Generalized local-world models for weighted networks

Zaofeng Pan, Xiang Li,* and Xiaofang Wang
Complex Networks and Control Lab, Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China
(Received 30 November 2005; revised manuscript received 1 March 2006; published 10 May 2006)

Based on the weighted scale-free network model proposed by Barrat, Barthélemy, and Vespignani [Phys. Rev. Lett. **92**, 228701 (2004)] and enlightened by our local-world concept [Li and Chen, Physica A **328**, 274 (2003)], we propose two generalized local-world (GLW) models for weighted complex networks. Theoretical analysis and numerical simulations show that the GLW models generate weighted networks as a crossover between exponential and scale-free weighted networks, and exhibit an alteration from assortative networks to disassortative networks.

DOI: 10.1103/PhysRevE.73.056109 PACS number(s): 89.75.Fb, 89.75.Da, 89.75.Hc

I. INTRODUCTION

From food webs [1], the Internet [2], and traffic networks [7,8], networked structures appear in a wide array of complex systems. Although Boolean structures are quite informative, which have led to a number of important progresses in the understanding of complex systems, including small-world and scale-free phenomena [3,4], one may put a step forward to consider physical features more realistically. A weighted network, in which connections between nodes in the network display heterogeneity in the capacity and intensity, is one of the most straightforward generalizations in this direction. For example, in the scientific collaboration network (SCN) [5,6], nodes are identified as authors, and weights depend on the frequencies of their collaborations, i.e., the number of coauthored papers. In the world-wide airport network (WAN) [7,8], each given weight is the number of available seats on the direct flight between two connected cities.

Mathematically, a weighted network is characterized by a generalized adjacency matrix W, whose element w_{ij} denotes the weight of the edge between node i and node j. Here, we restrict our interest to undirected networks where weights are symmetric, and assume that $w_{ii}=0$. Naturally, as the generalization of degree k_i of node i, the strength of node i is defined as $s_i=\Sigma_{j\in\Gamma(i)}w_{ij}$, where $\Gamma(i)$ denotes the set of neighbors of node i. Recently, Barrat *et al.* [4] made a detailed analysis on the structure of real-life weighted networks, including the SCN and the WAN, and stated that both weighted networks not only exhibit power-law degree distributions, but also have the power-law form of the weight distribution $P(w)\sim w^{-\theta}$ and the strength distribution $P(s)\sim s^{-\alpha}$. Moreover, the strengths are highly correlated with the degrees, which display the scale-free property of $s\sim k^{\beta}$ with $\beta\geq 1$ [9,10]. In order to explain these prominent properties, Barrat, Barthélemy, and Vespignani (BBV) built a simple evolving network model to study the dynamical evolution of weighted networks based on two basic ingredients of the exponential growth and preferential attachment mechanism [11]. Fundamentally, the BBV model can be considered as a generalization of the Barabási-Albert (BA) unweighted scale-free network model [4], and successfully yields scale-free properties of degrees, weights, and strengths.

On the other hand, although many unweighted networks are indeed scale-free [12,13], there are several examples of complex networks, such as the small networks of interactions among plants and animals [14,15], that exhibit an exponential truncation of the power-law behavior for large degrees. These broad-scale networks [15] are more homogeneous than scale-free networks. Many explanations are provided for such phenomena, including the small size of networks [16,17], and the mechanisms of the addition of edges determined by aging or connection costs [18], forbiddance [19], information filtering [18,20], local and global information [21,26], and the recent "initial core" theory [15]. In particular, we presented a local-world (LW) unweighted evolving network model [21] after the study of the World Trade Web (WTW) [22]. Theoretical analysis and numerical simulations show that the LW model represents a crossover between exponential and power-law scalings, which has been applied to model the Internet [25]. Can a generalized local-world model be built for weighted networks? How much effect does a local-world model have on the degrees, weights, and strengths in the evolution of weighted networks? In this work, we attempt to answer these questions in order to understand how the local-world phenomenon affects the dynamical evolution of weighted networks.

The rest of this paper is organized as follows. In Sec. II, we introduce the original local-world model, and propose two generalized local-world (GLW) models for weighted networks. In Secs. III and IV, we analytically study two GLW models with numerical simulations, whose properties of cluster coefficients and degree correlations are further studied in Sec. V. Finally, in Sec. VI, we conclude the whole work.

II. THE GENERALIZED LOCAL-WORLD MODELS

A. The local-world model for unweighted networks

A local-world is a small community with a few nodes in a network. There are numerical examples of a local-world in

*Correspondence author. FAX: +86-21-34205322. Email address: xli@sjtu.edu.cn

various real-life complex networks, which include, for instance, the families in protein-protein interaction networks, the regional economy cooperative organizations in the world trading networks, and the domains in computer networks. We argued that each node in a network only has local connections, and only has local information on the entire network. Consequently, the preferential attachment mechanism does not work on the global network, which was adopted in the BA scale-free model, but does work in a local-world scale. For example, it was observed in the WTW that many countries accelerate their economy collaborations in various regional economy cooperative organizations, such as the EU, ASEAN, and NAFTA [22]. On the Internet, a new host prefers to join the network by connecting with hosts in the same domain. In the social network, a newcomer is willing to contact with his neighbors first.

The local-world model for unweighted networks starts from an initial configuration of m_0 nodes connected by a few edges, and evolves based on two dominating mechanisms: the determination of a local world, and the topological growth [21].

(i) ***Determination of a local-world.*** Randomly choose M nodes from the existing network, which constitute the local-world of the new coming node at every time step.

(ii) ***Topological growth.*** Add a new node with m edges to m previously existing nodes in its local-world determined in step (i), where the nodes are preferentially chosen with the probability $\Pi_{\text{local}}(n\to i)$,

$$\Pi_{\text{local}}(n \to i) = \Pi'(i \in \text{local-world})\frac{k_i}{\sum_{j^{\text{local}}} k_j},$$

where $\Pi'(i\in\text{local-world})=M/(t+m_0)$.

After t time steps, this procedure yields a network with $N=t+m_0$ nodes and mt edges.

