Regulating noise-induced spiking using feedback

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We report successful manipulation of the noise provoked spiking behavior using delayed feedback control. Experiments were performed in a three electrode electrochemical cell under potentiostatic conditions. The uncontrolled system exhibited noise invoked oscillations whose regularity was quantified using normalized variance (NV). Superimposing delayed feedback, for appropriate values of delay (τ), an enhancement in the regularity of the spike sequence was attained. Numerical simulations corroborated experimental observations.

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The term stochastic resonance (SR) [1–4], as it is now widely used, describes the constructive role of noise in nonlinear dynamical systems. Initially, within the framework of SR, detection of subthreshold periodic signals (PSR) [1,4–8] in the presence of superimposed noise was studied. Subsequently, the field has expanded immensely and at present numerous new applications of noise in diverse natural processes exist [9–14]. Due to the ubiquity of both noise and nonlinear systems, the effects that arise as a consequence of their mutual interactions are frequent in occurrence and generic in nature. One of these noise provoked effects, namely coherence resonance (CR) [12,13], has been observed experimentally in excitable optical [15] and chemical [16,17] systems. CR involves the inception of almost periodic oscillations supported by purely stochastic fluctuations and is attributed to an interplay between the nonlinear nature of deterministic systems and random driving forces. The properties of the spike trains in CR systems are, by definition, a function of the superimposed noise intensity.

However, the CR effect in nonlinear systems possess the following drawback: In CR systems, by virtue of the underlying mechanisms, regularity of the induced spike sequences is significantly less in comparison to that for PSR systems. Even for the optimum noise level the maximal regularity obtained is quite low. It is possible to envisage situations where higher regularity is required for optimum functioning. Therefore, there exists a need to regulate the characteristics/properties of these noise provoked dynamics using appropriate control techniques [18].

Control theory is a field adopted by physicists from engineers and has been extensively used to tame complex deterministic behavior [19–21]. The types of control normally implemented can be broadly divided into two categories: forcing and feedback. Both these strategies have proven effective in controlling nonlinear deterministic dynamics. However Janson *et al.* [18], have shown recently that it is also possible to control noise induced dynamics. In their paper they show, using numerical simulations, that the delayed feedback technique [22,23] is a viable option for enhancing the regularity of noise induced motion.

In this work, we report experimental manipulation of the spike sequence regularity using delayed feedback method. Using appropriate values of the control parameters (amplitude γ and delay τ), the regularity of the spike trains provoked by different values of superimposed noise amplitudes could be augmented. This points to the robustness and the effectiveness of the delayed feedback technique in controlling noise induced motion for excitable systems.

Experiments were carried out in a three-electrode electrochemical cell, configured to study the potentiostatic electrodissolution of iron in a mixture of copper sulfate and sulfuric acid. The anode was a pure iron (Sigma Aldrich 99.98% purity) disk (6.3 mm diameter) shrouded by epoxy. The electrolyte solution was a mixture of 1.0 molar sulfuric acid and 0.4 molar copper sulfate. A volume of about 500 ml was maintained in the cell. The anodic potential (V), measured relative to a saturated calomel reference electrode (SCE), was used as the control (bifurcation) parameter on to which the external perturbations were superimposed. The cathode was a 5 mm diameter copper rod. Oscillations in the anodic current I (the current between the anode and the cathode) were recorded using a 12-bit data acquisition card at a sampling rate of 250 Hz. The external noise used in the experiments was derived from a random number generator consistent to white noise with a Gaussian distribution [14]. This output was converted to an analog signal and superimposed on to the anodic voltage via a potentiostat (PINE Model AFRDE5). The frequency at which the noise amplitude was varied is about $\approx$1.25 Hz.

The details of the autonomous behavior exhibited by the electrochemical cell have been reported previously [14,17] and can be summarized as follows: Varying anodic voltage (V) as the bifurcation parameter, two different dynamical responses of the anodic current I were observed, namely, a stationary state behavior (constant current response), and period-1 oscillations emerging from a supercritical Hopf bifurcation at $\approx$175 mV. At anodic voltages slightly above the Hopf bifurcation, small, harmonic oscillations were observed. At higher voltages the experimental system exhibited relaxation oscillations whose period augmented with increasing voltage. In our earlier studies [14,17], we noted that this period lengthening occurs until the oscillations died at the homoclinic bifurcation point V_{hc} at about 216 mV.

