Enhancement and sustainment of internal stochastic resonance in unidirectional coupled neural system

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In this study, the collective dynamical behavior of two unidirectionally, linearly coupled neurons was investigated. Our investigation illustrates that, depending on the coupling strength, the internal stochastic resonance (ISR) effect observed in one of the two neurons could be amplified or sustained by the other. The amplification of ISR is enhanced as the coupling strength is increased from 0. However, when the coupling strength is increased above a certain level, the amplification of ISR is reduced, implicating that there exists an optimal coupling strength for the information flow between two neurons. As the coupling strength is increased further, i.e., above the critical level, synchronization of the two subsystems is achieved, yet the two subsystems exhibit the same magnitude response to the external noise, suggesting that the information transmission among coupled subunits could not be improved by further enhancing their coupling strength. And similar phenomena could also be obtained for the nonidentical case.

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I. INTRODUCTION

Coupled systems are ubiquitous in nature ranging from physical to biological systems. The coupling can be realized, for example, by direct exchange of mass or heat among subunits or via a common external forcing. The external driving can be either a periodic or random signal. Recent studies show that random forcing, commonly referred to as noise, could induce a number of interesting dynamical behavior including synchronization and suppression of chaos [1–3]. Since the discovery of stochastic resonance (SR) [4], the beneficial role of external noise has attracted increasing attention in different fields of science [5–9]. Because a nonlinear system cannot ordinarily respond to a subthreshold weak signal, SR is a process where the response of a system to a weak, external, periodic modulation is enhanced in the presence of background noise. Later, coherence resonance (CR) [10], or internal stochastic resonance (ISR) [11], and explicit internal signal stochastic resonance [12] were studied by replacing external signals with internal signals coming from noise-induced oscillation or intrinsic periodic oscillation of the systems.

Since coupling is of fundamental importance in a variety of complex nonlinear physical, chemical, and biological systems, the dynamical behaviors of noisy coupled systems have been extensively investigated [13–15]. Lindner *et al.* [16] exhibited the importance of coupling to the global spatiotemporal dynamics of an array of one-dimensional overdamped oscillators. Zhou *et al.* [17] found that coupling could enhance the coherence resonance in a chain of noisy neurons. These studies indicate that couplings have an important influence on the dynamic behaviors of nonlinear systems. The coupling of dynamical systems can also lead to synchronization, which could be induced either by coupling the systems or by forcing them. Various synchronization have recently been found in coupled systems [18–23].

It is well known that neurons are important information transmission channels in the living body and play the role of transferring the response of biological systems to the external fluctuation from environment to neural centers. Various dynamical phenomena have been explored in coupled neural systems [24–29]. It has been shown that the response of neurons could be improved by background noise. However, to the best of our knowledge, not much research has been focused on how the coupling influences the intrinsic periodic behavior and the information transmission in noisy coupled neurons. The motivation of this paper is, therefore, to study the influence of coupling on the internal information flow (measured by signal-to-noise ratio) transduction with the phenomenon of internal stochastic resonance in two-coupled neurons. To address the problem, two unidirectional coupled periodical oscillators are considered. It is different than our other work [30], where two subsystems were both subject to coupling and the energy was transferred from one to the other. Here, one of the oscillators is subject to external noise without coupling, so the energy of this subsystem is invariable and the other inherits the noise through a linear coupling between. The system with noise is called the "driver" and the one with coupling is called the "receiver." Because of this coupling, it is found that the system has many interesting dynamic behaviors, for example, coupling optimal enhancement of ISR effect of the receiver, synchronization between the subsystems, and array enhanced stochastic resonance (AESR) without tuning. When we change one parameter slightly, similar phenomena could also be obtained.