B. Generalized local-world models for weighted networks

Inspired by the LW model, we first present a generalized local-world model (GLW-I) for weighted networks based on the BBV model, which starts from an initial configuration of m_0 nodes connected by a few edges with their assigned weights $\omega_0=1$. This model evolves with three mechanisms: the determination of a local world, the topological growth, and the weight's dynamics.

(I) ***Determination of a local-world.*** Randomly choose M nodes from the existing network, which constitute the local-world of the new coming node at every time step.

(II) ***Topological growth.*** Add a new node with m edges to m nodes in its local-world, according to the probability

$$\Pi_{\text{local}}(n \to i) = \Pi'(i \in \text{local-world})\frac{s_i}{\sum_{j^{\text{local}}} s_j}, \tag{1}$$

where $\Pi'(i\in\text{local-world})=M/(t+m_0)$.

(III) ***Weight's dynamics-I.*** The weight of each new edge $(n\to i)$ is initially set to a given value $\omega_0=1$. The creation of this edge will introduce variations of the traffic across the network. For the sake of simplicity, we assume that the addition of a new edge on node i only triggers local rearrangements of weights on its neighbors $j\in\Gamma(i)$, according to the rule

$$w_{ij} \to w_{ij} + \Delta w_{ij}. \tag{2}$$

Here, Δw_{ij} depends on the local dynamics, which can be a function of different parameters such as the weight w_{ij}, the connectivity, or the strength of node i. In the following, we assume that the addition of a new edge induces an increase of δ (δ=const) of the total outgoing traffic, where the perturbation is proportionally distributed among the edges according to their weights,

$$\Delta w_{ij} = \delta_i \frac{w_{ij}}{s_i}. \tag{3}$$

Therefore, this rule yields a strength increase of $\delta+\omega_0$ for node i, implying that $s_i\to s_i+\delta+\omega_0$.

After the weights have been updated, the growing process is repeated by introducing a new node, and returns to step (I) until the desired size of the network is reached.

Recently, Hu *et al.* [23] introduced a traffic-driven (TR) model for technological networks. They argued that the traffic as well as its dynamics plays a key role in understanding the evolution of technological networks, which drives us to propose another new generalized local-world model (GLW-II) for weighted networks. The only difference between the GLW-II model and the GLW-I model lies in the weight's dynamics, which is stated for the GLW-II model as follows:

(III′) ***Weight's dynamics-II.*** From the start of network growing, the traffic of all nodes in the local-world chosen in (I) will constantly increase with the probability proportional to the node strength $s_i(\sum_{j^{\text{local}}} s_j)^{-1}$ at every time step. We assume that the growing speed of the total traffic in the network is a discrete constant W. Therefore, at every time step, the newly created traffic of node i is

$$\Delta W_i = W\frac{s_i}{\sum_{j^{\text{local}}} s_j} \tag{4}$$

which will be preferentially arranged to those neighbors having larger edge weights with probability w_{ij}/s_i. After the weights have been updated, the growing process is repeated by adding a new node, and returns to step (I) until the desired size of the network is reached.

III. ANALYTICAL RESULTS OF THE GLW-I MODEL

At every time step t, $m\le M\le m_0+t$. There are two limiting cases in the GLW-I model.

A. Case A: $M=m$

In this case, the new node connects to every node in its local-world, which means the preferential attachment selection is not effective during the network evolution. In this case, the distributions of degrees, weights, and strengths decay exponentially (Fig. 1).

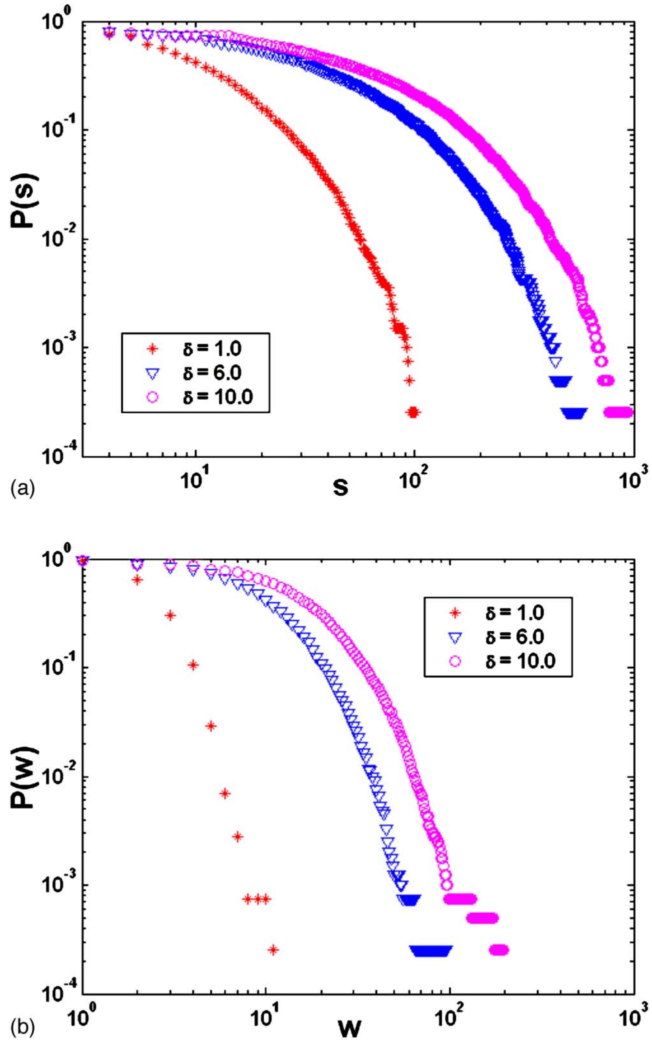

FIG. 1. (Color online) (a) The cumulative strength distribution $P(s)$ and (b) the cumulative weight distribution $P(w)$ of the GLW-I model in Case A. The distributions decay exponentially. All networks are generated with $m=3$, the size of local-world $M=3$, and the networks size $N=4000$. Each curve in the figure is the average result of five groups of networks.