The first set of experiments involved searching for the CR phenomena in our experimental system [17]. For this purpose, the set point for the control parameter (anodic voltage) V was chosen such that $V_0 > V_{hc}$. Consequently anodic cur-

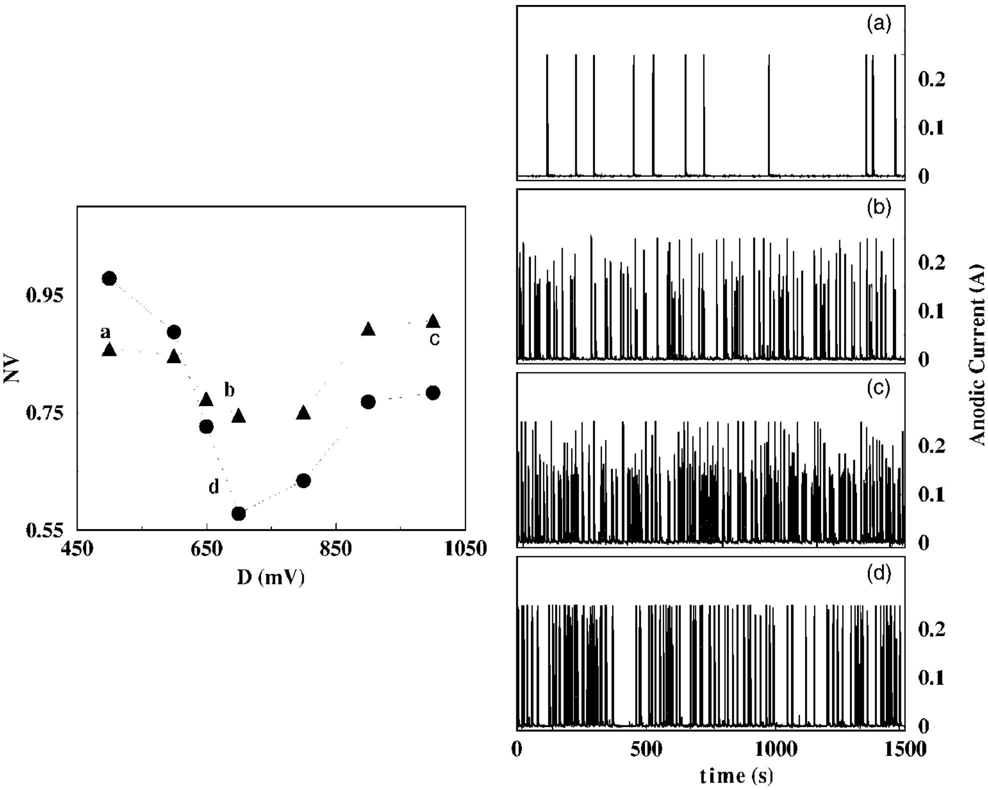

FIG. 1. The left panel shows the experimentally computed NV curves without (dotted) and with (solid) delayed feedback control. The set point (V_0) was chosen to be 360 mV such that the autonomous dynamics exhibited excitable fixed point behavior. The amplitude γ of the delayed feedback control term was fixed at 100 mV and the τ was calculated, as explained in the text, to be $\tau = 16.428$ s. The upper three traces of the right panel shows experimental time series of the anodic current (I) provoked by noise amplitudes labeled a, b, c of the dotted curve, whereas the bottom trace shows the noise induced time series, in the presence of the delayed feedback control, for the noise amplitude corresponding to label d of the solid curve.

rent (I), the system observable, exhibited excitable fixed point behavior. The anodic voltage V was then defined as $V = V_0 + D\xi$, where the amplitude of the imposed Gaussian white noise ξ is D. We studied the system response as a function of the noise amplitude D. Normalized variance (NV) was used to quantify the extent of induced regularity. It is defined as $\mathrm{NV} = \frac{\sqrt{\mathrm{Var}(t_p)}}{\langle t_p \rangle}$, where t_p is the time between successive peaks. It is evident, that more regular the dynamics lower the value of the computed NV. The dotted curve in Fig. 1 was obtained by plotting the NV values, calculated using experimental data, as a function of the noise amplitude D. Time series for three of the imposed noise amplitudes are also shown. The first point (label a) corresponds to a low level of noise for which the bifurcation point was seldom crossed. Label b correspond to the optimum noise level for whom maximal regularity of the generated spike sequence was observed. As the amplitude of superimposed noise was increased further, the observed regularity was destroyed manifested by an increase in the NV (label c). This was a consequence of the dynamics being contaminated by high levels of noise.

