)$, with $b_f=0.03$, $k_f=k_0$, $\omega_f=1.2$. (b) Space-uniform modulation: $b=b_0+b_f\cos(\omega_f t)$, with $b_f=0.03$, $\omega_f=1.2$. (c) Traveling-wave modulation: $b=b_0+b_f\cos(k_f y-\omega_f t)$, with $b_f=0.03$, $k_f=k_0$, $\omega_f=1.0$. Here all forcings are applied at time $t=224$, corresponding to the Winfree turbulence of state Fig. 1(c).

lead to irregular dynamics of filaments and to the emergence of Winfree turbulence, i.e., a highly disordered pattern of wave propagation.

When an external forcing is switched on with a purely temporal term $b_f\neq 0$, $k_f=0$ (the excitability of the system is uniformly modulated under periodic forcing), the filament tension coefficient α will be modified into an effective filament coefficient $\alpha_{eff}=\alpha+\zeta(b_f^2/\Delta\omega)$ [10], where ζ is a positive coefficient determined by the properties of the media only and $\Delta\omega=\omega_f-\omega_0$ with ω_0 being the frequency of the corresponding two-dimensional spiral waves. Hence, this theory predicts that with the negative filament tension ($\alpha<0$) of the unperturbed system the effective filament tension coefficient can be changed from negative to positive if the forcing frequency is larger than the rotation frequency ($\Delta\omega>0$), which, as a result, prevents the expansion of scroll rings. Thus it is possible to suppress the turbulent waves by shrinking the existing scroll filaments.

Now, let us consider a more general case: $b_f\neq 0$, $k_f\neq 0$. The system is now under the influence of a spatiotemporal perturbation. The numerical results are obtained through an integration of Eq. (1) with the explicit Euler algorithm on a regular rectangular grid inside a cube of $L^3=60^3$, space steps $\Delta x=\Delta y=\Delta z=0.375$, time step $\Delta t=0.01$, and no-flux boundary condition on each surface. In our simulations, for generating Winfree turbulence from an expanding scroll ring, we initiate the evolution by a half spherical wave. Throughout the paper, we fix the parameters to be $a=1.1$, $b_0=0.19$, $\varepsilon=0.02$. The small positive ε value makes it possible to have a unique steady state locally stable but excitable. The values of a and b_0 guarantee that the system is inside the region where Winfree turbulence of scroll waves dominates generically without external forcing. In the unperturbed case, the spiral of the two-dimensional system has its rotation period $T_0=5.28$, frequency $\omega_0=1.19$, wavelength $\lambda_0=23.2$, and wave number $k_0=0.271$.

We begin our simulations with the case of exact 1:1 spatial resonance $k_f=k_0$. To demonstrate the whole processes of evolution and suppression of Winfree turbulence, we plot the

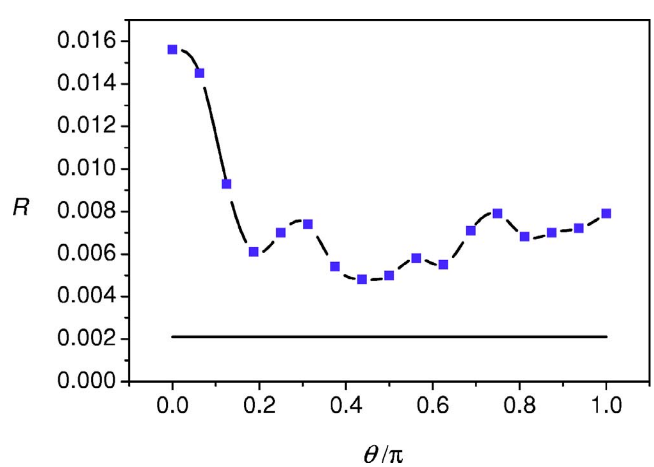

FIG. 3. (Color online) The variation of R of a scroll ring [Fig. 1(a)] versus the angle θ between the forcing wave vector $\vec{k}_f$ and the z axis. $b=b_0+b_f\cos(\vec{k}_f\cdot\vec{r}-\omega_f t)$, $b_f=0.03$, $k_f=k_0$, $\omega_f=1.2$. The solid line shows the value of R of space-uniform modulation: $b=b_0+b_f\cos(\omega_f t)$, with $b_f=0.03$, $\omega_f=1.2$. For any orientation the suppression rate for traveling-wave modulation is higher than that of the space-uniform modulation.

