Formation and propagation of coupled ultraslow optical soliton pairs in a cold three-state double-Λ system

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We investigate the simultaneous formation and propagation of coupled ultraslow optical soliton pairs in a cold, lifetime-broadened three-state double-Λ atomic system. Starting from the equations of motion of atomic response and two-mode probe-control electromagnetic fields, we derive coupled nonlinear Schrödinger equations that govern the nonlinear evolution of the envelopes of the probe fields in this four-wave mixing scheme by means of the standard method of multiple scales. We demonstrate that for weak probe fields and with suitable operation conditions, a pair of coupled optical solitons moving with remarkably slow propagating velocity can be established in such a highly resonant atomic medium. The key elements to such a shape preserving, well matched yet interacting soliton pair is the balance between dispersion effect and self- and cross-phase modulation effects of the system.

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I. INTRODUCTION

Resonant interaction between electromagnetic radiation and multilevel atomic media has been the center of many recent studies. One of the important subjects in this field is how to achieve loss-free and distortion-free propagation of optical pulses in an optical thick medium. In the past decades, significant progress has been made in this research direction, including self-induced transparency in two-level atoms [1], simultons (simultaneous different-wavelength optical solitons), and counterintuitive pulse sequences in three- and multilevel systems [2–7]. In all these works, the optical fields involved are always intense enough that in general the optical absorption and pulse distortion can be neglected. When the fields are weak, however, strong one-photon absorption often significantly attenuates signal field, leading to weak and distorted optical signal waves.

In recent years, the technique of electromagnetically induced transparency (EIT) [8–11] has attracted a great deal of attention mainly because it can render an otherwise opaque medium transparent to a signal field even when the field is tuned on to a very strong one-photon transition. It has been shown that the wave propagation in such a highly resonant optical medium possesses many striking features [12–16]. One of these features is the change of dispersion (or index) property of the medium by a control field that produces the transparency. Such a significant change of material dispersion properties naturally leads to a modification of propagating velocity of the signal field. This effect is particularly significant under weak driving conditions where the reduction of group velocity can be very substantial [17,18]. Such ultraslow propagation of optical fields may have important applications in the field of optical telecommunications where devices such as high fidelity optical buffers, phase shifts, transmission lines, switches, routers, and wavelength converters [19], etc., are highly desirable.

In addition to efficient suppression of absorption and modification of dispersion properties, the EIT technique can be used to significantly enhance the Kerr nonlinearities of optical media. In particular, it has been proposed for achieving a large nonlinear phase shift with a very weak control optical field [12,14]. Furthermore, this technique has been shown to be beneficial to certain nonlinear optical processes under weak driving conditions, where the ultraslow propagation [17–20] is a dominant feature.

However, one must notice that the significant modification of the material dispersion properties inevitably leads to significant increase of signal field attenuation and distortion [21]. This is why most of the recent studies on ultraslow wave propagation also exhibit substantial signal field loss and deformation. Thus it is necessary to seek an effective remedy to reduce such a distortion yet to preserve important features such as ultraslow propagation. It has been proposed recently [22–25] to tailor the nonlinear properties of a highly resonant medium to balance detrimental effects of attenuation and distortion. This proposal leads to a new class of optical solitons, i.e., *ultraslow optical solitons*, achievable using highly resonant optical media under weak driving condition. Such shape preserving optical pulse propagation may have potential applications in optical information processing and engineering. It is for this reason that ultraslow optical solitons deserve to be pursued in both fields of fundamental research and technological development [22–25].

In the early study of ultraslow solitons [22], only one probe field is used and hence one obtains a single nonlinear Schrödinger (NLS) equation that admits a single component soliton under the condition where the dispersion effect can be balanced by the self-phase modulation (SPM) effect. In many cases that are of interest to both fundamental physics and technical applications, however, one often encounters propagation of multiple fields in a single medium. In these cases, the interaction of the two or multiple fields via the atomic medium will lead to coupled field propagation where both SPM and cross-phase modulation (CPM) effects of the input fields are of importance. Thus it is necessary to inves-

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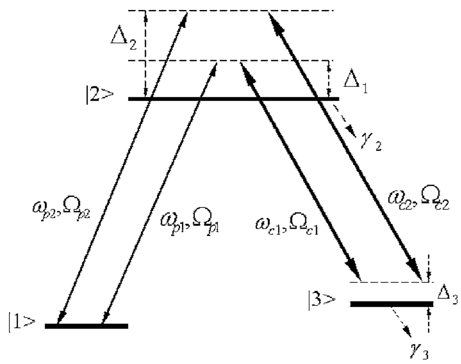

FIG. 1. Lifetime-broadened three-level double-Λ atomic system interacting with a two-mode probe field with angular frequencies ω_{pn} and a two-mode control field with angular frequencies ω_{cn} ($n=1,2$). Δ_1 and Δ_2 are the two one-photon detunings, and Δ_3 is the two-photon detuning. Ω_{pn} and Ω_{cn} are half of the Rabi frequencies for the probe and control fields.

tigate the effects of both SPM and CPM on ultraslow propagation. It is this topic that will be addressed in the present study. We shall consider simultaneous formation and propagation of two coupled ultraslow optical solitons in a cold, lifetime-broadened simple three-level double-Λ atomic system. This scheme is simple and requires fewer resources; one laser with a few acousto-optical modulators is sufficient to generate all fields needed yet preserve the required phase relations. It has been shown recently [26] that such a simple scheme permits, under suitable conditions and by neglecting higher nonlinear contributions, two independent input fields to evolve into a pair of temporally, amplitude, and group-velocity (TAG) matched ultraslow optical pulses. We will show here that when nonlinear effects are included, the coupled optical fields can evolve into a pair of ultraslow optical solitons with remarkable propagating characteristics. Such optical soliton pairs may have potential applications in high-fidelity information storage, photon pair entanglement, and quantum computing [26]. The paper is arranged as follows. In Sec. II, we describe a three-state two-mode model and discuss its solution in a linear regime. In Sec. III, we derive two coupled NLS equations controlling the evolution of two wave envelopes of optical fields by using a method of multiple scales. In Sec. IV, we provide the optical soliton solutions of the coupled NLS equations and discuss their physical properties. Section V contains a discussion and summary of our results.

