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Generalized model and optimum performance of an irreversible quantum Brayton engine with spin systems

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The purpose of this paper is to establish a model of an irreversible quantum Brayton engine using many noninteracting spin systems as the working substance and consisting of two irreversible adiabatic and two isomagnetic field processes. The time evolution of the total magnetic moment M is determined by solving the generalized quantum master equation of an open system in the Heisenberg picture. The time of two irreversible adiabatic processes is considered based on finite-rate evolution. The relationship between the power output P and the efficiency η for the irreversible quantum Brayton engine with spin systems is derived. The optimally operating region (or criteria) for the engine is determined. The influences of these important parameters on the performances (P and η) of the engine are discussed. The results obtained herein will be useful for the further understanding and the selection of the optimal operating conditions for an irreversible quantum Brayton engine with spin systems.

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I. INTRODUCTION

Since the 1970s, the investigation on the performance characteristics of thermodynamic processes and optimization of thermodynamic cycles has made tremendous progress from classical to quantum processes by scientists and engineers with finite time thermodynamics [1–7]. In recent years, the performance optimization of the quantum cycles (quantum engines, quantum refrigerators, and quantum heat pumps) has become one of the interesting research subjects for people working in thermodynamics and statistical physics. Many authors have studied the performance of the quantum engines based on the semigroup approach and the generalized quantum master equation [8–17]. The quantum degeneracy effect on the work from a quantum cycle with quantum ideal gases has been analyzed, too [18–20].

Similar to classical engines, quantum engines using harmonic oscillator or spin systems as the working fluid may have a set of typical cycle models. The quantum Brayton cycle is one of these models. Much work has been performed for the performance analysis and optimization of either classical [21–25] or quantum [26] Brayton engine. For an endoreversible quantum Brayton cycle with spin systems, the magnetic moment of the system in two adiabatic branches is a constant. However, for an irreversible quantum Brayton cycle, the magnetic moment of the system in two irreversible adiabatic branches is variable due to the increase of entropy generation. The time of two irreversible adiabatic processes must be considered due to finite-rate evolution.

In this paper, we discuss a technique that, we believe, offers several important advantages. The technique is called as the generalized quantum master equation. From a quantum Liouville's equation, it can describe the time evolution and relaxation of the density matrix in an open system. It can be used for nearly any open dynamical quantum system interacting with its environment without the need for detailed derivation in each case. Its physical picture on the link between the system and its environment is very clear. Finally, and most importantly, under some conditions, the dynamic parameters which describe the system can be obtained by solving the equation.

The thermodynamic parameters of the cycle are calculated by using phenomenological laws such as Newton's heat transfer law, radiation heat transfer law, nonlinear heat transfer law, and so on, in the classical analysis. It is calculated by solving the generalized quantum master equation of an open system in the Heisenberg picture for quantum analysis in this paper. The latter makes the quantum picture of the coupling action between the working fluid and the source (or sink) more clear and simple. The quantum analysis provides the application foundation of the nonequilibrium statistical mechanics to the practical engineering cycles.

Although there are a number of papers on the thermodynamic cycle in which quantum analysis has been used [8–17,26], in most of them, the description on the irreversibilities, the coupling action and the time evolution picture of dynamic operator in the system is abbreviated. In the present paper, we will solve the generalized quantum master equation to obtain the expression of time evolution on the cycle. A cycle model of an irreversible quantum Brayton engine with spin systems (IQBESS) is used, which consists of two irreversible adiabatic and two isomagnetic field branches. For the previous published models, two adiabatic branches are isentropic adiabatic branches, the heat leakage between the source and the sink, and coupling between the orbital angular momentum $\hat{L}$ and the spin angular momentum $\hat{S}$ are

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not considered. For the present cycle model, two adiabatic branches are nonisentropic adiabatic branches, the heat leakage is considered and $\hat{L}$-$\hat{S}$ coupling is adopted. The general optimal relationship between the power output and thermal efficiency is derived using the generalized quantum master equation in an open system. The important characteristic parameters such as the power output, the efficiency and the magnetic moment are optimized. The optimization zone on the performance of the engine is determined in this paper. Planck's constant $h=6.63\times10^{-34}$ Js and $\hbar=h/(2\pi)=1.05\times10^{-34}$ Js in the international system, but $\hbar=1$ is set in the quantum calculation of the paper for simplicity. (It is equivalent to use the different unit system.)