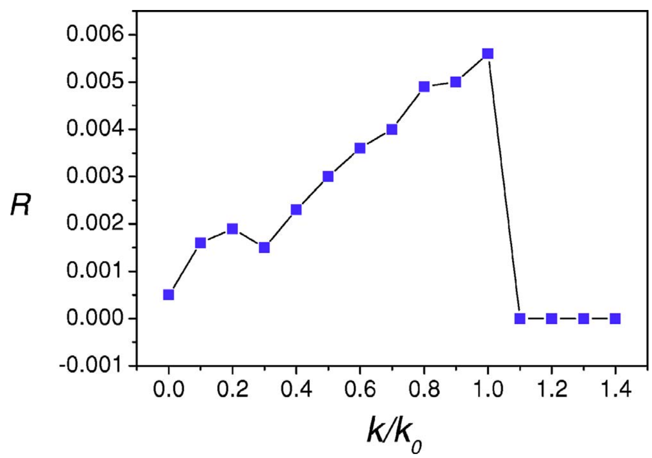

FIG. 4. (Color online) Dependence of R on the wave number k_f with b_f=0.03, ω_f=1.2.

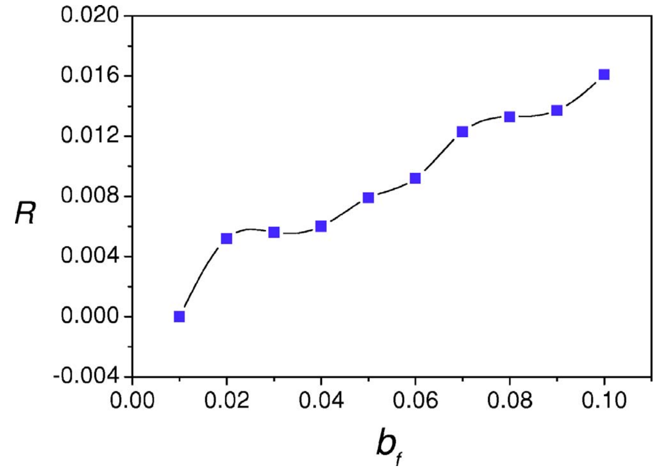

FIG. 5. (Color online) Variation of R with forcing amplitude b_f with k_f=k_0, ω_f=1.2.

three-dimensional graphs. As a starting point of the numerical simulation, the expanding filament is initiated from a ring. There is a translational symmetry along the filament; each slice of a scroll wave perpendicular to the filament contains a two-dimensional rotating spiral wave [see Figs. 1(a) and 1(b)]. As the ring grows, transverse deformations develop. Soon the length of the vortex filament increases rapidly to form an irregular expanding tangle, and the tangle consists of only one connected filament until it touches a boundary plane. After this process, the filament gets fragmented into many pieces that span the whole volume of the medium, and Winfree turbulence emerges as shown in Fig. 1(c). Then, we switch on the modulation of the parameter b in the form of a traveling wave that propagates in the medium, $b=b_0+b_f\cos(k_f y-\omega_f t)$, with b_f=0.03, k_f=k_0, ω_f=1.2. Figure 1(d) shows that the application of traveling-wave forcing leads to suppression of Winfree turbulence: all curved filaments shrink and disappear.

Comparing the space-uniform modulation [10] with the present traveling-wave modulation, the latter method shows much better control efficiency. In Figs. 2(a) and 2(b), we plot the time dependence of the total filament lengths L under a traveling-wave modulation and a space-uniform modulation. The filament length under the traveling-wave modulation [Fig. 2(a)] shrinks to zero about 12 times faster than that under the space-uniform modulation [Fig. 2(b)] with the same combination of two parameters $(b_f,\omega_f)=(0.03,1.2)$. We also give the most optimal forcing frequency ω_f=1.0 still with b_f=0.03 and k_f=k_0: the filament length in the course of time [Fig. 2(c)] shrinks to zero 36 times faster than the uniform one of Fig. 2(b). Thus a considerable advance is achieved by the traveling-wave modulation method when the rate of filament suppression R is considered. R is defined as $R=1/T$, where T is the time when the filament length shrinks to zero. Here, we would like to emphasize that, in practice, this rate is of crucial importance in cardiac defibrillation since long-time cardiac fibrillation can lead to sudden cardiac death.