II. DYNAMICAL MODELS

The model used in the present work was proposed by FitzHugh [31]. It is a typical model of excitable systems, nerve pulses, and CRs [7]. The related dynamic equations are expressed as

$$\varepsilon \frac{dx}{dt} = x - x^3/3 - y,$$

$$\frac{dy}{dt} = x + a,$$

where $\varepsilon = 0.01$. For a single FitzHugh-Nagumo (FHN) neuron, if $|a| > 1$, the system has only a stable fixed point, while for $|a| < 1$, a limit cycle occurs. To study the transduction of the information flow in noisy neurons, we adopt unidirectional and linear coupling between, respectively, the driver and the receiver are displayed as follows:

$$\varepsilon \frac{dx_1}{dt} = x_1 - x_1^3/3 - y_1 + \beta\xi(t),$$

$$\frac{dy_1}{dt} = x_1 + a. \quad (1)$$

The receiver is

$$\varepsilon \frac{dx_2}{dt} = x_2 - x_2^3/3 - y_2 + K_c(x_1 - x_2),$$

$$\frac{dy_2}{dt} = x_2 + a, \quad (2)$$

where K_c is the coupling strength and β is the intensity of Gaussian white noise $\xi(t)$ with zero mean value $\langle\xi(t)\rangle = 0$ and unit variance $\langle\xi(t)\xi(t+\tau)\rangle = \delta(\tau)$. Here, the electrical coupling is chosen to study the information transmission. Another kind of coupling similar to reciprocally synaptic coupling is also under consideration [32,33].

III. RESULTS AND DISCUSSIONS

In the present work, a is set to 0.5 for a periodic oscillatory state to investigate the response of the intrinsic periodic signal to the external fluctuation in coupled systems. Equations (1) and (2) are solved numerically by using the Euler method. To quantify the SR effect, 16 384 points at intervals of 0.01 are used to obtain frequency spectra by fast Fourier transform. Based on the frequency spectrum, signal-to-noise ratio (SNR) is defined as $H(\Delta\omega/\omega_f)^{-1}$ as in Ref. [34]. Here, H is the maximum peak height of the spectrum, ω_f is the principal peak frequency, and $\Delta\omega$ is the width of the peak at its half height, which is slightly different than Ref. [34] for the convenience of computation. Each plot of SNR versus noise intensity or coupling strength is obtained by averaging 40 runs.

When the driver system is subject to external noise, from Fig. 1, it is seen that the driver exhibits the phenomenon of the ISR; the internal periodic signal and noise reach the optimal match at a certain noise intensity. Since the behavior of the driver system is not affected by the coupling, one would expect that the ISR effect of the driver system is fixed. Figure 2 displays the power spectral density (PSD) of each oscillator, it could be seen that the contour of the receiver is smoother compared with that of the driver system, which illustrates that the coupling could also play the role of noise filter in addition to the role of transferring the internal energy. With the increase of the coupling strength, as mentioned above, the ISR of the driver system would not change with the coupling strength as the intrinsic periodic dynamic behaviors of the driver cannot be affected by modulating the coupling levels. However, the effect of the ISR for the receiver differs in the variable coupling level. Figure 3(a) shows the SNR versus the noise intensity for x_2 on different coupling strengths. A trendency of the ISR effect, which appears to increase first and then decrease with the increment of coupling, can be observed in this figure. Neural networks are generally conceived considering interactions between two neurons, one that is releasing information and another that is receiving it. The importance of the intensity of the interactions cannot be ignored. The tendency mentioned above indicates that at an appropriate coupling level, the energy in the coupled neurons could be optimally transferred and the response of the second neuron to the inherited random information can be most effectively amplified. It is important to study the role of coupling in the information processing and transduction in neural systems under noisy background.