II. THE MODEL AND SOLUTION IN LINEAR REGIME

The lifetime-broadened three-level double-Λ atomic system under consideration is shown in Fig. 1. Technically, this system is very simple and a single laser with suitable acousto-optical modulators is sufficient to generate all needed fields. In this model, there is a pulsed two-frequency-mode probe field [with center angular frequencies ω_{pn} ($n=1,2$)] and two-frequency-mode continuous-wave (cw) control field [with center angular frequencies ω_{cn} ($n=1,2$)] couple states $|1\rangle$ and $|2\rangle$, and $|2\rangle$ and $|3\rangle$, respectively. Thus, under suitable driving condition, we have a double-Λ four-wave mixing (FWM) scheme [27]. The probe field has a pulse length τ_0 at the entrance of the medium. In the interaction picture, the material equations of motion for atomic response and the wave equation for the two-component probe field are given by [26]

$$\left(i\frac{\partial}{\partial t}+d_{pn}\right)A_{2n}+\Omega_{cn}A_3+\Omega_{pn}A_1=0, \tag{1a}$$

$$\left(i\frac{\partial}{\partial t}+d_3\right)A_3+\Omega_{c1}^*A_{21}+\Omega_{c2}^*A_{22}=0, \tag{1b}$$

$$i\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{pn}+\frac{c}{2\omega_{pn}}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Omega_{pn}+\kappa_{12}A_{2n}A_1^*=0, \tag{1c}$$

where $A_{2n}(n=1,2)$ is the part of state $|2\rangle$'s amplitude with the polarization at angular frequency ω_{pn}, $d_{pn}=\Delta_{pn}+i\gamma_2$ with Δ_{pn} being the one-photon detuning of the probe laser (ω_{pn}) from $|1\rangle\to|2\rangle$ resonance and γ_2 being the decay (homogeneous relaxation) rate of state $|2\rangle$. In addition, A_3 is the amplitude of state $|3\rangle$, $d_3=\Delta_{pn}+i\gamma_3$ with Δ_3 being the two-photon detuning between the states $|1\rangle$ and $|3\rangle$ and γ_3 being the decay rate of state $|3\rangle$. Finally, $2\Omega_{pn}$ ($2\Omega_{cn}$) is the Rabi frequency of the probe (control) field corresponding to the relevant frequency mode, $\kappa_{12}=2\pi N\omega_{pn}|D_{12}|^2/(\hbar c)$ with N and D_{12} being atomic concentration and the dipole moment of the transition $|1\rangle\to|2\rangle$, respectively. The main approximation when obtaining Eqs. (1a)–(1c) is the neglect of far off-resonant terms such as cross-mode emission with nonvanishing two-photon detuning. It should always be accurate if the probe field at ω_{pn} is sufficiently weak [26].

We first consider the linear theory of wave propagation in the system. When the probe field is very weak, the ground state is not depleted and hence one has $A_1\simeq 1$. Assuming that Ω_{pn}, A_{2n}, and A_3 are proportional to $\exp[i(Kz-\omega t)]$ and neglecting the transverse diffraction effect, we obtain the linear dispersion relation

$$K=K^{\pm}=\frac{\omega}{c}+\frac{\kappa_{12}}{2D}\{-(D_1+D_2)\pm[(D_1-D_2)^2+4|\Omega_{c1}\Omega_{c2}|^2]^{1/2}\}, \tag{2}$$

where

$$D_n=|\Omega_{cn}|^2-(\omega+d_{pn})(\omega+d_3),$$

$$D=|\Omega_{c1}|^2(\omega+d_{p2})+|\Omega_{c2}|^2(\omega+d_{p1})-(\omega+d_{p1})(\omega+d_{p2})(\omega+d_3).$$

We see that the system displays two branches of dispersion curve, $K=K^{\pm}(\omega)$. With these results we can construct a general solution for the linear wave by a Fourier superposition,

$$\Omega_{p1}(z,t)=\int_{-\infty}^{\infty}d\omega[F_p^+e^{i\theta_+}+F_p^-e^{i\theta_-}], \tag{3a}$$

$$\Omega_{p2}(z,t) = \frac{1}{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}\int_{-\infty}^{\infty} d\omega[G_+F_p^+e^{i\theta_+} + G_-F_p^-e^{i\theta_-}] \tag{3b}$$

with similar expressions for A_{2n} and A_3, where $\theta_\pm = K^\pm z - \omega t$ and $G_\pm = (K^\pm - \omega/c)D + \kappa_{12}D_2$. The expressions for F_p^+ and F_p^- are given by

$$F_p^+ = \frac{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}{G_+ - G_-}\tilde{\Omega}_{p2} - \frac{G_-}{G_+ - G_-}\tilde{\Omega}_{p1}, \tag{4a}$$

$$F_p^- = \frac{G_+}{G_+ - G_-}\tilde{\Omega}_{p1} - \frac{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}{G_+ - G_-}\tilde{\Omega}_{p2}, \tag{4b}$$

where $\tilde{\Omega}_{pn} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\,\Omega_{pn}(0,t)e^{i\omega t}$ is the Fourier transform of the probe field at the entrance of the medium. Thus one has generally electromagnetic excitations with both K^+ and K^- dispersion branches, with amplitudes being characterized by the envelope functions F_p^+ and F_p^-, respectively. However, note that in some particular cases one can get excitations from only one dispersion branch. For example, if the boundary condition is chosen such that $G_+\tilde{\Omega}_{p1} = \kappa_{12}\Omega_{c1}\Omega_{c2}^*\tilde{\Omega}_{p2}$, one has $F_p^{(-)} = 0$ and hence only the K^+ branch appears; this case will be discussed in Sec. IV A.

Equation (2) shows that the system displays dispersion, which will result in a broadening of the input pulse. To demonstrate this, we make the Taylor expansion at the center frequency of the probe field (i.e., $\omega = 0$),

$$K^\pm(\omega) = K_0^\pm + K_1^\pm\omega + \frac{1}{2}K_2^\pm\omega^2 + \cdots, \tag{5}$$

where $K_j^\pm = (\partial^j K^\pm/\partial\omega^j)|_{\omega=0}$ $(j = 0,1,2,\ldots)$. For input Gaussian wave packets $\Omega_{pn}(0,t) = \Omega_{pn}(0,0)\exp(-t^2/\tau_0^2)$, we obtain the following expression [also a similar expression for $\Omega_{p2}(z,t)$]:

$$\Omega_{p1}(z,t) = \frac{\Lambda_p^+(0,0)}{\sqrt{b_1^+ - ib_2^+}}\exp\left[iK_0^+z - \frac{(K_1^+z - t)^2}{\tau_0^2(b_1^+ - ib_2^+)}\right] + \frac{\Lambda_p^-(0,0)}{\sqrt{b_1^- - ib_2^-}}\exp\left[iK_-^0z - \frac{(K_1^-z - t)^2}{\tau_0^2(b_1^- - ib_2^-)}\right],$$

where $b_1^\pm = 1 + 2z\,\mathrm{Im}(K_2^\pm)/\tau_0^2$, $b_2^\pm = -2z\,\mathrm{Re}(K_2^\pm)/\tau_0^2$, $\Lambda_p^+(0,0) = [\Omega_{p2}(0,0) - G_-(0)\Omega_{p1}(0,0)]/[G_+(0) - G_-(0)]$, and $\Lambda_p^-(0,0) = [G_+(0)\Omega_{p1}(0,0) - \Omega_{p2}(0,0)]/[G_+(0) - G_-(0)]$. Clearly, each input field will break into two propagation components, propagating with the different group velocities $1/\mathrm{Re}[K_1^+]$ and $1/\mathrm{Re}[K_1^+]$, respectively. Due to nonvanishing $K_2^\pm$, the wave packets will spread and attenuate as the distance z increases, and will eventually separate from each other.