II. IRREVERSIBLE QUANTUM BRAYTON ENGINE WITH SPIN SYSTEMS

Consider an irreversible closed quantum Brayton engine cycle with spin systems consisting of two isomagnetic field branches connected by two irreversible adiabatic branches as mentioned above. All four branches of the cycle can be described by quantum equations of motion. The engine operating between two heat reservoirs at constant temperatures T_h and T_c. In two isomagnetic field branches $B=B_1$ and $B=B_2$ with $B_1>B_2$, the spin systems used as the working substance, which obey Fermi-Dirac statistics, are not only coupled mechanically with the given "magnetic field" $B(t)$, but also with a heat source or a heat sink at temperature T. The source and sink are infinitely large and their internal relaxation is very strong. Therefore, the source and the sink are assumed to be in thermal equilibrium. There is heat leakage Q_r between the source and the sink. It is the result of the coupling action between the source and the sink by the medium of the engine. The rate of heat leakage $\dot{Q}_r$ between the source and the sink is assumed to be a constant for simplicity.

The working substance is an ideal paramagnetic material. One expresses mechanical quantities by operators in the quantum mechanics [27,28]. $\hat{L}$ and $\hat{S}$ denote the orbital angular momentum and the spin angular momentum of the atom in this paper. The total angular momentum $\hat{J}$ can be expressed as $\hat{J}=\hat{L}+\hat{S}$ when $\hat{L}$-$\hat{S}$ coupling is adopted. The Hamiltonian of the interaction between a magnetic moment $\hat{M}$ and an external magnetic field is given by [27]

$$\hat{H}_b=-\hat{M}\cdot\vec{B}, \tag{1}$$

where the magnitude of the magnetic field can change over time, but is not allowed to reach zero. The total magnetic moment operator $\hat{M}$ of the system is defined as

$$\hat{M}=-\frac{ne}{2m(4\pi\epsilon_0)^{0.5}}(\hat{L}+2\hat{S}), \tag{2}$$

where n, m, ϵ_0, and e are the density of atom number, the reduced mass of the atom, the vacuum dielectric constant, and the electronic electric quantity, respectively. Assuming the magnetic induction $\vec{B}$ is aligned with the z axis and ne-

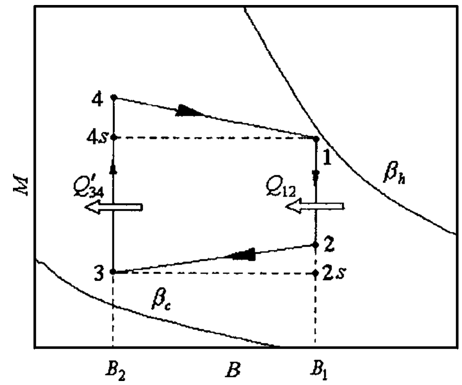

FIG. 1. M-B diagram of an irreversible quantum Brayton engine cycle.

glecting the relativistic correction, the disturbing Hamiltonian of the system is given by

$$\hat{H}=\eta(r)\hat{S}\cdot\hat{L}+\frac{ne}{2m(4\pi\epsilon_0)^{0.5}}(\hat{L}+2\hat{S})\cdot B \tag{3}$$

with

$$\eta(r)=\frac{1}{2\mu^2C^2r}\frac{dV}{dr},$$

where V=V(r) is the potential function, μ the magnetic permeability, and C the Curie's constant. If the external magnetic field is strong, the first term in equation (3) can be neglected. Thus, one has

$$\hat{H}=-\frac{ne}{2m(4\pi\epsilon_0)^{0.5}}(\hat{L}+2\hat{S})\cdot B=-\hat{M}_z\cdot B \tag{4}$$

with

$$\hat{M}_z=-\frac{ne}{2m(4\pi\epsilon_0)^{0.5}}(\hat{L}_z+2\hat{S}_z)=-E(\hat{L}_z+2\hat{S}_z), \tag{5}$$

where $E=ne\big/2m(4\pi\epsilon_0)^{0.5}$ is constant.