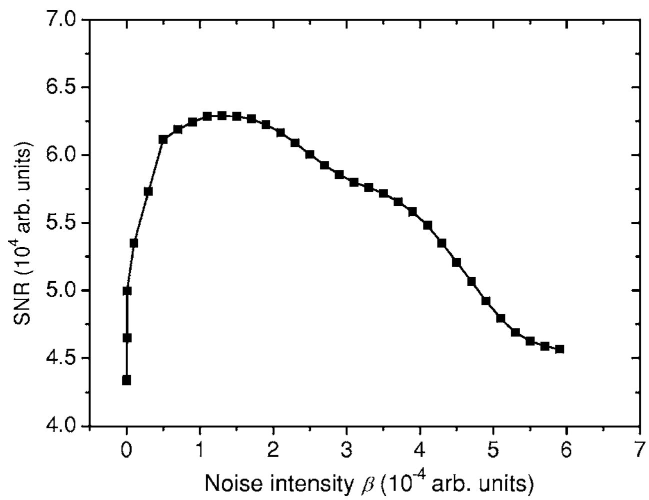

FIG. 1. The SNR for the driver system versus noise intensity. Parameters: $\varepsilon = 0.01$, $a = 0.5$.

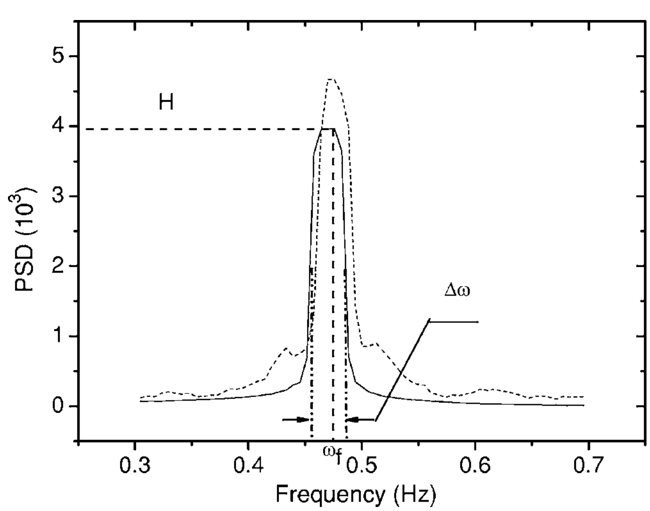

FIG. 2. The power spectral density (PSD) for x_1 (dashed line) and x_2 (solid line) at $K_c = 0.001$, $\beta = 0.0001$. Other parameters as in Fig. 1.