III. ASYMPTOTIC EXPANSION AND NONLINEAR ENVELOPE EQUATIONS

In this section, we apply a weak nonlinear perturbation theory to the three-state double Λ system and search for the formation and propagation of shape-preserving probe pulses. We note that nonvanishing one- and two-photon detunings are necessary to introduce group-velocity dispersion and also induce SPM and CPM effects, which can provide effective means to balance the detrimental dispersion effect, leading to stable formation and propagation of solitonlike probe pulses. To make the nonlinear effect of the system significant, the intensity of the probe field should be increased to deplete the population of the ground state $|1\rangle$. We go beyond the linear theory by systematically including the depletion of the ground-state population and the nonadiabatic corrections of the atomic response. We assume that the dispersion and nonlinearity of the system are not strong so that a standard method of multiple scales [28] can be used to derive the nonlinearly coupled envelope equations for both dispersion branches. For this purpose, we make the following asymptotic expansion $(n = 1,2)$:

$$A_1 = 1 + \varepsilon^2 a_1^{(2)} + \varepsilon^3 a_1^{(3)} + \cdots, \tag{6a}$$

$$A_{2n} = \varepsilon a_{2n}^{(1)} + \varepsilon^2 a_{2n}^{(2)} + \varepsilon^3 a_{2n}^{(3)} + \cdots, \tag{6b}$$

$$A_3 = \varepsilon a_3^{(1)} + \varepsilon^2 a_3^{(2)} + \varepsilon^3 a_3^{(3)} + \cdots, \tag{6c}$$

$$\Omega_{pn} = \varepsilon\Omega_{pn}^{(1)} + \varepsilon^2\Omega_{pn}^{(2)} + \varepsilon^3\Omega_{pn}^{(3)} + \cdots, \tag{6d}$$

where ε is a small parameter characterizing the small population depletion in the ground state. To obtain a divergence-free expansion, we assume all quantities on the right-hand side (rhs) of Eqs. (6a)–(6d) are the functions of the multiscale variables $z_l = \varepsilon^l z (l = 0,1,2)$, $t_l = \varepsilon^l t (l = 0,1)$, $x_1 = \varepsilon x$, and $y_1 = \varepsilon y$. Then Eqs. (1a)–(1c) are converted into

$$\left(i\frac{\partial}{\partial t_0} + d_{pn}\right)a_{2n}^{(j)} + \Omega_{cn}a_3^{(j)} + \Omega_{pn}^{(j)} = M_n^{(j)}, \tag{7a}$$

$$\left(i\frac{\partial}{\partial t_0} + d_3\right)a_3^{(j)} + \Omega_{c1}^*a_{21}^{(j)} + \Omega_{c2}^*a_{22}^{(j)} = N^{(j)}, \tag{7b}$$

$$i\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\Omega_{pn}^{(j)} + \kappa_{12}a_{2n}^{(j)} = P_n^{(j)}, \tag{7c}$$

together with the condition $a_1^{(2)} + (a_1^{(2)})^* = -(a_{21}^{(1)} + a_{22}^{(1)})(a_{21}^{(1)} + a_{22}^{(1)})^* - a_3^{(1)}(a_3^{(1)})^*$ [higher-order $a_1^{(j)}$ $(j \geq 3)$ are not needed and thus neglected]. The explicit expressions of $M_n^{(j)}$, $N^{(j)}$, and $P_n^{(j)}$ $(n = 1,2)$ are omitted to save space.

Equations (7a)–(7c) can be solved order by order. The case for $j = 1$ is just the linear problem solved in the preceding section and hence one can obtain the linear dispersion relation, given by Eq. (2), and the solution for $a_{2n}^{(1)}$, $a_3^{(1)}$, and $\Omega_{pn}^{(1)}$ in a general form. However, here we are interested in the case in which only one dispersion branch is excited. Thus we take

$$\Omega_{p1}^{(1)} = F_p^+e^{i\theta_+} + F_p^-e^{i\theta_-}, \tag{8}$$

where $\theta_\pm = K^\pm z_0 - \omega t_0 = K^\pm z - \omega t$. The solutions for $\Omega_{p2}^{(1)}$, $a_{2n}^{(1)}$, and $a_3^{(1)}$ are given by

$$\Omega_{p2}^{(1)} = \frac{1}{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}\left[G_+F_p^+e^{i\theta_+} + G_-F_p^-e^{i\theta_-}\right], \tag{9a}$$

$$a_{21}^{(1)} = \frac{1}{\kappa_{12}}\left[\left(K^{+} - \frac{\omega}{c}\right)F_p^{+}e^{i\theta_+} + \left(K^{-} - \frac{\omega}{c}\right)F_p^{-}e^{i\theta_-}\right], \quad (9b)$$

$$a_{22}^{(1)} = \frac{1}{\kappa_{12}^2\Omega_{c1}\Omega_{c2}^*}\left[\left(K^{+} - \frac{\omega}{c}\right)G_+F_p^{+}e^{i\theta_+} + \left(K^{-} - \frac{\omega}{c}\right)G_-F_p^{-}e^{i\theta_-}\right], \quad (9c)$$

$$a_3^{(1)} = H_+F_p^{+}e^{i\theta_+} + H_-F_p^{-}e^{i\theta_-}, \quad (9d)$$

where $H_{\pm}=-[1+(\omega+d_3)(K^{\pm}-\omega/c)/\kappa_{12}]/\Omega_{c1}$, and $F_p^{\pm}$ are yet to be determined envelope functions depending on the slow variables z_j and t_j.

In the next order ($j=2$), a solvability condition for the second-order solution of Eq. (7) requires

$$i\left(\frac{\partial}{\partial z_1} + \frac{1}{V_g^{\pm}}\frac{\partial}{\partial t_1}\right)F_p^{\pm} = 0, \quad (10)$$

where $V_g^{\pm}=K_1^{\pm}$ is the group velocity of the wave envelope $F_p^{\pm}$.

