PHYSICAL REVIEW E **73**, 045601(R) (2006)

Fluctuations of local density of states and C_0 speckle correlations are equal

B. A. van Tiggelen* and S. E. Skipetrov†

Laboratoire de Physique et Modélisation des Milieux Condensés/CNRS, Maison des Magistères/Université Joseph Fourier, Boîte Postale 166, F-38042 Grenoble, Cedex 9, France

(Received 26 August 2005; published 10 April 2006)

We establish a conceptual relation between the fluctuations of the local density of states (LDOS) and the intensity correlations in speckle patterns resulting from the multiple scattering of waves in random media. We show that among the known types of speckle correlations (C_1, C_2, C_3, and C_0) only C_0 contributes to LDOS fluctuations in the infinite medium. We propose to exploit the equivalence of LDOS fluctuations and the C_0 intensity correlation as a "selection rule" for scattering processes contributing to C_0.

DOI: 10.1103/PhysRevE.73.045601 PACS number(s): 42.25.Dd

The local density of states (LDOS) of waves $\rho(\mathbf{r},\omega)$ is an important concept that regularly turns up in discussions of waves in the interaction with media. The number $\rho(\mathbf{r},\omega)dV\,d\omega$ represents the local weight of all eigenfunctions in the frequency interval $d\omega$ around frequency ω inside a small volume dV around position $\mathbf{r}$. In homogeneous media it is independent of $\mathbf{r}$ and just equal to the "density of states per unit volume" found in all textbooks. Near boundaries the LDOS exhibits Friedel-type oscillations on the scale of the wavelength [1]. In band-gap materials the LDOS was shown to govern the spontaneous emission of an atom at position $\mathbf{r}$ [2,3]. In random media, where wave propagation is diffuse, the equipartition principle attributes an *average* local energy density of radiation that is directly proportional to the ensemble-averaged LDOS. This apparently simple principle can have surprising consequences, for instance, when waves with different velocities participate in the diffusion process, as is the case for seismic waves [4]. For disordered band-gap materials [5] the equipartition principle is surprising in the sense that the multiple scattering process, with a typical length scale equal to the mean free path ℓ, that is, in general, much larger than the wavelength or the lattice constant, distributes energy with subwavelength structure. From a fundamental point of view, the LDOS is also the crucial quantity in the recent studies on "passive imaging" [6]. Its basic principle is that for a homogeneous distribution of sources—such as noise—the field correlation function (with time and space) is essentially proportional to the (Fourier transform of) LDOS, and thus sensitive to local structure, random or not.

In random media the LDOS is a random quantity. Its statistical distribution has been studied previously within the framework of the nonlinear sigma model [7,8], random matrix theory [9], and the optimal fluctuation method [10]. The purpose of this Rapid Communication is to establish a relation between the fluctuations of the LDOS—within the ensemble of random realizations—and the intensity correlations in speckle patterns. Several contributions to intensity correlations have been identified. The "standard," Gaussian correlation C_1 is the best known [11], but non-Gaussian correlations C_2 and C_3 have been predicted [12] and observed [13,14], mostly in the transmitted flux. Recently C_0 has been added [15,16]. The C_0 correlation is caused by scatterers close to either the receiver or the source and is, surprisingly, of *infinite* spatial range. Contrary to the other correlations, C_0 is nonuniversal and highly dependent on details of the scatterers, such as their phase function. The total transmission coefficient is known to be dominated by C_2, and the conductance by C_3. Unfortunately, the basic observable variable whose fluctuations are dominated by C_0 has never been identified. This is perhaps why only one observation has been reported so far, in the polarization correlation of microwaves [14].