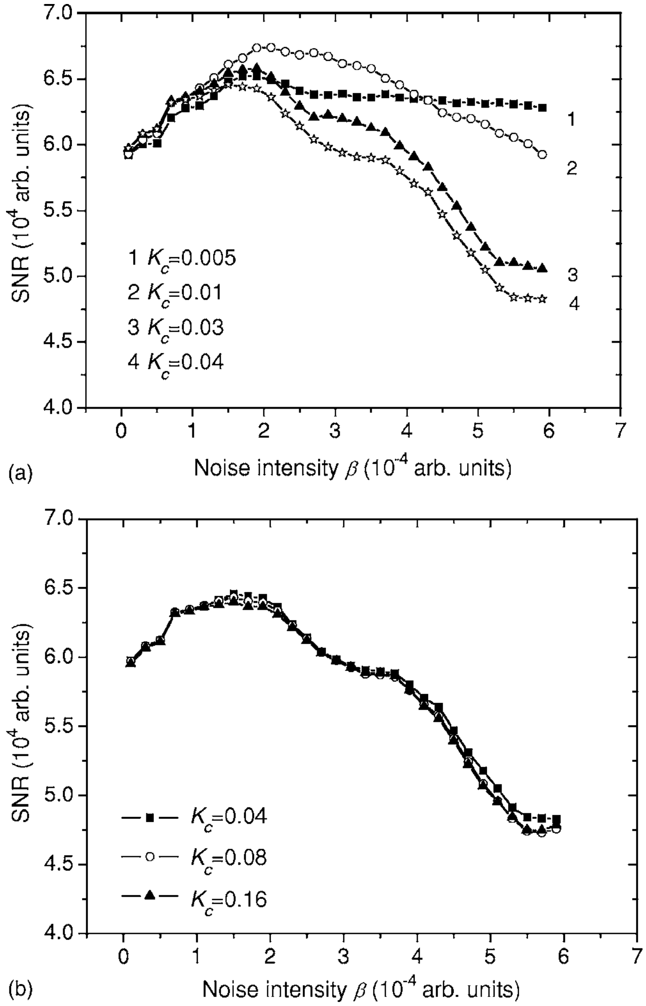

FIG. 3. The SNR against noise intensity for x_2 on various coupling strength: (a) small coupling level; (b) large coupling level. Other parameters as in Fig. 1.

As the coupling strength is stronger, we find that the responses of x_2 to noise in different coupling cases coincide with each other. The SNR of the receiver at stronger coupling, respectively, are presented in Fig. 3(b), which exhibits the corresponding ISR effect at various coupling levels. It indicates that the energy transferred from the driver to the receiver would not be improved, the whole system could resist to the influence of coupling and sustain the intrinsic periodic oscillatory state.

To further confirm the optimal coupling at which the energy transduction is most effective and the critical coupling level where the ISR effect of the receiver would not change any more, the SNR of x_2 against the coupling strength is displayed for different noise intensities in Fig. 4. It is easy to observe the two coupling levels from this figure. The optimal coupling strength is about 0.01, where the global system's, coupling and noise reach the optimal match. The critical coupling strength is about 0.04, as seen intuitively in this figure, so the modulation of coupling would not change the ISR effect, which could be called AESR without tuning. It seems that the cooperative effect between the system and noise is robust and cannot be influenced by the coupling, indicating

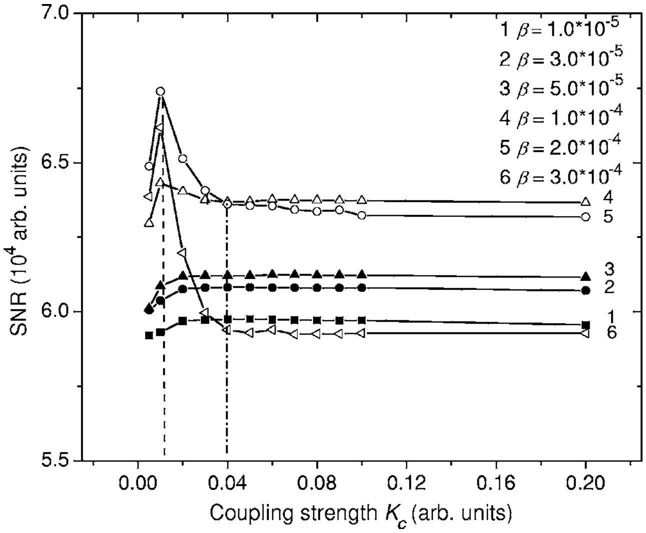

FIG. 4. The SNR versus coupling strength for x_2 at different noise disturbance β=(1) 1.0×10^{-5}, (2) 3.0×10^{-5}, (3) 5.0×10^{-5}, (4) 1.0×10^{-4}, (5) 2.0×10^{-4}, (6) 3.0×10^{-4}. Other parameters are as in Fig. 1.

that the information transmission between reaches the saturation. Furthermore, the contour curves of SNR are drawn in Fig. 5. The optimal combination for β and K_c can be easily observed in this figure. And at a stronger coupling level, it shows that the SNR will be constant at the same disturbance, which corresponds with the case in Fig. 4. It is very important to study the role of coupling in coupled bioexcitable systems and how to adjust the connectivity to obtain the optimal or stable system exhibition to the external stimuli.