In the order $j=3$, the solvability conditions for the third-order solution of Eq. (7) yield to the closed equations,

$$i\frac{\partial F_p^{+}}{\partial z_2} - \frac{1}{2}K_2^{+}\frac{\partial^2 F_p^{+}}{\partial t_1^2} + \alpha_+\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}\right)F_p^{+} - (\beta_{11}|F_p^{+}|^2 + \beta_{12}|F_p^{-}|^2)F_p^{+} = 0, \quad (11a)$$

$$i\frac{\partial F_p^{-}}{\partial z_2} - \frac{1}{2}K_2^{-}\frac{\partial^2 F_p^{-}}{\partial t_1^2} + \alpha_-\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}\right)F_p^{-} - (\beta_{21}|F_p^{+}|^2 + \beta_{22}|F_p^{-}|^2)F_p^{-} = 0, \quad (11b)$$

where the explicit expressions of the coefficients have been given in Appendix A. Combining Eqs. (10) and (11) and returning to original variables, we obtain

$$i\frac{\partial U^{+}}{\partial z} + i\delta\frac{\partial U^{+}}{\partial \tau} - \frac{1}{2}K_2^{+}\frac{\partial^2 U^{+}}{\partial \tau^2} + \alpha_+\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)U^{+} - (\beta_{11}|U^{+}|^2 + \beta_{12}|U^{-}|^2)U^{+} = 0, \quad (12a)$$

$$i\frac{\partial U^{-}}{\partial z} - i\delta\frac{\partial U^{-}}{\partial \tau} - \frac{1}{2}K_2^{-}\frac{\partial^2 U^{-}}{\partial \tau^2} + \alpha_-\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)U^{-} - (\beta_{21}|U^{+}|^2 + \beta_{22}|U^{-}|^2)U^{-} = 0, \quad (12b)$$

where $U^{\pm}=\varepsilon F_p^{\pm}$, $\tau=t-z/V_g$, and $\delta=(1/V_g^{+}-1/V_g^{-})/2$ with $V_g=2V_g^{+}V_g^{-}/(V_g^{+}+V_g^{-})$. Equations (12a) and (12b) are the key coupled nonlinear equations controlling the propagation of the two-frequency-mode probe field. In the following sections, we will discuss various cases where stable formation and propagation of an optical soliton pair can be achieved.

IV. COUPLED OPTICAL SOLITON SOLUTIONS

Equations (12a) and (12b) are two coupled NLS equations, with parameters δ, $K_2^{\pm}$, $\alpha_{\pm}$, $\beta_{n,n}$, $\beta_{n,3-n}$ $(n=1,2)$ characterizing, respectively, the group-velocity mismatch, dispersion, diffraction, SPM, and CPM effects of the system. Although there is a large body of research on coupled solitons (also called vector solitons [29]) in the fields of optical fibers [30], crystal lattices [31], and others [32–34], as we shall show below, the coupled optical solitons predicted here are characteristically very different from those obtained in solid media under very different excitation conditions. Indeed, in contrast to the conventional optical soliton generation techniques, which can be classified as far-off resonance techniques, the scheme described in the present study is qualified as a *near-resonance technique*. Obviously, the latter entertains the possibility of tunability and dynamic switching, which is inherently lack in the conventional methods.

A. Single-dispersion-branch excitation

We first consider a single-dispersion-branch excitation, which is possible in two cases. One is that when the input condition of the two-frequency-mode probe field is chosen such that $G_+\tilde{\Omega}_{p1}=\kappa_{12}\Omega_{c1}\Omega_{c2}^*\tilde{\Omega}_{p2}$. In this case, one has $F_p^{(-)}=0$ [see Eq. (4a) in Sec. II] and hence only a K^+ branch exists. Another case corresponds to small values of Δ_3 and γ_3. In this situation, a detailed analysis reveals readily that the K^- branch has a larger imaginary part than the K^+ branch, thus after a short propagation distance the K^--branch component has decayed away. Note that small detuning Δ_3 is easy to realize in experiment, and in an ultracold atomic gas (such as ultracold ^{87}Rb vapor) where the Doppler broadening is negligible and the homogeneous relaxation rate of the energy state $|3\rangle$ can be made very small, e.g., $\gamma_3=10^4$ s^{-1} or less. Thus a single-dispersion-branch excitation is realistic in our three-state double-Λ system.

In order to make an estimation for the relative importance of various terms in the equation for U^+, we write it in dimensionless form,

$$i\frac{\partial u^{+}}{\partial s} - d_D\frac{\partial^2 u^{+}}{\partial \sigma^2} - d_{\mathrm{NL}}|u^{+}|^2u^{+} + d_F\left(\frac{\partial^2 u^{+}}{\partial x'^2} + \frac{\partial^2 u^{+}}{\partial y'^2}\right) = 0, \quad (13)$$

where we have scaled the variables by using $\sigma=(t-z/V_g^{+})/\tau_0$, $s=z/L_D$, $(x',y')=(x,y)/R_{\perp}$, and $u^{+}=U^{+}/U_0$. Here $L_D=2\tau_0^2/|K_2^{+}|$ is the dispersion length and $R_{\perp}$ is the beam radius of the probe field pulse. In order to favor the formation of a soliton, we have assumed L_D is equal to L_{NL} (the nonlinear length), i.e., the balance of the dispersion and nonlinearity of the system, where $L_{\mathrm{NL}}=1/(U_0^2|\beta_{11}|)$ is the nonlinear length, with $2U_0=\tau_0^{-1}(2|K_2^{+}|/|\beta_{11}|)^{1/2}$ being the typical Rabi frequency of the probe field. The dimensionless coefficients of Eq. (13), are given by $d_D=\mathrm{sgn}(K_2^{+})$, $d_{\mathrm{NL}}=\mathrm{sgn}(\beta_{11})$, and $d_F=L_D/L_F$, with $L_F=R_{\perp}^2/\alpha_+$ (diffraction length). Note that when getting the dimensionless Eq. (13), we have assumed that the imaginary parts of its coefficients are much less than the relevant real parts, thus they can be neglected. The rationality and reality of this assumption will be discussed below.