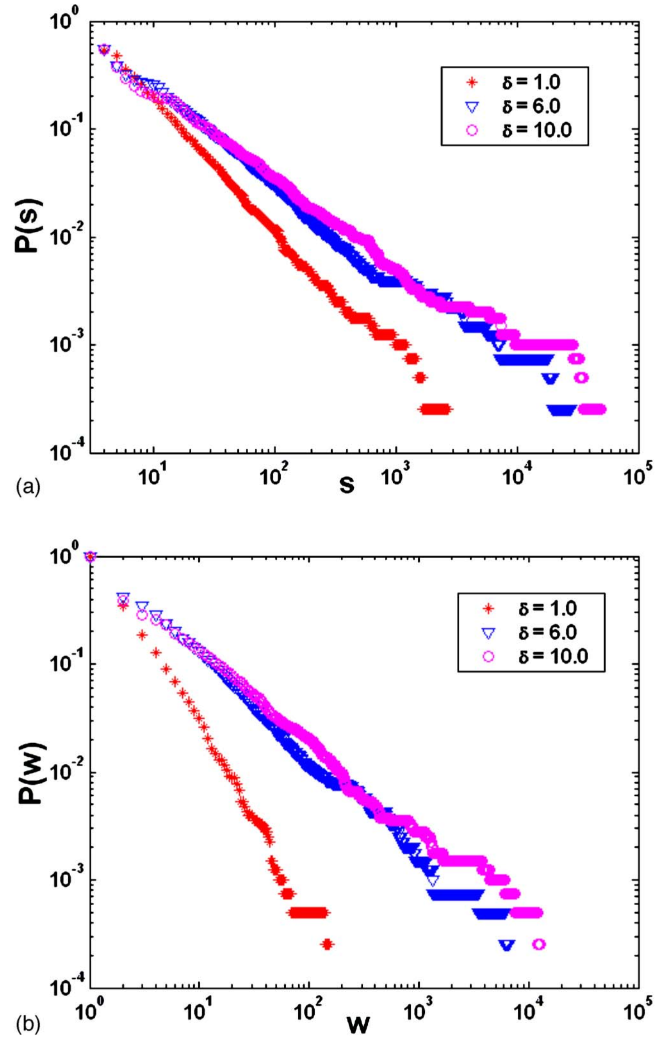

FIG. 2. (Color online) (a) The cumulative strength distribution $P(s)$ and (b) the cumulative weight distribution $P(w)$ of the GLW-I model in Case B. The distributions are of power-law form. All networks are generated with $m=3$, the size of local-world $M=t+m_0$, and the network size $N=4000$. Each curve in the figure is the average result of five groups of networks.

B. Case B: $M=t+m_0$

In this case, the local-world is the same as the whole network, which keeps growing with the time evolution. Hence, the local-world model in this case is exactly the same as the BBV model (Fig. 2).

When $M \approx m$, the distributions of degrees, weights, and strengths are very close to those of Case A, and decay exponentially. When $M \approx m_0+t$, the distributions of degrees, weights, and strengths are similar to those of Case B, following power-law distributions. Therefore, as M increases from m to m_0+t, the GLW-I model represents a crossover between the exponential and power-law scalings, as illustrated in Fig. 3.

Assume that the addition of m edges at every time step t is uncorrelated. Using the mean-field method [11], we can obtain an analytic result for the general case of $m \ll M < t+m_0$. Since the random selection of M nodes as the local-world at time step t in the network having $t+m_0$ nodes, we can treat k, w, s and the time t as continuous variables using continuous approximations. Therefore, the evolution of weight w_{ij} is

$$\begin{aligned}\frac{dw_{ij}(t)}{dt} &= \Pi_{\text{local}}(n \to i) m\delta \frac{w_{ij}}{s_i(t)} + \Pi_{\text{local}}(n \to j) m\delta \frac{w_{ij}}{s_j(t)} \\ &= \frac{M}{t+m_0}\left(m \frac{s_i(t)}{\sum_{\text{local}} s_l(t)} \delta \frac{w_{ij}}{s_i(t)} \right. \\ &\quad \left. + m \frac{s_j(t)}{\sum_{\text{local}} s_l(t)} \delta \frac{w_{ij}}{s_j(t)} \right) \\ &= \frac{2Mm\delta}{t+m_0} \frac{w_{ij}}{\sum_{\text{local}} s_l(t)}.\end{aligned}$$