The ISR effect between the two subsystems is also compared when coupling increases. As mentioned above, the ISR effect of the driver system is uniform in various coupling levels. With the increasing of the coupling, there is the enhancement and suppression of the ISR effect for the receiver. Eventually the receiver system synchronizes with the driver system. From Fig. 6, we can observe that x_2 synchronizes the response to noise with x_1 at coupling strengths of 0.04 and 0.1. Figure 7 displays the noise intensity versus coupling

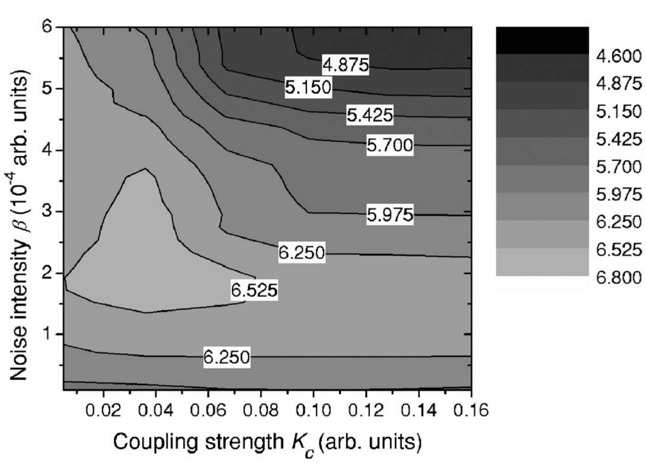

FIG. 5. The contour plot of SNR (10^4 arb. units) as a function of β and K_c. Other parameters are as in Fig. 1.

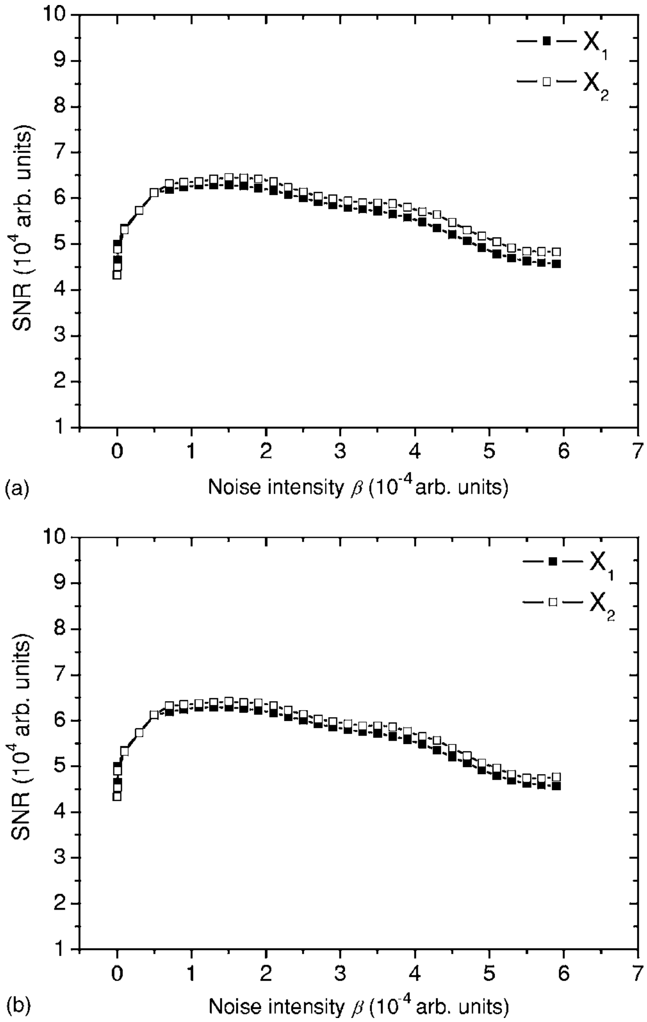

FIG. 6. The SNR versus noise intensity for x_1 and x_2, respectively, on the coupling strength K_c=0.04 (a), K_c=0.1 (b). Other parameters as in Fig. 1.