If $d_F \ll 1$, the diffraction term can be neglected. Then Eq. (13) is reduced to the well known NLS equation that is completely integrable and allows bright and dark soliton solutions, depending on the sign of d_Dd_{NL}. When $d_Dd_{\mathrm{NL}}>0$, one has the single-soliton solution $u^{+}=\sqrt{2}$ sech $\sigma\exp(-is)$, or in terms of probe field (near the center frequency, i.e., $\omega=0$),

$$\Omega_{p1}(z,t)=\frac{1}{\tau_0}\left(\frac{|K_2^+|}{|\beta_{11}|}\right)^{1/2}\mathrm{sech}\left[\frac{1}{\tau_0}\left(t-\frac{z}{V_g^+}\right)\right]\times\exp\left[iK^+(0)z-i\frac{z}{L_D}\right], \tag{14a}$$

$$\Omega_{p2}(z,t)=\frac{G_+(0)}{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}\frac{1}{\tau_0}\left(\frac{|K_2^+|}{|\beta_{11}|}\right)^{1/2}\mathrm{sech}\left[\frac{1}{\tau_0}\left(t-\frac{z}{V_g^+}\right)\right]\times\exp\left[iK^+(0)z-i\frac{z}{L_D}\right]=\frac{G_+(0)}{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}\Omega_{p1}(z,t), \tag{14b}$$

when returning to original variables. Equation (14) represents typical bright solitons traveling with common propagating velocity V_g^+.

Before getting into specific parameters, let us consider a few particular cases.

(i) *The limit of single-frequency-mode probe field.* If we remove the second control field at ω_{c2}, the system is reduced to a single-frequency-mode probe field limit. In this case, one has no internally generated field Ω_{p2} and from the discussion given after Eq. (3a) one has $G_+=0$ and hence only the solution given by Eq. (14a), which describes a single soliton in a three-state single-Λ system, as has been studied before [24].

(ii) *The case of* $\Delta_3=0$ *and* $\Delta_{pn}=0$ *(*$n=1,2$*)*. In this case, it is clear from Eq. (2) and the definition of D_n and D that under the adiabatic condition used in our derivation, D_n is largely a real quantity whereas D is purely imaginary (assuming $\gamma_3\simeq 0$). It follows that $K^{\pm}$ are imaginary and the exponential factors in Eqs. (14) indicate there will be no soliton formation. This is in accord with the previous findings that the two-photon detuning is necessary to bring out possible soliton behavior [23,24].

We now give a set of experimental realistic numerical parameters for the formation of the solitons given above. For ultracold ^{87}Rb atomic vapors, we choose $|1\rangle=5S_{1/2}$ $(F=1,M_F=-1)$, $|2\rangle=5P_{1/2}$ $(F=2,M_F=0)$, and $|3\rangle=5S_{1/2}$ $(F=2,M_F=1)$. Since the lifetime of the $5P_{1/2}$ level is 27.7×10^{-9} s, we have $\gamma_2=1.805\times10^{-7}$ s^{-1}. Taking $D_{12}=2.20\times10^{-18}$ esu cm and $N=10^{14}$, we have $\kappa_{12}=2.28\times10^{11}$ cm s^{-1}. Other parameters are selected as $\gamma_3=1.0\times10^4$ s^{-1}, $\Delta_1=1.0\times10^8$ s^{-1}, $\Delta_2=2.0\times10^8$ s^{-1}, $\Delta_3=1.0\times10^6$ s^{-1}, $\Omega_{c1}=\Omega_{c2}=1.0\times10^8$ s^{-1}, $\tau_0=3.0\times10^7$ s^{-1}, $\lambda_{p1}\approx\lambda_{p2}=0.8\times10^{-4}$ cm, and $R_\perp=0.1$ cm. With the above parameters one gets $K^+=(11.49+0.13i)$ cm^{-1}, $K_1^+=(1.15+0.0023i)\times10^{-5}$ cm^{-1} s, $K_2^+=(8.83+1.08i)\times10^{-14}$ cm^{-1} s^2, and $\beta_{11}=(1.15+0.013i)\times10^{-15}$ cm^{-1} s^2. Note that the imaginary parts of the above quantities are much less than the relevant real parts, thus they can be safely neglected (i.e., the damping due to the small imaginary parts can be taken as a perturbation on solitons). The reason for the small imaginary parts in these quantities is due to one- and three-photon destructive interferences and hence a kind of induced transparency for the probe field [26]. In terms of these quantities, we obtain $U_0=2.07\times10^7$ s^{-1}, $L_D=L_{\mathrm{NL}}=2.02$ cm, $L_{\gamma1}\equiv1/\mathrm{Im}K^+$ (linear absorption length)=7.93 cm, and $L_F=792.4$ cm. Since $L_{\gamma1}$ and L_F are much larger than L_D, the damping and diffraction effects can be safely neglected. When the light pulse propagates to distance $z=2.02$ cm, the solitons (14) (with spatial width 0.026 cm) form, and their propagating velocity is given by $V_g^+=2.88\times10^{-6}$ c. Thus the solitons obtained have indeed an ultraslow propagating velocity comparing with the light speed in vacuum.

Using the above parameters, it is easy to show that $G_+(0)/(\kappa_{12}\Omega_{c1}\Omega_{c2}^*)=1$. We then arrive at

$$\Omega_{p1}(z,t)=\Omega_{p2}(z,t). \tag{15}$$

Thus we have obtained an ultraslowly propagating optical soliton pair, which is completely matched in amplitude, wave form, and propagating velocity, i.e., TAG matched nonlinear pulse pair [35]. Such a completely matched ultraslow optical soliton pair in a single-species three-state medium may have important applications in high-fidelity quantum information storage, photon pair entanglement, and quantum computing [26].

We now discuss the formation condition of the ultraslow optical soliton pair. The flux of energy of the probe field is given by the Poynting vector integrated over the cross section of the sample,

$$P=\int\int dS(\mathbf{E}_p\times\mathbf{H}_p)\cdot\mathbf{e}_z, \tag{16}$$

where $\mathbf{e}_z$ is the unit vector in the propagation direction. To leading order, the field is transverse and one has $\mathbf{E}_p=(E_{p1}+E_{p2},0,0)$, then $\mathbf{H}_p=(0,H_{p1}+H_{p2},0)$ with $H_{pn}=\varepsilon_0 cn(\omega_{pn})E_{pn}$, where $n(\omega_{pn})$ is the refractive index of the probe field at frequency ω_{pn}. Note that when $E_{pn}=(\hbar/D_0)\Omega_{pn}\exp[i(\omega_{pn}z/c-\omega_{pn}t)]+\text{c.c.}$, one obtains the average flux of energy over the carrier-wave period,

$$\bar{P}=\bar{P}_{\max}\mathrm{sech}^2\left[\frac{1}{\tau_0}\left(t-\frac{z}{V_g^+}\right)\right], \tag{17}$$

where the peak power reads

$$\bar{P}_{\max}=2\varepsilon_0 cS_0\left[n(\omega_{p1})|E_{p1}|^2+n(\omega_{p2})|E_{p2}|^2\right]=4\varepsilon_0 cS_0\left[n(\omega_{p1})+n(\omega_{p2})\times\left(\left|\frac{|G_+|}{\kappa_{12}|\Omega_{c1}\Omega_{c2}|}\right|\right)^2\right]\left(\frac{\hbar}{D_{12}}\right)^2\frac{1}{\tau_0^2}\frac{|K_2^+|}{|\beta_{11}|}, \tag{18}$$