Considering

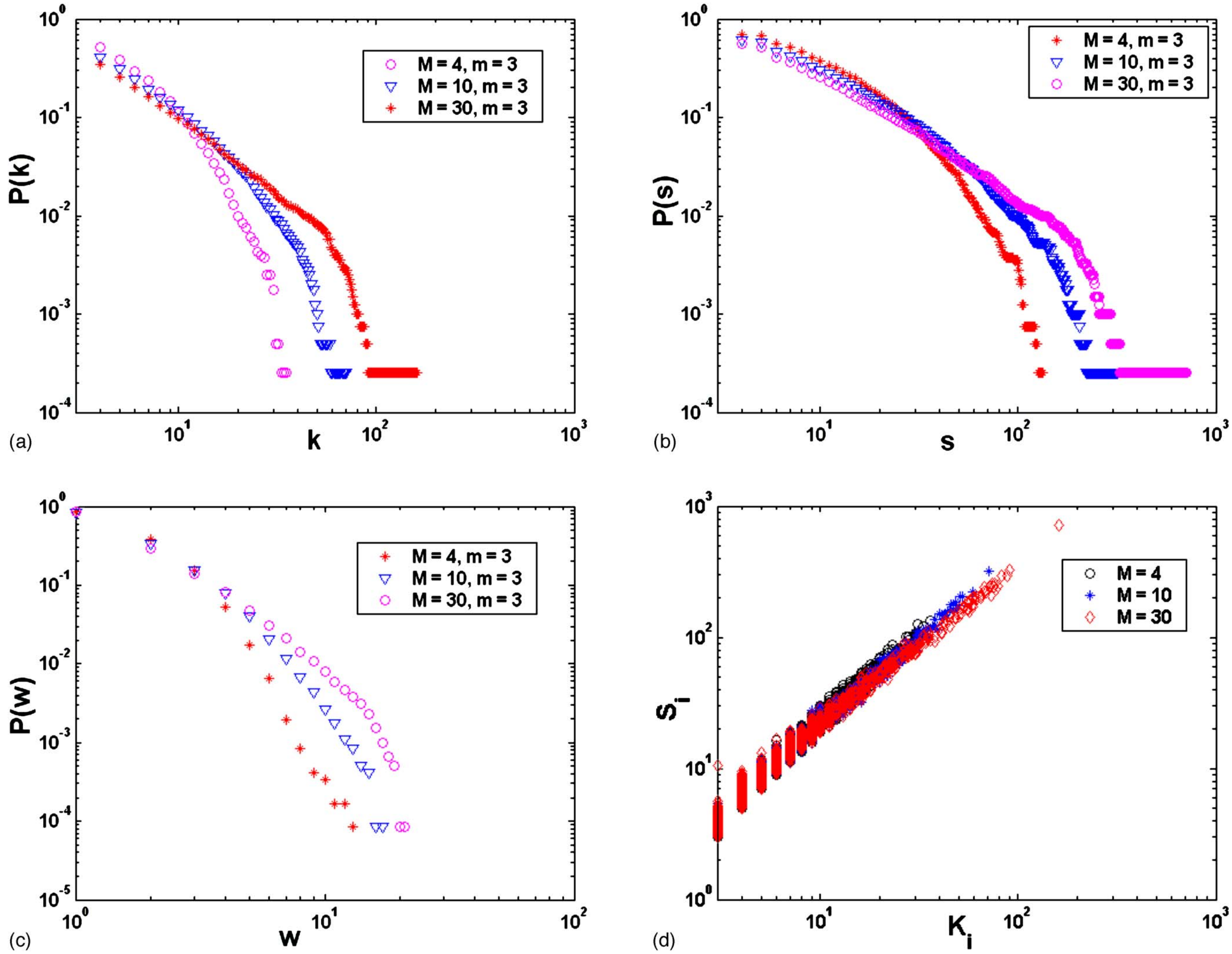

FIG. 3. (Color online) Properties of the GLW-I model. (a) The cumulative degree distribution $P(k)$, (b) the cumulative strength distribution $P(s)$, (c) the cumulative weight distribution $P(w)$, and (d) the strength s_i of node i with degree k_i. Networks are generated with $M=4$, 10, and 30, respectively, and $m=3$, $\delta=6$, and the network size $N=4000$. Each curve in the figure is the average result of five groups of networks.

$$\langle s_l \rangle = \sum\nolimits_l s_l(t)/(t+m_0) \approx 2m(1+\delta)t/(t+m_0), \tag{5}$$

$$\sum_{1} {}_{\text{local}}\, s_l(t) = \langle s_l \rangle M, \tag{6}$$

we rewrite the evolution equation as

$$\frac{dw_{ij}}{dt} = \delta \frac{w_{ij}}{(1+\delta)t} = \theta \cdot \frac{w_{ij}}{t}, \tag{7}$$

where $\theta = \delta/(1+\delta)$. The edge $(i \rightarrow j)$ is created at $t_{ij} = \max(i,j)$ with the initial condition $w_{ij}(t_{ij}) = 1$. Therefore, we have

$$w_{ij}(t) = (t/t_{ij})^{\theta}. \tag{8}$$

Furthermore, the evolution equations for s_i and k_i are

$$\begin{aligned} \frac{ds_i}{dt} &= \sum_j \frac{dw_{ij}}{dt} + m\Pi_{\text{local}}(n \rightarrow i) \\ &= \delta \frac{s_i(t)}{(1+\delta)t} + \frac{M}{t+m_0} m \frac{s_i(t)}{\sum_{\text{local}} s_l(t)} \\ &= \left[\frac{\delta}{1+\delta} + \frac{1}{2(1+\delta)} \right] \frac{s_i(t)}{t} \\ &= \frac{2\delta+1}{2\delta+2} \frac{s_i(t)}{t} \\ &= \lambda \frac{s_i(t)}{t}, \end{aligned} \tag{9}$$

where $\lambda = (2\delta+1)/(2\delta+2)$, and

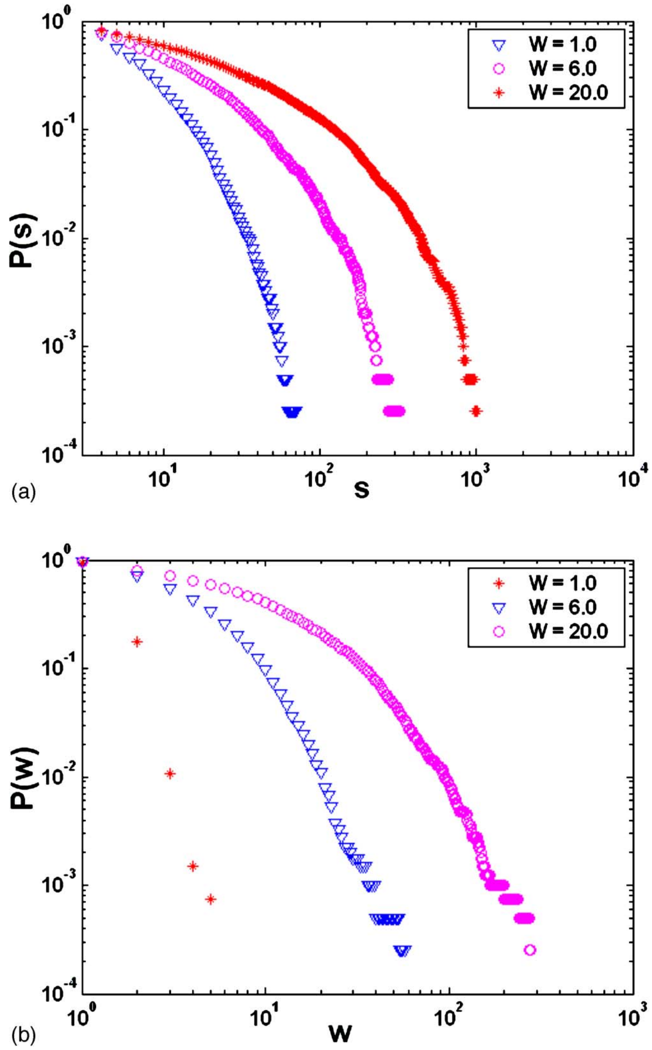

FIG. 4. (Color online) (a) The cumulative strength distribution $P(s)$ and (b) the cumulative weight distribution $P(w)$ of the GLW-II model in Case A. The distributions decay exponentially. All networks are generated with $m=3$, the size of local-world $M=3$, and the networks size $N=4000$. Each curve in the figure is the average result of five groups of networks.