The fluctuations of the LDOS can, in principle, be found from the average (Bethe-Salpeter) two-particle Green's function, but the diffusion approximation that is usually employed for this object [17] is not valid on length scales of the order of the wavelength, which appear to give an important contribution. Mirlin [8] noticed that in three dimensions (3D) the result is dominated by nearby scattering and is of the order $1/k\ell$, where k is the wave number and $\ell \gg 1/k$ is the mean free path. An exact calculation in the infinite medium with Gaussian white-noise disorder gives

$$\frac{\mathrm{Var}[\rho(\mathbf{r})]}{\langle\rho(\mathbf{r})\rangle^2} = \left(\frac{4\pi}{k}\right)^2 \frac{1}{2}[\langle G(\mathbf{r},\mathbf{r})G^*(\mathbf{r},\mathbf{r})\rangle_c - \mathrm{Re}\langle G^2(\mathbf{r},\mathbf{r})\rangle_c]$$
$$\approx \left(\frac{4\pi}{k}\right)^2 \frac{1}{2}\frac{4\pi}{\ell}\int d\mathbf{x}\frac{1-\cos(4kx)}{(4\pi x)^4} = \frac{\pi}{k\ell}. \qquad (1)$$

Here $G(\mathbf{r},\mathbf{r})$ is the Green's function of the wave equation describing the waves in the random medium. In the second equality we are restricted to single scattering in the Born approximation [see Fig. 1(a)]. The value $\pi/k\ell$ agrees *exactly* with the one found for the C_0 speckle correlation [lower right diagram in Fig. 1(b)] [15]. Going beyond single scattering, i.e., replacing the dotted lines in Fig. 1 by diffusion ladders, yields small corrections $\sim 1/(k\ell)^2$ to both the variance of the LDOS and C_0. A *deeper, generally valid* relation between the fluctuations of the LDOS and the C_0 correlation is suggested by the above observations. This is the principal subject of the present work.

Let us consider the simplest model possible (scalar waves

*Electronic address: Bart.Van-Tiggelen@grenoble.cnrs.fr
†Electronic address: Sergey.Skipetrov@grenoble.cnrs.fr

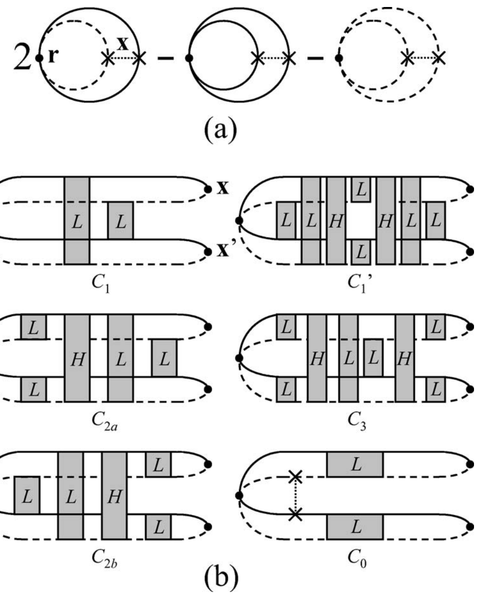

FIG. 1. (a) Two-field intensity diagrams that give the leading order to the variance of the LDOS in a random medium with Gaussian uncorrelated disorder. (b) Typical four-field diagrams that contribute to speckle correlations in a random medium. Solid and dashed lines denote retarded and advanced Green's functions, respectively, shaded boxes are ladder propagators (L) and Hikami boxes (H), the dotted line with crosses denotes scattering of two wave fields on the same heterogeneity. The variance of the LDOS can be obtained by integrating over $\mathbf{x}$ and $\mathbf{x}'$. Only the lower right diagram yields a nonvanishing contribution to the variance of the LDOS in the infinite medium.

in an infinite random medium with white-noise disorder) and leave more complicated situations for future work. Our assumptions are as follows. (1) At long distances, the diffusion approximation for the correlation $\langle G(\mathbf{r},\mathbf{x})G^*(\mathbf{r}',\mathbf{x}')\rangle$ of two Green's functions is valid. (2) The correlation $\langle I(\mathbf{r},\mathbf{x})I(\mathbf{r}',\mathbf{x}')\rangle$ of two intensities $I(\mathbf{r},\mathbf{x})=|G(\mathbf{r},\mathbf{x})|^2$ propagating from the source $\mathbf{r}$ to the receiver $\mathbf{x}$ is composed of terms belonging to *only* four different classes, referred to as C_1, C_2, C_3, and C_0, distinguished by a different correlation range. The first assumption excludes one- and two-dimensional random media that are subject to localization effects. We will thus concentrate on 3D random media. The classification into C_1, C_2, C_3, and C_0 summarizes the outcome of numerous theoretical approaches and experiments [11–17,20,21]. The class C_1 has a short-range correlation in *both* the source positions $\mathbf{r},\mathbf{r}'$ and the receiver positions $\mathbf{x},\mathbf{x}'$ (with a range at most equal to the mean free path). C_2 has two parts. The first part has a long-range correlation (typically a power law) in source positions and short-range correlations in receiver positions, and vice versa for the second part. C_3 has long-range correlation in *both* the source and the receiver positions. The class of terms described by C_0 exhibits an *infinite* range correlation in *either* the source or the receiver positions [15,16]. The classes C_2, C_3, and C_0 imply non-Gaussian statistics of the wave field. For weak disorder ($k\ell \gg 1$) these statistics are Gaussian and C_1 dominates.