where S_0 is the cross-section area of the sample. We see that the peak power is directly proportional to the dispersion coefficient $|K_2^+|$ and inversely proportional to the square of the pulse width τ_0 as well as the self-phase modulation coefficient $|\beta_{11}|$. Using the above numerical example and taking $D_{12}=2.1\times10^{-27}$ cm C and $S_0=1.0\times10^{-2}$ cm^2, we obtain $|E_{p1}|_{\max}=|E_{p2}|_{\max}=2.07\times10^2$ V/m and $\bar{P}_{\max}=4.53\times10^{-4}$ W. Thus, very low field intensity and input power are required for generating an ultraslow optical soliton pair using a highly resonant atomic medium. This is drastically different from the conventional optical soliton generation technique using optical fibers where ps or fs laser pulses are needed to reach very high peak power in order to bring out the nonlinear effect needed for soliton formation.

B. Two-dispersion-branch excitations

We now consider the case in which both dispersion branches (i.e., K^+ and K^- branches) are excited in the system. We show that it is still possible to generate soliton pairs from each probe wave. That is, it is possible to generate two simultaneous solitons (simultons [2,36]) that move with the same propagating velocity. To this end we rewrite Eqs. (12a) and (12b) in the following dimensionless form:

$$i\frac{\partial u^+}{\partial s} + id_\delta\frac{\partial u^+}{\partial \sigma} - d_{1\mathrm{D}}\frac{\partial^2 u^+}{\partial \sigma^2} - d_{11}|u^+|^2u^+ - d_{12}|u^-|^2u^+ = 0, \tag{19a}$$

$$i\frac{\partial u^-}{\partial s} - id_\delta\frac{\partial u^-}{\partial \sigma} - d_{2\mathrm{D}}\frac{\partial^2 u^-}{\partial \sigma^2} - d_{21}|u^+|^2u^- - d_{22}|u^-|^2u^- = 0. \tag{19b}$$

Here, $s=z/L_D$, $\sigma=\tau/\tau_0$, $u^\pm=U^\pm/U_0$, $d_{1\mathrm{D}}=K_2^+/|K_2^-|$, $d_{2\mathrm{D}}=\mathrm{sgn}(K_2^-)$, $d_{ln}=\beta_{ln}/|\beta_{22}|$ $(l,n=1,2)$, and $d_\delta=\mathrm{sgn}(\delta)L_D/L_\delta$, with $L_D=2\tau_0^2/|K_2^-|$ (dispersion length) and $L_\delta=\tau_0/|\delta|$ (group-velocity mismatch length). Again, in favoring the formation of solitons, we assume that the imaginary parts of the coefficients in Eq. (19) are small in comparison with their corresponding real parts (see the discussion below). At the same time, we also set $L_D=L_{\mathrm{NL}}$, where $L_{\mathrm{NL}}=1/(|\beta_{22}|U_0^2)$ is the nonlinear length, which results in $U_0=\tau_0^{-1}[|K_2^-|/(2|\beta_{22}|)]^{1/2}$. We will also neglect the diffraction effect, which is valid for a larger beam radius of the probe pulse.

In general, Eq. (19) admits various coupled soliton solutions [31,33,34], including bright-bright, bright-dark, dark-bright, and dark-dark soliton pair solutions, as will be seen below.

(i) *Bright-bright solitons*. If the parameters d_{ln} $(l,n=1,2)$ fulfill the conditions $d_{11}d_{22}-d_{12}d_{21}>0$, $d_{22}d_{1\mathrm{D}}-d_{12}d_{2\mathrm{D}}>0$, and $d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}}>0$, or $d_{11}d_{22}-d_{12}d_{21}<0$, $d_{22}d_{1\mathrm{D}}-d_{12}d_{2\mathrm{D}}<0$, and $d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}}<0$, we have the bright-bright soliton pair solution,

$$u^+ = \sqrt{2}u_0\mathrm{sech}\,\sigma\exp[i(k_1\sigma+\Omega_1 s)], \tag{20a}$$

$$u^- = \sqrt{2}v_0\mathrm{sech}\,\sigma\exp[i(k_2\sigma+\Omega_2 s)], \tag{20b}$$

where $u_0=[(d_{22}d_{1\mathrm{D}}-d_{12}d_{2\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $v_0=[(d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $k_1=d_\delta/(2d_{1\mathrm{D}})$, $k_2=-d_\delta/(2d_{2\mathrm{D}})$, $\Omega_1=-d_\delta^2/(4d_{1\mathrm{D}})-d_{1\mathrm{D}}$, and $\Omega_2=-d_\delta^2/(4d_{2\mathrm{D}})-d_{2\mathrm{D}}$.

(ii) *Bright-dark solitons*. When d_{ln} satisfies $d_{11}d_{22}-d_{12}d_{21}>0$, $d_{12}d_{2\mathrm{D}}-d_{22}d_{1\mathrm{D}}>0$, and $d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}}>0$, or $d_{11}d_{22}-d_{12}d_{21}<0$, $d_{12}d_{2\mathrm{D}}-d_{22}d_{1\mathrm{D}}<0$, and $d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}}<0$, one obtains the bright-dark soliton pair,

$$u^+ = \sqrt{2}u_0\mathrm{sech}\,\sigma\exp[i(k_1\sigma+\Omega_1 s)], \tag{21a}$$

$$u^- = \sqrt{2}v_0\tanh\sigma\exp[i(k_2\sigma+\Omega_2 s)], \tag{21b}$$

with $u_0=[(d_{22}d_{1\mathrm{D}}-d_{12}d_{2\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $v_0=[(d_{21}d_{1\mathrm{D}}-d_{11}d_{2\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $k_1=d_\delta/(2d_{1\mathrm{D}})$, $k_2=-d_\delta/(2d_{2\mathrm{D}})$, $\Omega_1=-k_1d_\delta-d_{1\mathrm{D}}(1-k_1^2)-d_{12}v_0^2$, and $\Omega_2=k_2d_\delta+d_{2\mathrm{D}}(2+k_2^2)-d_{21}u_0^2$.