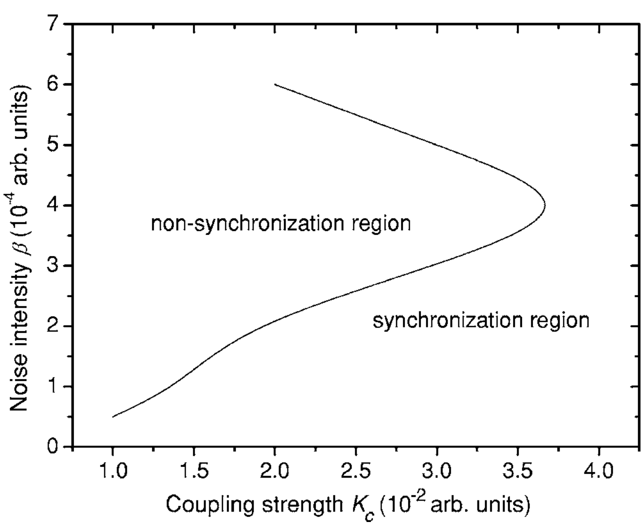

FIG. 7. The noise intensity versus coupling strength for x_2. Other parameters are as in Fig. 1.

strength, which indicates the synchronization and nonsynchronization regions. Otherwise, since the ISR effect of the receiver is uniform over the critical coupling level as mentioned above, the coupled neural subunits with various stronger coupling levels have similar responses to the external fluctuation, i.e., the ISR effects are sustained though the coupling level changes.

The above investigation is for identical subunits. A nonidentical case is also studied in this paper. Here, we slightly change the parameter a in Eq. (1) to see the response of the receiver to the noise transferred by coupling. We set a in Eq. (1) to 0.51 and keep it as 0.5 in Eq. (2). The contour curves of SNR and the diagram on β versus K_c are displayed in Fig. 8, from the two plots we can observe the optimal combination and the critical coupling level for this nonidentical case. We can also find that the SNR value on the optimal combination is larger than that in Fig. 5. It seems that there might be an optimal a where the response to external noise could be optimally amplified. Note that we have investigated other cases with different a and obtain similar phenomena (the data are not shown here).

Hou *et al.* [35] explored the noise-induced oscillation (NIO) in a one-way coupled chemical system, and found that the stochastic oscillation could be propagated and enhanced by coupling. Here, the ISR effect can also be transmitted and enhanced in unidirectional coupled neurons in an intrinsic periodic oscillatory state. The amplification of the ISR for the second oscillator could be modulated by changing the coupling strength, nevertheless, the internal information transmission would not be improved with the increment of coupling after a critical coupling level, which means AESR without tuning. Synchronization between the two subsystems also occurs. When the parameter is changed slightly, similar phenomena can also be obtained. The SNR of the receiver on the optimal combination changes with the variable parameter, implying that there may be an optimal value for the parameter. The ISR and coupling are significant to the internal signal processing, transferring, and maintaining in coupled systems, especially coupled biological systems, we expect that our work could be helpful to the relevant experiment or practical application.

IV. SUMMARY

The internal SR of two unidirectionally linearly coupled neurons is investigated when the first neuron is subject to environmental noise. We find that coupling is significant to the study of the dynamic behaviors in coupled systems. The ISR effect and the phenomenon of synchronization can be influenced when the system are in different coupling levels. As the coupling strength is in an appropriate level, the ISR performance of the receiver can be optimal, indicating the most effective information transmission between the two neural oscillators. If the coupling strength is enhanced above some level, the synchronization of the two subsystems could occur, and the information transmission among coupled subunits could not be improved. Most research on SR show the significance of noise to the enhancement of the transmission of information in neural system. The present work exhibits