The random dielectric constant is denoted by $\varepsilon(\mathbf{r})$, and we shall add a fictitious, *homogeneous* dissipation ε_a and later consider $\varepsilon_a \downarrow 0$. The Green's operator for scalar waves is $G(\mathbf{r},\mathbf{p},\omega)=[\{\varepsilon(\mathbf{r})+i\varepsilon_a\}\omega^2/c^2-\mathbf{p}^2]^{-1}$. The resolvent identity states that $G-G^*=-2i\varepsilon_a(\omega^2/c^2)GG^*$. In real space this translates to the identity

$$-\operatorname{Im} G(\mathbf{r},\mathbf{r},\omega,\varepsilon_a=0)=\frac{\omega^2}{c^2}\lim_{\varepsilon_a\downarrow 0}\varepsilon_a\int d\mathbf{x}\, I(\mathbf{r},\mathbf{x}), \qquad (2)$$

where the integral extends over the whole space The intensity $I(\mathbf{r},\mathbf{x})$ was defined in assumption (2) above. We recall that the (radiation) LDOS $\rho(\mathbf{r},\omega)$ is equal to $-(\omega/\pi c^2)\operatorname{Im} G(\mathbf{r},\mathbf{r},\omega)$ [2]. Thus, Eq. (2) physically expresses that for a homogeneous distribution of sources, the local radiation density is directly proportional to the LDOS. For brevity we shall drop the frequency reference. The second moment of the LDOS can be expressed as

$$\langle \rho(\mathbf{r})^2\rangle=\frac{\omega^6}{\pi^2 c^8}\lim_{\varepsilon_a\downarrow 0}\varepsilon_a^2\int d\mathbf{x}\int d\mathbf{x}'\langle I(\mathbf{r},\mathbf{x})I(\mathbf{r},\mathbf{x}')\rangle. \qquad (3)$$

Equation (3) establishes a conceptual relation between the variance of the LDOS at $\mathbf{r}$, $\operatorname{Var}[\rho(\mathbf{r})]=\langle\rho(\mathbf{r})^2\rangle-\langle\rho(\mathbf{r})\rangle^2$, given by its left-hand side, and intensity correlations in a speckle pattern created by a point source at $\mathbf{r}$ (the integrand of the right-hand side). This facilitates a direct correspondence between the various contributions to the LDOS variance and speckle correlations. We demonstrate in the Appendix that, among the four classes of speckle correlations, only C_0 contributes to the LDOS variance, because of its *infinite* range in the receiver positions $\mathbf{x}$ and $\mathbf{x}'$. The others cancel for different though fundamental reasons, such as current conservation [18].

We conclude that the normalized fluctuations of the LDOS and the C_0 speckle correlation are one and the same,

$$\frac{\operatorname{Var}[\rho(\mathbf{r})]}{\langle\rho(\mathbf{r})\rangle^2}=C_0, \qquad (4)$$

and that observational attempts to confirm the existence of C_0 should focus on the LDOS, either probed by spontaneous emission [2] or by using evanescent waves [19]. It follows from our analysis that only correlations with infinite spatial range contribute to $\operatorname{Var}[\rho(\mathbf{r})]$, and Eq. (4) might serve as a definition for C_0. Alternately, any nonzero variance of LDOS implies the existence of spatial correlations of the intensity I with infinite range.