To give an understanding of the high efficiency in controlling Winfree turbulence, we investigate the action of the traveling-wave forcing on a scroll ring [i.e., on Fig. 1(a)]. Let us consider the influence of θ (θ is the angle between $\vec{k}_f$ and the z axis) on the rate of filament suppression R of a scroll ring. Figure 3 shows that R varies with θ: R reaches its maximal value at θ=0 ($\vec{k}_f$ parallel to the z axis), and it reaches its minimal value around $\theta=\pi/2$ ($\vec{k}_f$ perpendicular to the z axis). Compared with the solid line shown in Fig. 3 (the value of R of space-uniform forcing), the controlling speed of the traveling-wave forcing is faster than that of the space-uniform forcing for all angles. Winfree turbulence can be approximately viewed as a combination of many small sections of rings oriented in different directions; therefore, the traveling-wave forcing can control Winfree turbulence more effectively than the space-uniform one.

Now let us consider the influence of wave number k_f with b_f=0.03, ω_f=1.2. To reveal the response of the scroll filaments to the misfit between the forcing wave number and the characteristic spiral wave number, we plot R versus k_f/k_0 in Fig. 4. It is shown that Winfree turbulence fails to be controlled for $k_f/k_0>1$. We define R as zero if the filament length is still larger than 50 after 500 rotation periods of the original spiral wave T_0. Roughly speaking, R increases with k_f/k_0 until it reaches the upper limit $k_f/k_0=1$, i.e., the exact spatial resonance, where the maximum R is observed. The speed of the space-uniform modulation is given in Fig. 4 by the result of k_f=0, which is small.

Since the forcing amplitude b_f is an important parameter for spatiotemporal control, let us study how the rate of filament suppression R varies on changing b_f. In Fig. 5 we give the influence of the forcing amplitude on R under exact spatial 1:1 resonance (k_f=k_0) with ω_f=1.2: R increases with the forcing amplitude.

Finally, we search for the most effective forcing frequency ω_f with b_f=0.03 and k_f=k_0. A detailed simulation shows that the most effective and applicable frequencies span a small region near ω_f=1.0. In Fig. 6, we focus on this small region and give R for different forcing frequencies ω_f with b_f=0.03 and k_f=k_0. The dependence of R on ω_f ($\omega_f>\omega_0=1.19$) for space-uniform modulation is also given in Fig. 6; in Ref. [10], it is shown that R of space-uniform modulation is zero for $\omega_f<\omega_0$. From Fig. 6, one can see that

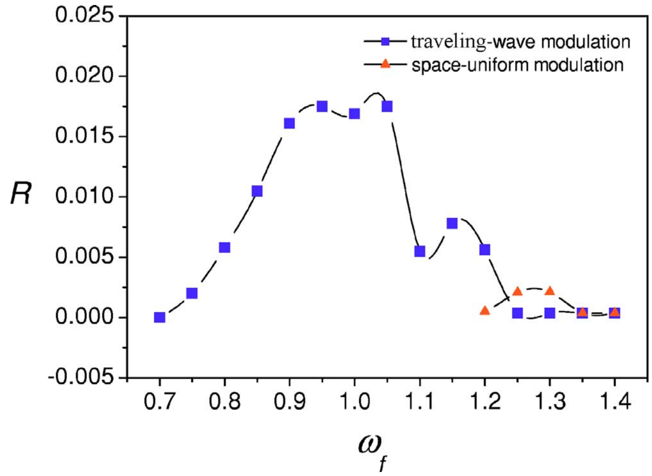

FIG. 6. (Color online) Dependence of R on the forcing frequency ω_f. For traveling-wave modulation, $b_f=0.03$, $k_f=k_0$; for space-uniform modulation, $b_f=0.03$.

the control efficiency of traveling-wave modulation is incomparably higher than that of space-uniform modulation.