Figure 1 shows schematically the M-B diagram for an IQBESS cycle. Processes 2-3 and 4-1 are nonisentropic adiabatic branches, and processes 1-2 and 3-4 are isomagnetic field branches. Along processes 2-3 and 4-1, the working medium is coupled with the external magnetic field $B(t)$, but is not coupled with the external heat baths. Along processes 1-2 and 3-4, the working medium is coupled with both the heat baths and the constant external magnetic field B (B_1 and B_2). Processes 2s-3 and 4s-1 are isentropic adiabatic branches. Hence, cycle 1-2-3-4 is an irreversible one, and cycle l-2s-3-4s is an endoreversible one. The irreversible and endoreversible cycles are distinguished by using the factors ϕ_1 and ϕ_2 called the factor of internal irreversible degree in the adiabatic processes. The factors ϕ_1 and ϕ_2 are defined as

$$M_2=\phi_1M_{2s}=\phi_1M_3 \tag{6}$$

$$M_4=\phi_2M_{4s}=\phi_2M_1=\phi_2yM_3 \tag{7}$$

with $y=M_1/M_3>1$. Where M_1, M_2, M_{2s}, M_3, M_{4s}, and M_4 with $M_i=\langle\hat{M}_i\rangle$ ($i=1,2,2s,3,4s$, and 4) are magnetic moments of the system at state 1, 2, 2s, 3, 4s, and 4, respectively. It is seen from Fig. 1 that $M_{2s}=M_3$ and $M_{4s}=M_1$. The factors ϕ_1 and ϕ_2 denote the internal irreversibilities in the irreversible adiabatic branches, which is the result of miscellaneous factors such as friction and nonequilibrium activities inside the engine. They generate the heat and make the internal temperature in the irreversible adiabatic branches increase over endoreversible cycle. It results in magnetic moment and entropy of the system decreasing. So $\phi_1>1$ and $\phi_2>1$ correspond to irreversible adiabatic branches and $\phi_1=\phi_2=1$ corresponds to endoreversible adiabatic branches.

The internal temperatures of the working substance at state 1, 2, 2s, 3, 4s, and 4 are, respectively, T_1, T_2, T_{2s}, T_3, T_{4s} and T_4. The second law of thermodynamics requires $\beta_c>\beta_3>\beta_1>\beta_h$, $M_2>M_{2s}$, and $M_4>M_{4s}$, where $\beta_i=1/T_i$ ($i=h,c,1,2,2s,3,4s$, and 4).

The model mentioned above is similar to a generalized irreversible Carnot engine cycle model with the losses of heat resistance, heat leakage and internal irreversibility [29,30].

III. CYCLE PERIOD TIME

Defining non-Hermitian operators as [27,28]

$$\hat{J}_+=\hat{J}_x+i\hat{J}_y, \tag{8}$$

$$\hat{J}_-=\hat{J}_x-i\hat{J}_y, \tag{9}$$

where $i=\sqrt{-1}$ is the imaginary unit. The components $(\hat{J}_x,\hat{J}_y,\hat{J}_z)$ of the total angular momentum operator $\hat{J}(\hat{J}=\hat{L}+\hat{S})$ obey the commutation relation in the following form:

$$[\hat{J}_j,\hat{J}_k]=i\hat{J}_m, \tag{10}$$

where [,] is the quantum Poisson's bracket and j, k, m are the cycle permutation of x, y, z.