Because C_0 correlation is nonuniversal and sensitive to the local, microscopic structure of the random medium, our Eq. (4) implies that the fluctuations of the LDOS are nonuniversal too, contrary to the universality of conductive fluctuations. In the context of imaging with noise [6] essentially relying on the measurement of the LDOS, the equivalence of the C_0 correlation and the LDOS fluctuations implies that only objects closer than a wavelength can affect the LDOS and can thus be imaged "passively."

The C_0 correlation determines the variance of the LDOS at a given frequency ω, and it continues to do so in the correlation of the LDOS at two frequencies differing by some $\Omega \ll \omega$. We obtain $\langle\rho(\mathbf{r},\omega)\rho(\mathbf{r},\omega+\Omega)\rangle_c/\langle\rho(\mathbf{r},\omega)\rangle^2 \simeq C_0$, independent of Ω. Similarly, if the disordered medium is not stationary, such as, e.g., a suspension of small particles in Brownian motion, the LDOS will fluctuate in time. The time correlation of these fluctuations, $\langle\rho(\mathbf{r},t)\rho(\mathbf{r},t+\tau)\rangle_c/\langle\rho(\mathbf{r},t)\rangle^2$, is again determined by C_0. According to Ref. [16], $C_0(\tau)$ decays as $\tau^{-3/2}$ for large enough τ. We conclude therefore that the LDOS exhibits long-range correlations in time and infinite-range correlations in frequency.

To summarize, our main conclusion is that fluctuations in the local density of states for waves in random media are conceptually equal to the recently predicted C_0 intensity correlation, and not to the other three types of intensity correlation. Crucial for this equivalence is the infinite spatial range of C_0. Observational evidence for C_0 has been reported so far only once, in the polarization of microwaves [14], whereas the other speckles are firmly established. In a finite medium the intensity correlations $C_{1,2,3}$ will emerge as *extensive* contributions to the LDOS variance, vanishing in some way as the medium scales upwards. With some minor modifications, our main conclusion should hold for infinite 3D disordered band-gap materials, where the LDOS is a much less trivial quantity.

We thank M. Campillo, R. Maynard, and R. Weaver for discussions.

APPENDIX

In this appendix we demonstrate that C_1, C_2, and C_3 correlation functions do not contribute to the fluctuations of the LDOS in Eq. (3), and that C_0 gives the only nonvanishing contribution. We restrict ourselves to the infinite, reciprocal media where $G(\mathbf{x},\mathbf{r})=G(\mathbf{r},\mathbf{x})$ and assume $k\ell \gg 1$.

We first consider Gaussian (C_1) statistics according to which $\langle G(1)G^*(2)G^*(3)G(4)\rangle=\langle G(1)G^*(2)\rangle\langle G^*(3)G(4)\rangle+\langle G(1)G^*(3)\rangle\langle G^*(2)G(4)\rangle$. The first term just gives the average LDOS squared. In the diffusion approximation [assumption (1)], the correlation of two Green's functions takes the form [22]

$$\langle G(\mathbf{r},\mathbf{x})G^*(\mathbf{r}',\mathbf{x}')\rangle = \langle -\operatorname{Im} G(\mathbf{r},\mathbf{r}')\rangle L(\mathbf{r},\mathbf{x})\langle -\operatorname{Im} G(\mathbf{x},\mathbf{x}')\rangle. \tag{A1}$$

In the infinite medium, the field propagator $\langle \operatorname{Im} G(\mathbf{r},\mathbf{r}')\rangle$ oscillates algebraically on the scale of the wavelength and decays exponentially beyond the extinction length ℓ. The ladder propagator $L(\mathbf{r},\mathbf{x})$, however, is very long range and decays significantly only by absorption. Therefore, for the purpose of this Rapid Communication we do not have to discriminate between $\mathbf{r}$ and $\mathbf{r}'$ or $\mathbf{x}$ and $\mathbf{x}'$ in L. On long length scales L obeys a diffusion equation with absorption time τ_a,

$$-D\nabla^2 L(\mathbf{r},\mathbf{x}) + \frac{1}{\tau_a}L(\mathbf{r},\mathbf{x}) = K\delta(\mathbf{r}-\mathbf{x}), \tag{A2}$$

where the factor $K=\lim_{\varepsilon_a\downarrow 0}[\pi\langle\rho(\mathbf{x})\rangle\omega\varepsilon_a\tau_a]^{-1}$ is imposed by the ensemble average of Eq. (2).[1]