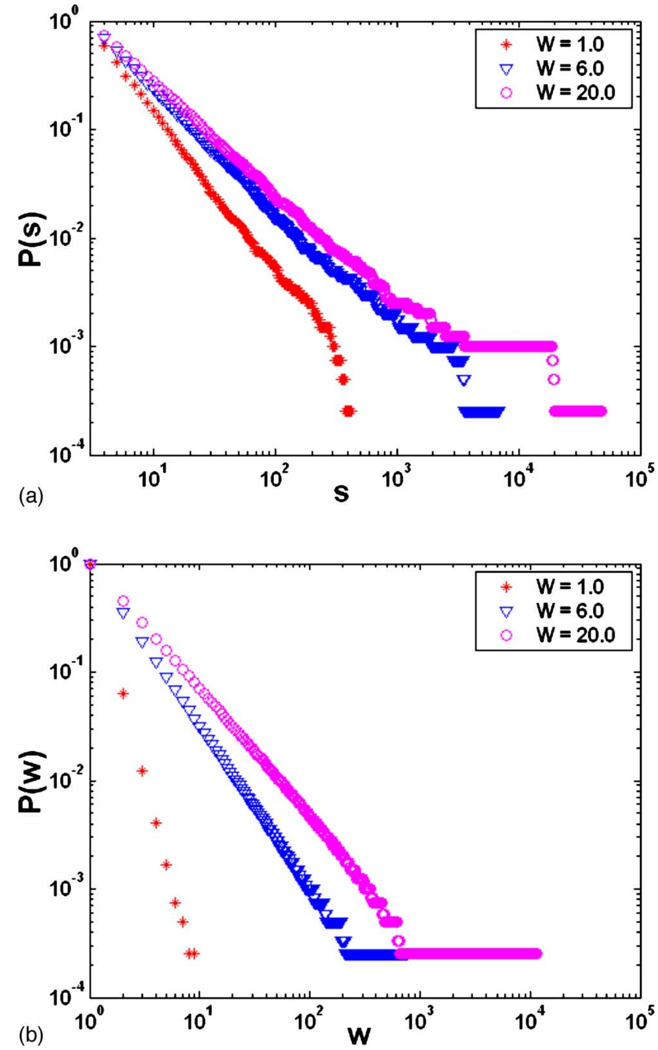

FIG. 5. (Color online) (a) The cumulative strength distribution $P(s)$ and (b) the cumulative weight distribution $P(w)$ of the GLW-II model in Case B. The distributions are of power-law form. All networks are generated with $m=3$, the size of local-world $M=t+m_0$, and the networks size $N=4000$. Each curve in the figure is the average result of five groups of networks.

$$\frac{dk_i}{dt} = m\Pi_{\mathrm{local}}(n \to i) = \frac{1}{2\delta+2}\frac{s_i(t)}{t}. \tag{10}$$

Considering that $k_i(t=i)=s_i(t=i)=m$, we finally obtain

$$s_i(t) = m(t/i)^{\lambda}, \tag{11}$$

$$k_i(t) = \frac{s_i(t) + 2m\delta}{2\delta+1}. \tag{12}$$

Equation (12) shows a linear relationship between the degrees and the strengths, whose scale-free distributions share the same power-law exponent

$$\gamma_I = 1 + \frac{1}{\lambda} = \frac{4\delta+3}{2\delta+1}. \tag{13}$$

That means, when $m \ll M < t+m_0$, the distributions of degrees, strengths, and weights follow the same power law as that of the BBV model, as portrayed in Fig. 3.

When $M=m$, i.e., without the mechanism of preferential attachment, the degrees, strengths, and weights of the GLW-I model are proportional, and the model generates a homogeneous weighted network with $k_i \sim \langle k_i \rangle$, $s_i \sim \langle s_i \rangle$, and $w_{ij} \sim \langle w_{ij} \rangle$, which becomes more heterogeneous as M increases from m. Thus the GLW-I model exhibits a crossover of weighted networks between exponential and power-law scalings.

IV. ANALYTICAL RESULTS OF THE GLW-II MODEL

Similarly, there are also two limiting cases in the GLW-II model.