The second set of experiments involved the inception of the CR effect in conjunction with delayed feedback control. The superimposed control should alter the dotted NV curve of Fig. 1. The anodic voltage for these set of experiments was modulated as $V = V_0 + D\xi + \gamma[I(t) - I(t-\tau)]$, where $\gamma[I(t) - I(t-\tau)]$ was the feedback term intended to enhance the regularity of the spike trains. γ was the control amplitude and τ was the delay time determined as follows [23]: Using the time series provoked by the noise amplitude of label b in Fig. 1, a return map (not shown) was constructed by plotting successive interspike intervals (t_{p+1} vs t_p). Subsequently, a linear regression of this apparently structureless return map was obtained. The intersection of this regression line with the line of identity, provides the appropriate delay τ used in the feedback term. The CR experiments performed in the presence of feedback control yielded the solid NV curve presented in Fig. 1. It is evident, by visual inspection, that the maximum attainable regularity of the spike train was enhanced (manifested by a deeper minima) due to the superimposed delayed feedback control. Moreover the NV values, for the controlled system, calculated at other noise amplitudes were also reduced. This lowering of the NV curve points to the effectiveness of the delayed feedback control strategy. Figure 1 also shows the noise invoked time series in the presence of delayed feedback control. The bottom trace of the right-hand panel shows the time series for the noise amplitude corresponding to label d of the solid curve (optimum noise level for the controlled NV curve). It, when compared to the time series corresponding to label b of the dotted curve (optimum noise level for the uncontrolled NV curve), reveals the enhanced regularity observed by virtue of the superimposed feedback.

To corroborate our experimental findings, we performed numerical simulations in a two-dimensional model developed [24] using reaction rate kinetics. This model, reproduces the different dynamics and the underlying bifurcations observed experimentally and can be described as