The variance of the LDOS caused by C_1 becomes [see the upper left diagram in Fig. 1(b)],

$$\operatorname{Var}_1[\rho(\mathbf{r})] = \frac{\omega^4}{c^4}\lim_{\varepsilon_a\downarrow 0}\varepsilon_a^2\langle\rho(\mathbf{r})\rangle^2\int d\mathbf{x}\, L^2(\mathbf{r},\mathbf{x}) \times\int d\Delta\mathbf{x}\langle -\operatorname{Im} G(\Delta\mathbf{x})\rangle^2. \tag{A3}$$

The integrand of the second integral is $\sin^2(k\Delta x)\exp(-\Delta x/\ell)/(\Delta x)^2$, making the integral converge after typically the extinction length ℓ, without the need for absorption. The integrand of the first integral is typically $L(x)^2 \sim (K^2/D^2x^2)\exp(-2x/\sqrt{D\tau_a})$. The critical contribution of Eq. (A3) comes from large x, which justifies the diffusion approximation employed here. The first integral thus scales as $\sqrt{\tau_a}$. Since $\tau_a\sim 1/\varepsilon_a$, we conclude that as $\varepsilon_a\downarrow 0$, the C_1 contribution to the variance of the LDOS vanishes. All diagrams with short-range spatial correlations in both the source and the receiver positions have the same fate, in particular, the diagram C_1' in Fig. 1(b), that we discuss below.

We now turn to C_2, the first non-Gaussian contribution to the intensity correlation [12]. This is caused by a single crossing at an arbitrary point $\mathbf{s}$ in the medium [see the second and the third diagrams in the left column of Fig. 1(b)], and is described mathematically by the "Hikami box" vertex, with a scalar constant $\mathcal{H}$ that needs not be specified here. Two very similar contributions exist that differ only in selection rules [20]. The first is short range for $\mathbf{x}\neq\mathbf{x}'$, and equals

$$\langle I(\mathbf{r},\mathbf{x})I(\mathbf{r},\mathbf{x}')\rangle_{C_{2a}} = \mathcal{H}\pi^2\langle\rho(\mathbf{r})\rangle^2\langle \operatorname{Im} G(\mathbf{x},\mathbf{x}')\rangle^2 \times\int d\mathbf{s}\, L^2(\mathbf{r},\mathbf{s})(\nabla_1\cdot\nabla_2) \times L(\mathbf{r}_1=\mathbf{s},\mathbf{x})L(\mathbf{r}_2=\mathbf{s},\mathbf{x}). \tag{A4}$$

According to Eq. (3) we need the double integral $\varepsilon_a^2\int d\mathbf{x}\int d\mathbf{x}'$ of this object and let ε_a tend to zero. One integral converges again rapidly after one extinction length and is finite without absorption. We shall write $\int d\mathbf{x}'\langle \operatorname{Im} G(\mathbf{x},\mathbf{x}')\rangle^2=V_0$ and rearrange expression (A4) to

$$\operatorname{Var}_{2a}[\rho(\mathbf{r})] = \frac{\omega^4}{c^4}\mathcal{H}V_0\langle\rho(\mathbf{r})\rangle^2\lim_{\varepsilon_a\downarrow 0}\varepsilon_a^2\int d\mathbf{x}\int d\mathbf{s}\, L^2(\mathbf{r},\mathbf{s}) \times|\nabla L(\mathbf{s},\mathbf{x})|^2 \sim V_0\lim_{\varepsilon_a\downarrow 0}\varepsilon_a^2\int d\mathbf{x}|\nabla L(\mathbf{x})|^2\int d\mathbf{s}\, L^2(\mathbf{s}). \tag{A5}$$

In the second step we conveniently made use of the transla-

[1] In statistically homogeneous media, $\langle\rho(\mathbf{x})\rangle$ is independent of $\mathbf{x}$. In band-gap materials—in principle, beyond the scope of this work—the average over both disorder and unit cell should appear here, and is thus still independent of $\mathbf{x}$.