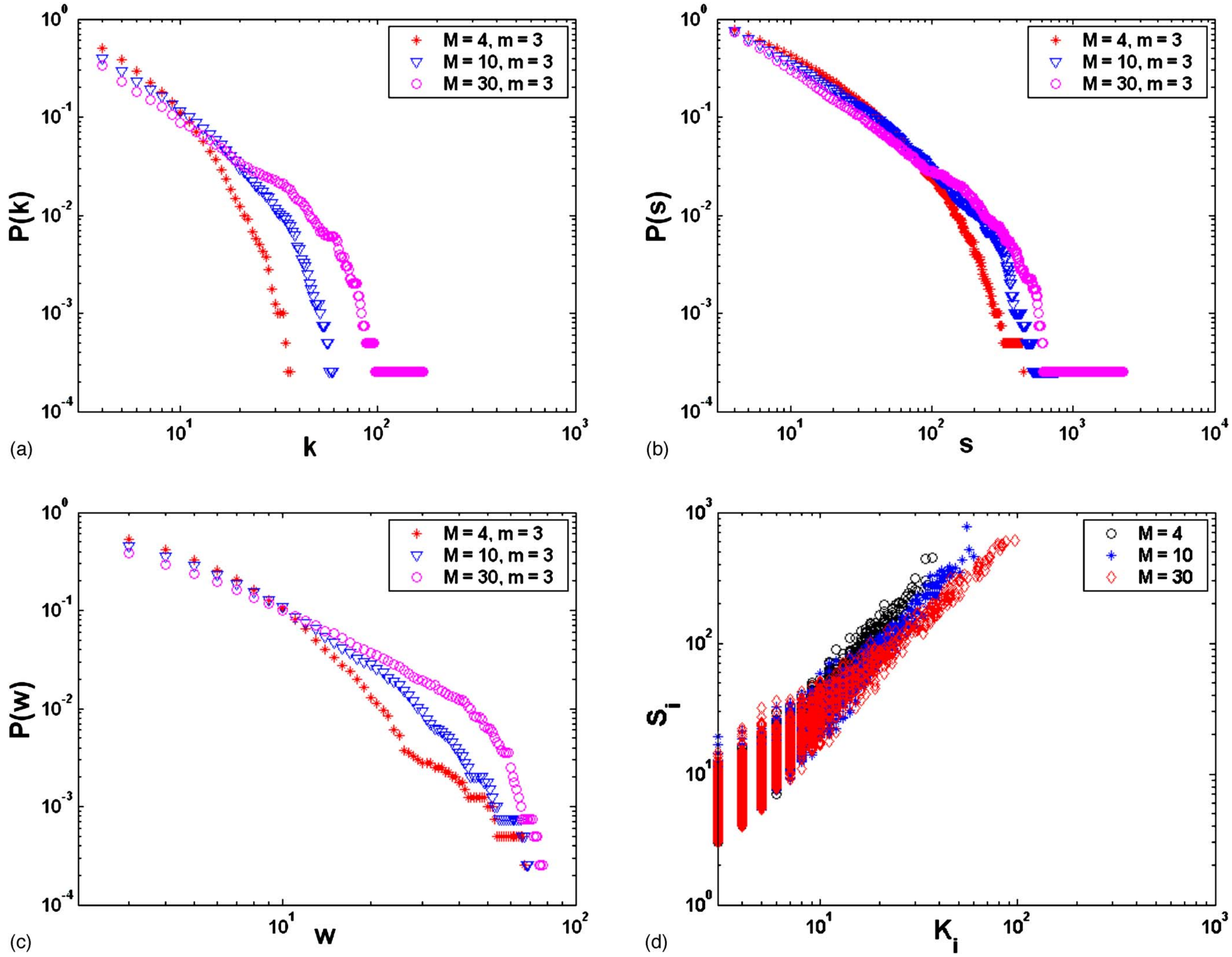

FIG. 6. (Color online) Properties of the GLW-II model. (a) The cumulative degree distribution $P(k)$, (b) the cumulative strength distribution $P(s)$, (c) the cumulative weight distribution $P(w)$, and (d) the strength s_i of node i with degree k_i. Networks are generated with $m=3$, $W=6$, and the networks size $N=4000$. Each curve in the figure is the average result of five groups of networks.

A. Case A: $M=m$

In this case, the distributions of degrees, weights, and strengths decay exponentially, as illustrated in Fig. 4.

B. Case B: $M=t+m_0$

In this case, the local-world is the same as the whole network, and the local-world model is exactly the same as the traffic-driven model, as shown in Fig. 5.

As in the case of the GLW-I model, when $M \approx m$, the distributions of degrees, weights, and strengths decay exponentially. When $M \approx m_0+t$, the distributions of degrees, weights, and strengths follow power-law distributions. Therefore, as M increases from m to m_0+t, the GLW-II model also represents a crossover between exponential and power-law distributions, as illustrated in Fig. 6.

We assume that the addition of m edges at every time step is uncorrelated. Similar to that of the GLW-I model, in the general case of $m \leqslant M < t+m_0$, we treat k, w, s, and the time t as continuous variables. Therefore, the weight w_{ij} evolves as

$$\frac{dw_{ij}(t)}{dt} = \Pi'(i \in \text{local-world})\Delta W_i \frac{w_{ij}}{s_i(t)} + \Pi'(j \in \text{local-world})\Delta W_j \frac{w_{ij}}{s_j(t)}$$

$$= \frac{M}{t+m_0} W\left(\frac{s_i(t)}{\sum_{\text{local}} s_l(t)} \frac{w_{ij}}{s_i(t)} + \frac{s_j(t)}{\sum_{\text{local}} s_l(t)} \frac{w_{ij}}{s_j(t)} \right)$$

$$= \frac{2MW}{t+m_0} \frac{w_{ij}}{\sum_{\text{local}} s_l(t)}.$$

Since

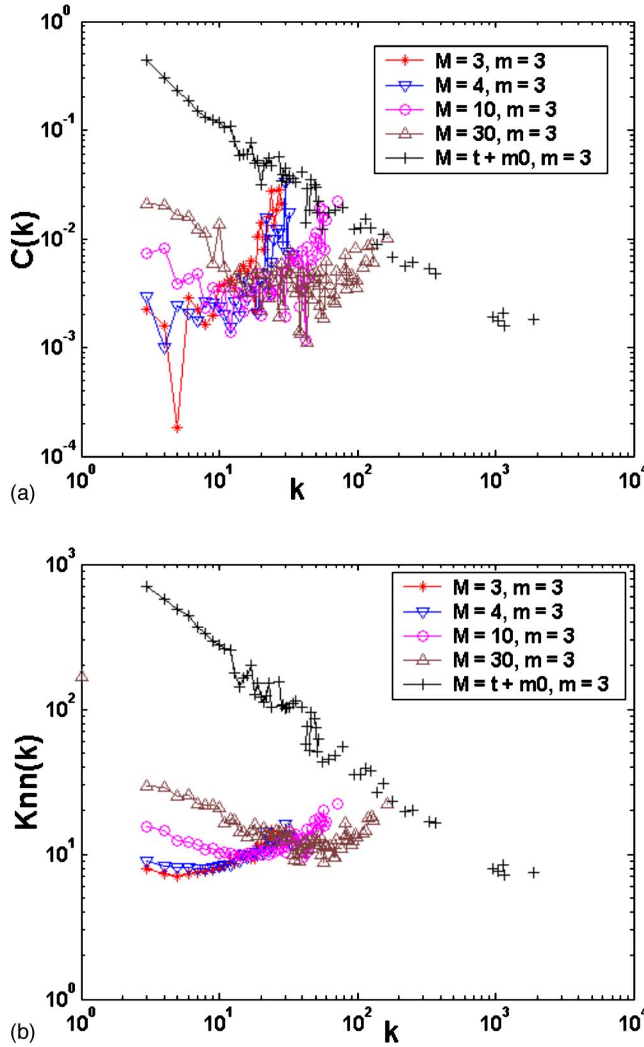

FIG. 7. (Color online) (a) Average clustering coefficient $C(k)$ and (b) average nearest-neighbor degree $k_{\mathrm{nn}}(k)$ of the GLW-I model. Networks are generated with $M=3,4,10,30,t+m_0$, respectively, $m=3$, $\delta=6$, and $N=4000$. Each curve in the figure is the average result of five groups of networks.