(iii) *Dark-bright solitons*. If the parameters d_{ln} fulfill the conditions $d_{11}d_{22}-d_{12}d_{21}>0$, $d_{12}d_{2\mathrm{D}}-d_{12}d_{2\mathrm{D}}>0$, and $d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}}>0$, or $d_{11}d_{22}-d_{12}d_{21}<0$, $d_{22}d_{1\mathrm{D}}-d_{12}d_{2\mathrm{D}}<0$, and $d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}}<0$, we have the dark-bright soliton pair solution,

$$u^+ = \sqrt{2}u_0\tanh\sigma\exp[i(k_1\sigma+\Omega_1 s)], \tag{22a}$$

$$u^- = \sqrt{2}v_0\mathrm{sech}\,\sigma\exp[i(k_2\sigma+\Omega_2 s)], \tag{22b}$$

where $u_0=[(d_{12}d_{2\mathrm{D}}-d_{22}d_{1\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $v_0=[(d_{11}d_{2\mathrm{D}}-d_{21}d_{1\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $k_1=d_\delta/(2d_{1\mathrm{D}})$, $k_2=-d_\delta/(2d_{2\mathrm{D}})$, $\Omega_1=-k_1d_\delta+(2+k_1^2)d_{1\mathrm{D}}-d_{12}v_0^2$, and $\Omega_2=k_2d_\delta-(1-k_2^2)d_{2\mathrm{D}}-d_{21}u_0^2$.

(iv) *Dark-dark solitons*. One can also get a dark-dark soliton pair if $d_{11}d_{22}-d_{12}d_{21}>0$, $d_{12}d_{2\mathrm{D}}-d_{22}d_{1\mathrm{D}}>0$, and $d_{21}d_{1\mathrm{D}}-d_{11}d_{2\mathrm{D}}>0$, or $d_{11}d_{22}-d_{12}d_{21}<0$, $d_{12}d_{2\mathrm{D}}-d_{22}d_{1\mathrm{D}}<0$, and $d_{21}d_{1\mathrm{D}}-d_{11}d_{2\mathrm{D}}<0$,

$$u^+ = \sqrt{2}u_0\tanh\sigma\exp[i(k_1\sigma+\Omega_1 s)], \tag{23a}$$

$$u^- = \sqrt{2}v_0\tanh\sigma\exp[i(k_2\sigma+\Omega_2 s)], \tag{23b}$$

with $u_0=[(d_{12}d_{2\mathrm{D}}-d_{22}d_{1\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $v_0=[(d_{21}d_{1\mathrm{D}}-d_{11}d_{2\mathrm{D}})/(d_{11}d_{22}-d_{12}d_{21})]^{1/2}$, $k_1=d_\delta/(2d_{1\mathrm{D}})$, $k_2=-d_\delta/(2d_{2\mathrm{D}})$, $\Omega_1=-k_1d_\delta+(2+k_1^2)d_{1\mathrm{D}}$, and $\Omega_2=k_2d_\delta+(2+k_2^2)d_{2\mathrm{D}}$.

As has been mentioned before and from the above results, we see that each probe pulse breaks into two solitons because of the two dispersion branches (i.e., K^+ and K^-). Correspondingly, one has two pairs of TAG matched solitons. In general, the dark soliton component of the soliton pairs given above can be replaced by a gray soliton with a different condition [34].

It should be pointed out that group velocity matching (i.e., $d_\delta=0$) is not a prerequisite of achieving stable soliton solutions. In practice, however, in order to form a soliton pair in a finite distance and time, one should choose parameters to make the length of significant group-velocity mismatch be larger than the dispersion and nonlinear lengths so that there is effective energy transfer between two wave components. Another key point to make soliton-pair propagation possible is that both absorption lengths of the K^+ and K^- branches, given, respectively, by $L_{\gamma1}(=1/\mathrm{Im}[K^+])$ and $L_{\gamma2}(=1/\mathrm{Im}[K^-])$, should be larger than the dispersion and nonlinear lengths.

We now present a practical numerical example to demonstrate the formation and propagation of soliton pairs, as described above. We consider an atomic alkali system where the decay rates are $\gamma_1=\gamma_2=5.0\times10^7\ \mathrm{s}^{-1}$ and $\gamma_3=1.0\times10^4\ \mathrm{s}^{-1}$. We choose the density of the medium, Rabi frequencies of the control fields, and detunings so that $\kappa_{12}=5.0\times10^{11}\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $\Omega_{c1}=\Omega_{c2}=2.0\times10^9\ \mathrm{s}^{-1}$, $\Delta_1=0.5\Delta_2=3.0\times10^9\ \mathrm{s}^{-1}$, and $\Delta_3=5.0\times10^8\ \mathrm{s}^{-1}$. The typical pulse

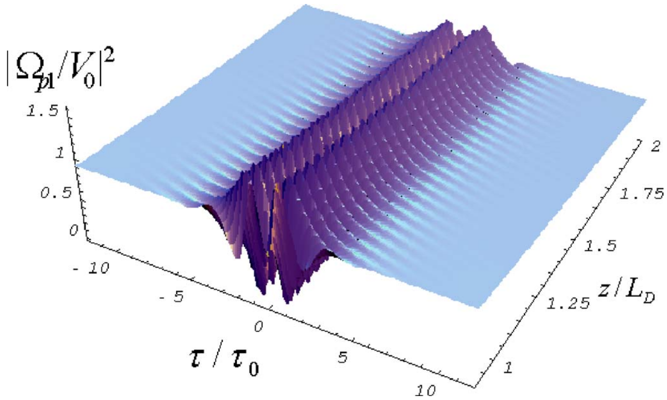

FIG. 2. (Color online) Evolution of the relative probe field intensity $|\Omega_{p1}/V_0|^2$ ($V_0=U_0u_0$) in the case of dark-bright soliton solution vs the dimensionless delay time τ/τ_0 and propagating distance z/L_D with the system parameters given in Sec. IV B and $R_\perp=0.05$ cm.

length will be about $\tau_0=1.0\times10^{-8}$ s^{-1}, and the excitation wavelengths are $\lambda_{p1}\approx\lambda_{p2}=c/\nu_p=0.8\times10^{-4}$ cm. With these parameters we obtain $K_2^+=(8.89+0.16i)\times10^{-17}$ cm^{-1} s^2, $K_2^-=(-8.45+0.27i)\times10^{-18}$cm^{-1} s^2, $\beta_{11}=(1.27+0.005i)\times10^{-17}$cm^{-1} s^2, $\beta_{12}=(8.63+0.37i)\times10^{-18}$ cm^{-1} s^2, $\beta_{21}=(-3.20+0.031i)\times10^{-17}$ cm^{-1} s^2, and $\beta_{22}=(-2.17+0.023i)\times10^{-17}$ cm^{-1} s^2. With these values we have $U_0=4.42\times10^7$ s^{-1}, $L_D=L_{\rm NL}=2.25$ cm, $L_{\gamma1}=10.1$ cm, $L_{\gamma2}=8.6$ cm, $d_{11}=0.59$, $d_{12}=0.40$, $d_{21}=-1.48$, $d_{22}=-1.0$, $d_{1\rm D}=10.52$, $d_{2\rm D}=-1.0$, and $d_\delta=9.36$. Using these parameters we obtain a dark-bright soliton-pair solution (22) with $u_0=71.13$ and $v_0=86.54$. We note that since $L_{\gamma n}\gg L_D$ ($n=1,2$) in this case, the damping effect can be safely neglected. When the light pulse propagates to a distance $z=2.25$ cm, the soliton pair (with spatial width 0.14 cm) forms, and their propagating velocity is given by $V_g=4.7\times10^{-4}$ c. The light intensity for each probe field at this depth of propagation can be calculated as follows. For Ω_{p1} we get