tional symmetry of the infinite medium. We see that $|\nabla L(\mathbf{x})| \sim 1/x^2$ for large x, making the integral converge without the need of absorption. Its divergence for $x < \ell$ is an artifact of the diffusion approximation, which is not valid at small length scales, and which we shall ignore. Hence, the volume integral over $\mathbf{x}$ is just finite, without absorption. The integral over the position $\mathbf{s}$ of the Hikami box scales as $\sqrt{\tau_a}$. As $\varepsilon_a \downarrow 0$ we conclude that the contribution of the first C_2 term to the LDOS, $\mathrm{Var}_{2a}[\rho(\mathbf{r})]$, vanishes.

The second contribution from C_2 is long range as a function of $\mathbf{x}-\mathbf{x}'$ [21]. Its expression reads

$$\begin{aligned}\langle I(\mathbf{r},\mathbf{x})I(\mathbf{r},\mathbf{x}')\rangle_{C_{2b}} &= \mathcal{H}\pi^4\langle\rho(\mathbf{r})\rangle^2\langle\rho(\mathbf{x})\rangle^2 \\ &\times \int d\mathbf{s}\, L^2(\mathbf{r},\mathbf{s})(\boldsymbol{\nabla}_1 \cdot \boldsymbol{\nabla}_2) \\ &\times L(\mathbf{r}_1=\mathbf{s},\mathbf{x})L(\mathbf{r}_2=\mathbf{s},\mathbf{x}'), \end{aligned} \quad \text{(A6)}$$

and a little rearranging shows that its contribution to the variance of the LDOS is

$$\mathrm{Var}_{2b}[\rho(\mathbf{r})] \sim \lim_{\varepsilon_a \downarrow 0} \varepsilon_a^2 \left| \int d\mathbf{x}\ \boldsymbol{\nabla} L(\mathbf{x}) \right|^2 \int d\mathbf{s}\, L^2(\mathbf{s}). \quad \text{(A7)}$$

We note that $\int_V d\mathbf{x} \nabla L(\mathbf{x}) = \int_{A(V)} d\mathbf{A} L(\mathbf{x})$, where $A(V)$ is the surface enclosing the volume V. The surface integral vanishes for any closed surface, because $L(\mathbf{x})$ does not depend on the direction of $\mathbf{x}$. Thus, $\mathrm{Var}_{2b}[\rho(\mathbf{r})]=0$.

The contribution of the C_3 correlation, the origin of universal conductance fluctuations, can be handled similarly. C_3 contains two Hikami boxes [C_3 in Fig. 1(b)], but that is a technical complication, and in just the same way as for C_2 it can be shown to vanish as $\varepsilon_a \downarrow 0$. The diagram C_1' in Fig. 1(b) looks very much like C_3 but has actually short spatial range in both the source and the receiver positions. As a result, it belongs to the class C_1, and its contribution to the variance of the LDOS vanishes for the same reason as was seen in Eq. (A3).

Finally, the C_0 correlation is given by [see the lower right diagram in Fig. 1(b) [15,16],

$$\langle I(\mathbf{r},\mathbf{x})I(\mathbf{r},\mathbf{x}')\rangle_{C_0} = \langle I(\mathbf{r},\mathbf{x})\rangle\langle I(\mathbf{r},\mathbf{x}')\rangle \times C_0, \quad \text{(A8)}$$

with C_0 a dimensionless scalar depending on the nature of the scatterers. For weak white-noise, uncorrelated disorder $C_0 = \pi/k\ell$ [15,16]. The essential property of C_0 that is important here is its *infinite* spatial range caused by the scattering of waves going to arbitrarily distant $\mathbf{x}$ and $\mathbf{x}'$ on a common scatterer in the vicinity of the source at $\mathbf{r}$. Inserting Eq. (A8) into the expression for the LDOS variance (3) and making use of Eq. (A1), we obtain

$$\begin{aligned}\mathrm{Var}_0[\rho(\mathbf{r})] &= \omega^2 C_0 \pi^2 \langle\rho(\mathbf{r})\rangle^2 \lim_{\varepsilon_a \downarrow 0} \varepsilon_a^2 \left(\int d\mathbf{x} \langle\rho(\mathbf{x})\rangle L(\mathbf{r},\mathbf{x}) \right)^2 \\ &= C_0 \pi^2 \langle\rho(\mathbf{r})\rangle^2 \langle\rho(\mathbf{x})\rangle^2 \lim_{\varepsilon_a \downarrow 0} \varepsilon_a^2 \omega^2 K^2 \tau_a^2 = C_0 \langle\rho(\mathbf{r})\rangle^2. \end{aligned} \quad \text{(A9)}$$

Hence, C_0 provides the only surviving contribution to $\mathrm{Var}[\rho(\mathbf{r})]$.