$$\epsilon \frac{du}{dt} = \frac{v - u}{R} - f(u,c), \tag{1}$$

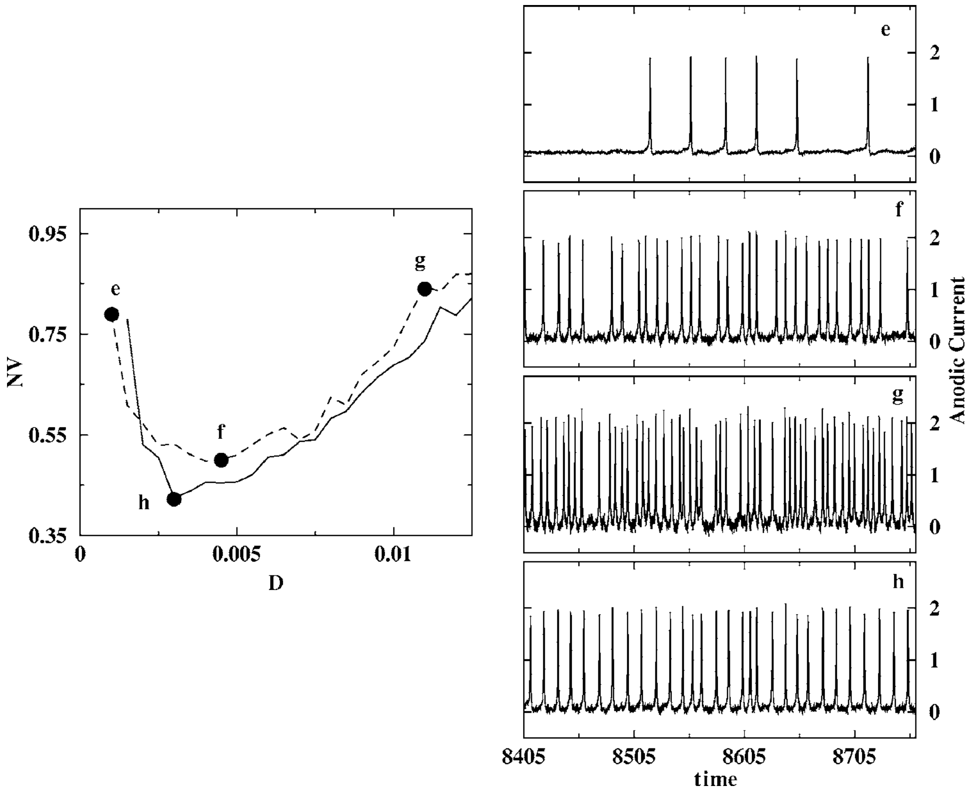

FIG. 2. The left panel shows the numerically computed NV curves without (dotted) and with (solid) delayed feedback control. It needs to be pointed out that all the quantities plotted are dimensionless. The set-point (V_0) was chosen to be 29.245 such that the autonomous dynamics exhibited excitable fixed point behavior. The amplitude of the delayed feedback control term was fixed at $\gamma=0.01$ and the τ was calculated, as explained in the text, to be $\tau=11.25$. The upper three traces of the right panel shows numerical time series of the anodic current (I) provoked by noise amplitudes labeled e, f, g of the dotted curve, whereas the bottom trace shows the noise induced time series, in the presence of the delayed feedback control, for the noise amplitude corresponding to label h of the solid curve.

$$\frac{dc}{dt} = \frac{u-v}{R} + (1-c) + f(u,c), \tag{2}$$

where

$$f(u,c) = c(a_1 u + a_2 u^2 + a_3 u^3). \tag{3}$$

The two independent variables u and c correspond to the electrode potential and the surface concentration. The three system parameters ϵ, R, and v, represent the double layer capacitance, Ohmic resistance, and the applied potential, respectively. Equation (2) represents the conservation of charge where Eq. (3) describes the mass balance. These dimensionless differential equations (1) and (2) were integrated using a second order Runge-Kutta method, specifically adapted for solving stochastic equations [25]. Dynamical behavior similar to experiments was found for the following parameter values [24]: $\epsilon=0.03$, $\alpha=0.1$, $R=10$, $a_1=1.125$, $a_2=-0.075$, and $a_3=0.00125$. Anodic voltage v, the bifurcation parameter, was varied to map out the different responses of the model.

For anodic voltage values $28.097 \leq v \leq 29.235$, limit cycle behavior was observed that gives way to stationary state behavior for $v > 29.235$. Similar to experiments, the value of the anodic voltage v was chosen such that the autonomous system exhibited excitable fixed point dynamics. Subsequently, noise ξ whose amplitude D was monotonically varied, was added onto the anodic voltage v to reveal the CR effect. Figure 2 shows the numerically generated NV curve (dotted) and the time series for three noise amplitudes. Labels e, f, and g of the dotted curve correspond to low, optimum, and high levels of superimposed noise, respectively. This noise invoked behavior was controlled using a delayed feedback term whose τ (the delay time) was calculated, as in experiments, using a return map of interspike intervals. The solid curve shown in Fig. 2 corresponds to the CR system in the presence of control. Consistent to experimental observations, an augmentation in the regularity of the noise induced spiking was observed in the presence of delayed feedback. The bottom trace of the right-hand panel in Fig. 2 shows the time series for the noise amplitude corresponding to label h of the solid NV curve. It, when compared to the time series corresponding to label f of the dotted NV curve in Fig. 2, reveals the increased regularity generated by virtue of the added control term.

Our experiments demonstrate unequivocally that it is indeed possible to regulate the noise provoked dynamics using a delayed feedback control strategy. The effectiveness of the feedback method is manifested by its ability to control the different spike sequences induced by distinct amplitudes of random forcings. In our experiments as well as in our simulations the control parameter τ was calculated using a return map of the spike sequence observed for a single noise amplitude (label b in experiments and label f in simulations). This value of τ was used thereafter to control the dynamics provoked by the remaining noise amplitudes. However, if one was to calculate τ for each individual value of the superimposed noise it would perhaps further improve the extent of regularity invoked. Control of noise induced motion apart, from being a challenging scientific problem, could be of relevance to threshold systems subjected to internal/external noise. For example, usually, in threshold systems with internal noise one has no control over the intensity of the existent noise. Consequently, regulating the characteristics of the noise provoked spike sequences is not possible. However, in such scenarios, judicious implementation (suitable choice of γ and τ) of the delayed feedback control strategy could alter/regulate the properties of the observed spike trains. Since it is possible to envisage numerous real systems with intrinsic noise, this problem entailing the control of the stochastically induced spike trains has tremendous validity and applicability.

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