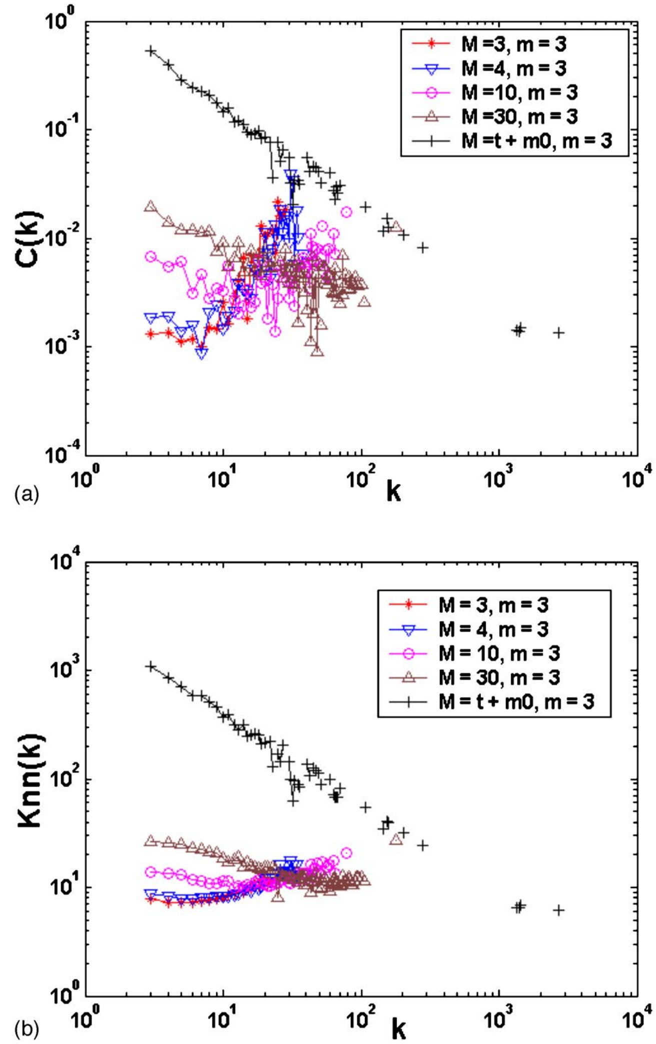

FIG. 8. (Color online) (a) Average clustering coefficient $C(k)$ and (b) average nearest-neighbor degree $k_{\mathrm{nn}}(k)$ of the GLW-II model. Networks are generated with $M=3,4,10,30,t+m_0$, respectively, $m=3$, $W=6$, and $N=4000$. Each curve in the figure is the average result of five groups of networks.

$$\sum\nolimits_{l^{\mathrm{local}}} s_l(t) \approx 2(W+m)Mt/(t+m_0) \tag{14}$$

we rewrite the evolution equation as

$$\frac{dw_{ij}}{dt} = \frac{W}{W+m}\frac{w_{ij}}{t} = \theta \cdot \frac{w_{ij}}{t}, \tag{15}$$

where $\theta = W/(m+W)$. The edge $(i \rightarrow j)$ is created at $t_{ij} = \max(i,j)$ with the initial condition $w_{ij}(t_{ij}) = 1$, which yields

$$w_{ij}(t) = (t/t_{ij})^{\theta}. \tag{16}$$

Therefore, the evolution equations for s_i and k_i are

$$\begin{aligned}\frac{ds_i}{dt} &= \sum_j \frac{dw_{ij}}{dt} + m\Pi_{\mathrm{local}}(n \rightarrow i) \\ &= \frac{2MW}{t+m_0}\frac{\sum_j w_{ij}}{\sum_{\mathrm{local}} s_l(t)} + \frac{M}{t+m_0} m \frac{s_i(t)}{\sum_{\mathrm{local}} s_l(t)} \\ &= \frac{(2W+m)M}{t+m_0}\frac{s_i(t)}{\sum_{\mathrm{local}} s_l(t)} = \frac{2W+m}{2W+2m}\frac{s_i(t)}{t} \\ &= \lambda \frac{s_i(t)}{t},\end{aligned} \tag{17}$$

where $\lambda = (2W+m)/(2W+2m)$, and

$$\frac{dk_i}{dt} = m\Pi_{\mathrm{local}}(n \rightarrow i) = \frac{m}{2(W+m)}\frac{s_i(t)}{t}. \tag{18}$$

Considering that $k_i(t=i) = s_i(t=i) = m$, we finally have

$$s_i(t) = m(t/i)^{\lambda}, \tag{19}$$

$$k_i(t) = m\frac{2W + s_i}{2W + m}. \tag{20}$$

Equation (20) shows a linear relationship between the degrees and the strengths, whose scale-invariant distributions share the same power-law exponent

$$\gamma_{II} = 1 + \frac{1}{\lambda} = 2 + \frac{m}{2W + m}. \tag{21}$$

Therefore, when $m \ll M < t+m_0$, the distributions of degrees, strengths, and weights follow the same power law as that of the traffic-driven model.

V. CLUSTERING AND CORRELATIONS

We now further investigate the topological properties of the GLW models in terms of clustering and degree correlations. The clustering coefficient c_i of node i is defined as the fraction of the neighbors of node i, which are also neighbors of each other, and the average clustering coefficient $C(k)$ with degree k is defined as

$$C(k) = \frac{1}{NP(k)} \sum_{i/k_i=k} c_i. \tag{22}$$

For some real-life networks, $C(k) \sim k^{-\alpha}$, which is associated with the hierarchy of network structure. Here, α is the hierarchical exponent [24].