$$\begin{aligned}|\Omega_{p1}/V_0|^2 &= \exp(-2d_{01}s)\tanh^2\sigma+(v_0/u_0)^2\\ &\quad\times\exp(-2d_{02}s)\mathrm{sech}^2\sigma+2(v_0/u_0)\\ &\quad\times\exp[-(d_{01}+d_{02})s]\mathrm{sech}\,\sigma\tanh\sigma\cos\Phi,\end{aligned} \tag{24}$$

where $V_0=U_0u_0$, $d_{0n}=L_D/L_{\gamma n}$, $\Phi=(k_2-k_1)\sigma+(\Omega_2-\Omega_1+L_D/L_{ga2}-L_D/L_{ga1})s$. In Fig. 2 we have shown the space-time evolution of the dimensionless probe field intensity $|\Omega_{p1}/V_0|^2$ for the dark-bright soliton solution versus the dimensionless delay time $\sigma=\tau/\tau_0$ and propagating distance $s=z/L_D$ using parameters given above. As expected, there is an internal oscillation due to the interference between the bright and dark soliton components.

We note that other types of soliton pair solutions described above can be obtained by choosing different sets of parameters. For instance, by choosing $\gamma_1=\gamma_2=2.0\times10^7$ s^{-1}, $\gamma_3=9.0\times10^5$ s^{-1}, $\Delta_1=3.2\times10^9$ s^{-1}, $\Delta_2=2.4\times10^9$ s^{-1}, $\Delta_3=-1.0\times10^8$ s^{-1}, and $\tau_0=3.0\times10^{-9}$ s^{-1}, with all other parameters being the same as those given above, we can get a bright-dark soliton pair solution (21).

V. DISCUSSION AND SUMMARY

Notice that another type of optical soliton with very low propagating velocity (i.e., slow-light solitons) in three-state Λ systems has been suggested in recent studies [37–39]. Especially, a slow-light soliton with controllable speed was constructed first in Ref. [37]. Although the slow-light solitons proposed by these authors possess some similar characteristics of the ultraslow optical solitons, suggested in Refs. [22–25] and this work, there are many differences between them. The most important one is that to produce the slow-light solitons, both strong probe and control fields are required. However, to produce the ultraslow optical solitons, one needs only very weak probe fields.

In conclusion, we have investigated the simultaneous formation and propagation of coupled ultraslow optical soliton pairs in a cold, lifetime-broadened three-state double-Λ atomic system. By using the standard method of multiple scales, we have derived coupled nonlinear Schrödinger equations that describe the weak nonlinear evolution of two wave envelopes of probe fields. In these envelope equations, both SPM and CPM effects and nonadiabatic corrections of atomic response are included in a systematic way. Such effects are important for the formation of shape-preserving, localized optical pulses in the system. We have shown that multiple coupled optical soliton pairs can be established in a short propagation distance and under low-level deriving conditions. These TAG-matched optical soliton pairs move with remarkably slow propagating velocity and have the dynamic characteristics that are not admitted in the conventional optical fiber based soliton generation schemes. The coupled ultraslow optical pair formation techniques discussed here may be applied to other multiwavelength experiments where significant index modification is a key feature. Because of their robust nature and ultraslow propagating velocity, the optical soliton pairs predicted in this study may have potential applications in modern optical information processing optical telecommunication engineering.

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APPENDIX A: THE COEFFICIENTS IN EQS. (11a) and (11a)

The explicit expressions of the coefficients of the Eqs. (11a) and (11b) are given by

$$\alpha_\pm=\frac{c}{2}\frac{1}{F_\pm+G_\pm}\left(\frac{F_\pm}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right), \tag{A1a}$$

$$\beta_{11} = \frac{1}{2\widetilde{D}_+} U_+[|J_0^+|^2 + |J_1^+|^2], \tag{A1b}$$

$$\beta_{12} = \frac{1}{2\widetilde{D}_+}\{U_+(|J_0^-|^2 + |J_1^-|^2) + U_-[J_0^+(J_0^-)^* + J_1^+(J_1^-)^*]\}, \tag{A1c}$$

$$\beta_{21} = \frac{1}{2\widetilde{D}_-}\{V_+[(J_0^+)^* J_0^- + (J_1^+)^* J_1^-] + V_-(|J_0^+|^2 + |J_1^+|^2)\}, \tag{A1d}$$

$$\beta_{22} = \frac{1}{2\widetilde{D}_-} V_-[|J_0^-|^2 + |J_1^-|^2]. \tag{A1e}$$

with

$$F_\pm = \left(K_\pm - \frac{\omega}{c}\right)D + \kappa_{12}D_1, \tag{A2a}$$

$$G_\pm = (K^\pm - \omega/c)D + \kappa_{12}D_2, \tag{A2b}$$

$$J_0^\pm = \frac{1}{\kappa_{12}}\left(K_\pm - \frac{\omega}{c}\right)\left[1 + \frac{G_\pm}{\kappa_{12}\Omega_{c1}\Omega_{c2}^*}\right], \tag{A2c}$$

$$J_1^\pm = -\frac{1}{\Omega_{c1}}\left[1 + \frac{\omega + d_3}{\kappa_{12}}\left(K_\pm - \frac{\omega}{c}\right)\right], \tag{A2d}$$

$$\widetilde{D}_\pm(\omega) = D\left[1 + \frac{G_\pm}{F_\pm}\right]. \tag{A2e}$$

$U_\pm$ and $V_\pm$ are defined as

$$U_\pm = 2D\left(K_\pm - \frac{\omega}{c}\right) + [-\kappa_{12}D_1G_\pm + \kappa_{12}^2|\Omega_{c1}|^2|\Omega_{c2}|^2 + D(K_\pm - \omega/c)G_\pm]/F_+, \tag{A3a}$$

$$V_\pm = 2D\left(K_\pm - \frac{\omega}{c}\right) + [-\kappa_{12}D_1G_\pm + \kappa_{12}^2|\Omega_{c1}|^2|\Omega_{c2}|^2 + D(K_\pm - \omega/c)G_\pm]/F_-. \tag{A3b}$$

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