In conclusion, with the Barkley model of a three-dimensional weakly excitable medium, we have studied the control of Winfree turbulence with a spatiotemporal forcing. Compared with the space-uniform modulation of the medium excitability [10], our numerical simulations show that turbulent waves can be controlled much more rapidly by traveling-wave modulation if the resonant wave number $k_f=k_0$ and a suitable frequency ω_f is chosen. In particular, the relations among the rate of filament suppression, R, the frequency of the forcing, ω_f, the wave number of the forcing, k_f, and the amplitude of the forcing, b_f are investigated in detail. Since Winfree turbulence is currently regarded as one of the possible mechanisms underlying cardiac fibrillation, we expect that the result in this Rapid Communication may suggest alternative approaches for using low-amplitude stimulations to stop fibrillation. Nevertheless, some problems of crucial importance for the applications to cardiac defibrillation remain open. The Barkley model is a simple monodomain model that ignores the flow of current in the extracellular fluid outside cardiac cells, and real hearts have a nonuniform complex geometry. So the first question is whether the method that has been shown effective in the simple Barkley model could keep its effectiveness in realistic bidomain cardiac models. Presently, it is only possible to perturb electrically the outer surface of the heart. Thus the second question is how one can realize the traveling-wave modulation in real hearts if the answer to the first question is positive. These will be further investigated in our future work.

This work was supported by the National Natural Science Foundation of China, and the Start-Up Fund for Returned Oversea Chinese Scholars.

[1] A. T. Winfree, *When Time Breaks Down* (Princeton University Press, Princeton, NJ, 1987); *The Geometry of Biological Time*, 2nd ed. (Springer-Verlag, Berlin, 2001).
[2] A. T. Winfree, Science **175**, 634 (1972); G. Li, Q. Ouyang, V. Petrov, and H. L. Swinney, Phys. Rev. Lett. **77**, 2105 (1996).
[3] S. Jakubith, H. H. Rotermund, W. Engel, A. von Oertzen, and G. Ertl, Phys. Rev. Lett. **65**, 3013 (1990); M. Kim, M. Bertram, M. Pollmann, A. von Oertzen, A. S. Mikhailov, H. H. Rotermund, and G. Ertl, Science **292**, 1357 (2001).
[4] J. M. Davidenko, A. V. Pertsov, R. Salomonsz, W. Baxter, and J. Jalife, Nature (London) **355**, 349 (1992); F. X. Witkowski, L. J. Leon, P. A. Penkoske, W. R. Giles, M. L. Spano, W. L. Ditto, and A. T. Winfree, *ibid.* **392**, 78 (1998).
[5] A. T. Winfree, Science **181**, 927 (1973).
[6] B. Welsh, J. Gomatam, and A. E. Burgess, Nature (London) **304**, 611 (1983).
[7] A. T. Winfree, Science **266**, 1003 (1994).
[8] V. N. Biktashev, A. V. Holden, and H. Zhang, Philos. Trans. R. Soc. London, Ser. A **347**, 611 (1994).
[9] I. Aranson and I. Mitkov, Phys. Rev. E **58**, 4556 (1998).
[10] S. Alonso, F. Sagués, and A. S. Mikhailov, Science **299**, 1722 (2003).
[11] S. Alonso, R. Kähler, A. S. Mikhailov, and F. Sagués, Phys. Rev. E **70**, 056201 (2004).
[12] H. Zhang, Z. Cao, N. J. Wu, H. P. Ying, and G. Hu, Phys. Rev. Lett. **94**, 188301 (2005).
[13] P. Y. Wang, P. Xie, and H. W. Yin, Chin. Phys. **12**, 674 (2003).
[14] S. Zykov, V. S. Zykov, and V. Davydov, Europhys. Lett. **73**, 335 (2006).
[15] D. Barkley, M. Kness, and L. S. Tuckerman, Phys. Rev. A **42**, R2489 (1990); D. Barkley, Phys. Rev. Lett. **68**, 2090 (1992).