[1] H. Koshravi and R. Loudon, Proc. R. Soc. London, Ser. A **433**, 337 (1991).

[2] R. Sprik, B. A. van Tiggelen, and A. Lagendijk, Europhys. Lett. **35**, 265 (1996).

[3] A. F. Koenderink, L. Bechger, H. P. Schriemer, A. Lagendijk, and W. L. Vos, Phys. Rev. Lett. **88**, 143903 (2002).

[4] R. Hennino, N. P. Trégourès, N. Shapiro, L. Margerin, M. Campillo, B. A. van Tiggelen, and R. L. Weaver, Phys. Rev. Lett. **86**, 3447 (2001).

[5] A. F. Koenderink and W. L. Vos, Phys. Rev. Lett. **91**, 213902 (2003).

[6] R. L. Weaver and O. I. Lobkis, Phys. Rev. Lett. **87**, 134301 (2001); N. M. Shapiro, M. Campillo, L. Stehly, and M. H. Ritzwoller, Science **307**, 1615 (2005).

[7] F. Wegner, Z. Phys. B **36**, 209 (1980); B. L. Al'tshuler, V. E. Kravtsov, and I. V. Lerner, JETP Lett. **43**, 441 (1986); I. V. Lerner, Phys. Lett. A **133**, 253 (1988); K. B. Efetov and V. N. Prigodin, Phys. Rev. Lett. **70**, 1315 (1993); A. D. Mirlin, Phys. Rev. B **53**, 1186 (1996).

[8] A. D. Mirlin, Phys. Rep. **326**, 259 (2000).

[9] C. W. J. Beenakker, Phys. Rev. B **50**, 15170 (1994).

[10] I. E. Smolyarenko and B. L. Altshuler, Phys. Rev. B **55**, 10451 (1997).

[11] B. Shapiro, Phys. Rev. Lett. **57**, 2168 (1986).

[12] S. Feng and R. Berkovits, Phys. Rep. **238**, 135 (1994).

[13] A. Z. Genack, N. Garcia, and W. Polkosnik, Phys. Rev. Lett. **65**, 2129 (1990); J. F. de Boer, M. P. van Albada, and A. Lagendijk, Phys. Rev. B **45**, 658 (1992); F. Scheffold, W. Härtl, G. Maret, and E. Matijević, *ibid.* **56**, 10942 (1997); F. Scheffold and G. Maret, Phys. Rev. Lett. **81**, 5800 (1998).

[14] A. A. Chabanov, N. P. Trégourès, B. A. van Tiggelen, and A. Z. Genack, Phys. Rev. Lett. **92**, 173901 (2004).

[15] B. Shapiro, Phys. Rev. Lett. **83**, 4733 (1999).

[16] S. E. Skipetrov and R. Maynard, Phys. Rev. B **62**, 886 (2000).

[17] E. Akkermans and G. Montambaux, *Physique Mésoscopique des Electrons et des Photons* (EDP Sciences, Paris, 2005).

[18] C. L. Kane, R. A. Serota, and P. A. Lee, Phys. Rev. B **37**, 6701 (1988).

[19] K. Joulain, R. Carminati, J. P. Mulet, and J. J. Greffet, Phys. Rev. B **68**, 245405 (2003).

[20] P. Sebbah, B. Hu, A. Z. Genack, R. Pnini, and B. Shapiro, Phys. Rev. Lett. **88**, 123901 (2002).

[21] M. J. Stephen and G. Cwilich, Phys. Rev. Lett. **59**, 285 (1987).

[22] A. Lagendijk and B. A. van Tiggelen, Phys. Rep. **270**, 143 (1996).