The correlation between degrees of neighboring nodes is another important concept. Since it is very difficult to calculate the conditional distribution $P(k'|k)$, which represents the probability of a given node with degree k connected to another node of degree k', a more convenient measurement is to calculate the average nearest-neighbor degree (ANND) of a given node with degree k as follows [10]:

$$k_{\mathrm{nn}}(k) = \sum_{k'} k' P(k'|k) = \frac{1}{NP(k)} \sum_{i/k_i=k} k_{\mathrm{nn}}^i. \tag{23}$$

A network is said to be assortative if $k_{\mathrm{nn}}(k)$ increases with k, which indicates that nodes having large degrees are preferentially connected with other nodes also having large degrees; on the other hand, a network is said to be disassortative if $k_{\mathrm{nn}}(k)$ decreases with k, indicating those nodes having large degrees are preferentially connected with other nodes having small degrees.

We fix $m=3$ and $N=4000$, and investigate the clustering and correlation properties of the GLW models with different local-world scales. We first focus on the GLW-I model. As shown in Fig. 7(a), for a small M, $C(k)$ is an increasing power-law function of k, which means low-degree nodes have small cluster coefficients and high-degree nodes have large cluster coefficients, i.e., high-degree nodes are close to each other. As M increases, cluster coefficients of low-degree nodes increase and cluster coefficients of high-degree nodes decrease. When $M=t+m_0$, $C(k)$ displays a power-law decay. Therefore, the GLW-I model exhibits a clustering transition depending on the parameter M, the scale of a local-world.

Analogous properties are observed in the spectrum of degree correlations. For a small M, $k_{\mathrm{nn}}(k)$ is an increasing function of k, indicating that high-degree nodes are preferentially connected with each other, i.e., the network is asssortative. As M increases, the ANNDs of small-degree nodes increase, and the curve of $k_{\mathrm{nn}}(k)$ becomes flat. When $M=t+m_0$, the disassortativeness finally emerges in the power-law $k_{\mathrm{nn}}=k^{-a}$, as shown in Fig. 7(b). Therefore, when $m<M<m_0+t$, the GLW-I model exhibits a transition of the degree correlation between the asssortativeness and disassortativeness. Similar transitions of $C(k)$ and $K_{\mathrm{nn}}(k)$ also exist in the GLW-II model, as shown in Fig. 8.

VI. CONCLUSIONS

In this work, we have proposed two generalized local-world (GLW) models for weighted networks based on a combination of two weighted scale-free models and the local-world concept. The GLW models exhibit a crossover between exponential and scale-free weighted networks, and an alteration from assortativeness to disassortativeness. The practical applications of the GLW models to real complex networks should be explored in the near future.

ACKNOWLEDGMENTS

This work was partly supported by the National Natural Science Fund for Distinguished Young Scholars of China under Grant No. 60225013, and the National Natural Science Foundation of China under Grants No. 90412004, No. 60504019, and No. 70431002.

[1] B. Drossel and A. J. McKane, Modeling Food Webs, in *Handbook of Graphs and Networks: From the Genome to the Internet*, edited by S. Bornholdt and H. G. Schuster (Wiley-VCH, Berlin, 2003).
[2] R. Pastor-Satorras and A. Vespignani, *Evolution and Structure of the Internet: A Statistical Physics Approach* (Cambridge University Press, Cambridge, England, 2004).
[3] D. J. Watts and S. H. Strogatz, Nature (London) **393**, 440 (1998).
[4] A. L. Barabási and R. Albert, Science **286**, 509 (1999).
[5] M. E. J. Newman, Phys. Rev. E **64**, 016132 (2001).
[6] A. L. Barabási, H. Jeong, Z. Néda. E. Ravasz, A. Schubert, and T. Vicsek, Physica A **311**, 590 (2002).
[7] R. Guimera, S. Mossa, A. Turtschi, and L. A. N. Amearal, Proc. Natl. Acad. Sci. U.S.A. **102**, 7794 (2005).
[8] A. Barrat, M. Barthélemy, R. Pastor-Satorras, and A. Vespignani, Proc. Natl. Acad. Sci. U.S.A. **101**, 3747 (2004).
[9] K. I. Goh, B. Kahng, and D. Kim, Phys. Rev. E **72**, 017103

(2005).
[10] R. Pastor-Satorras, A. Vázquez, and A. Vespignani, Phys. Rev. Lett. **87**, 258701 (2001).
[11] A. Barrat, M. Barthélemy, and A. Vespignani, Phys. Rev. Lett. **92**, 228701 (2004).
[12] H. Jeong, B. Tombar, R. Albert, Z. N. Oltvai, and A. L. Barabási, Nature (London) **407**, 651 (2000).
[13] F. L. Liljeros, C. R. Edling, N. Amaral, H. E. Stanley, and Y. Aberg, Nature (London) **411**, 907 (2001).
[14] P. Jordano, J. Bascompte, and J. M. Olesen, Ecol. Lett. **6**, 69 (2003).
[15] P. R. Guimmarães, Jr., M. A. M. de Aguiar, J. Bascompte, P. Jordano, and S. Furtado dos Reis, Phys. Rev. E **71**, 037101 (2005).
[16] S. N. Dorogovtsev, J. F. F. Mendes, and A. N. Samukhin, Phys. Rev. E **63**, 062101 (2001).
[17] T. H. Keitt and H. E. Stanley, Nature (London) **393**, 257 (1998).
[18] L. A. N. Amaral, A. Scala, M. Barthelemy, and H. E. Stanley, Proc. Natl. Acad. Sci. U.S.A. **97**, 11149 (2000).
[19] P. Jordano, J. Bascompte, and J. M. Olesen, Ecol. Lett. **6**, 69 (2003).
[20] S. Mossa, M. Barthelemy, H. E. Stanley, and L. A. N. Amaral, Phys. Rev. Lett. **88**, 138701 (2002).
[21] X. Li and G. Chen, Physica A **328**, 274 (2003).
[22] X. Li, Y. Y. Jin, and G. Chen, Physica A **328**, 287 (2003).
[23] B. Hu, G. Yan, W.-X. Wang, B.-H. Wang, and W. Chen, e-print nlin.AO/0505041.
[24] E. Ravasz and A.-L. Barabási, Phys. Rev. E **67**, 026112 (2003).
[25] G. Chen, Z. P. Fan, and X. Li, Modeling the complex Internet Topology, in *Complex Dynamics in Communication Networks*, edited by G. Vattay and L. Kocarev (Springer-Verlag, Berlin, 2005).
[26] J. Gomez-Gardense and Y. Moreno, Phys. Rev. E **69**, 037103 (2004).