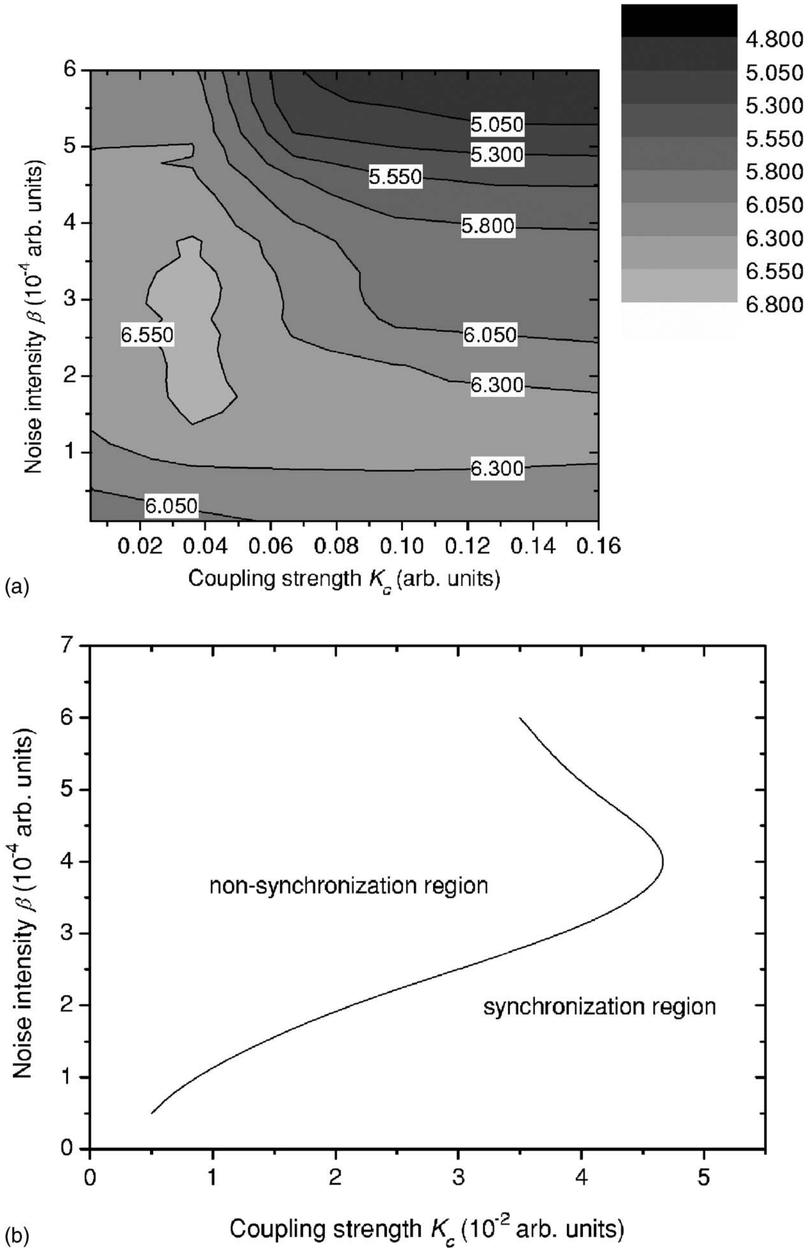

FIG. 8. The contour plot of SNR (10^4 arb. units) (a) and the synchronization diagram (b) for x_2 at $\varepsilon = 0.01$, a for the driver is 0.51, the receiver as 0.50.

that the coupling strength can be considered to be another tuning parameter of ISR in coupled systems. The amplification of ISR can be enhanced, suppressed, and sustained by modulating the coupling, i.e., the information transmission between coupled oscillators will experience an optimal amplification and reach the saturation eventually, which we call AESR without tuning. Furthermore, we also study the case the parameter is modulated slightly. The results imply that a certain value for the parameter may also exist where the internal information transmission could be optimized again on the combined noise and coupling. Since the internal information processing and transmission are essential in coupled biological systems, we expect our results could have relevant connections with various biological problems and have potential applicability in the study of the coupling role and ISR in many systems.

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