In two isomagnetic field processes, the working fluid system S becomes an open system owing to the coupling action between S and the heat reservoirs. Using the generalized quantum master equation of an open system, and in the Heisenberg picture, the motion equation of the operator $\hat{x}$ in the system S is obtained [11–13]

$$\frac{d\hat{x}}{dt}=i(\hat{L}_s+T_{rB}\hat{L}_{sB}\hat{\rho}_B)\hat{x}-\sum_{jk}\{\omega_{kj}^{+}[\hat{x},\hat{Q}_k]\hat{Q}_j-\omega_{jk}^{-}\hat{Q}_j[\hat{x},\hat{Q}_k]\}, \tag{11}$$

where $\hat{L}_s$ is the Liouville's operator of the system S, T_{rB} represents the trace of the operator of the heat source and the heat sink; $\hat{L}_{sB}$ represents the interaction between the working substance system S and the source or the sink; $\hat{\rho}_B$ is the density matrix of the source or the sink; ω_{kj}^{+} and ω_{jk}^{-} are constants related to the matrix elements of the density matrix $\hat{\rho}_B$; $\hat{Q}_k$ and $\hat{Q}_j$ are the operators of the working substance system S.

Setting

$$i(\hat{L}_s+T_{rB}\hat{L}_{sB}\hat{\rho}_B)\hat{x}=-\phi\sum_{jk}\{\omega_{kj}^{+}[\hat{x},\hat{Q}_k]\hat{Q}_j-\omega_{jk}^{-}\hat{Q}_J[\hat{x},\hat{Q}_k]\}, \tag{12}$$

where ϕ is the coefficient of the disturbance. Combining equations (11) and (2) yields

$$\frac{d\hat{x}}{dt}=-(1+\phi)\sum_{jk}\{\omega_{kj}^{+}[\hat{x},\hat{Q}_k]\hat{Q}_j-\omega_{jk}^{-}\hat{Q}_j[\hat{x},\hat{Q}_k]\}. \tag{13}$$

Solving equation (13) by setting $\hat{x}=\hat{M}_z$, $\hat{Q}_1=\hat{J}_+,\hat{Q}_2=\hat{J}_-$ gives $\dot{M}=\frac{d\langle\hat{M}_z\rangle}{dt}$ in following form:

$$\dot{M}=-(\gamma_1+\gamma_2)M-(\gamma_1-\gamma_2)[(l^2+l)-n_1+2(s^2+s-n_2)], \tag{14}$$

where l is the quantum number of an orbital angular momentum, s is the quantum number of a spin angular momentum, $n_1=\langle\hat{L}_z^2\rangle$, $n^2=\langle\hat{S}_z^2\rangle$, both $\gamma_1=E(\omega_{12}^{+}-\omega_{12}^{-})$ and $\gamma_2=E(\omega_{21}^{+}-\omega_{21}^{-})$ are constants related to the magnetic induction B, $\langle\rangle$ indicates the statistical average of the operator in the bracket. The crossed terms between $\hat{L}_z$ and $\hat{S}_j(j=x,y)$ are neglected in equation (14).

According to the molecular field theory, the magnetic moment is [27,28]

$$M=ng\mu_bJB_J\left[\frac{g\mu_bJ(B+\lambda M)}{kT}\right], \tag{15}$$

where g, μ_B, and λ are the Lande's g factor, the Bohr magneton, and molecular field coefficient, respectively, and $B_J=B_J(z)$ is the Brillouin function. For paramagnetic substance, $\lambda\to0$, at the limit $z=\frac{g\mu_bJB}{kT}\ll1$, equation (15) becomes Curie's law

$$M=\frac{CB\beta}{\mu} \tag{16}$$

with C being Curie's constant.

From equations (14) and (16), one finds

$$\dot{M}=2f_1(M-M_{e1})\quad(\text{Process 1-2}), \tag{17}$$

$$\dot{M}=2f_2(M-M_{e2})\quad(\text{Process 2-3}), \tag{18}$$

$$\dot{M}=2f_3(M-M_{e3})\quad(\text{Process 3-4}), \tag{19}$$

$$\dot{M}=2f_4(M-M_{e4})\quad(\text{Process 4-1}), \tag{20}$$

where

$$f_i=-\frac{(\gamma_{1i}-\gamma_{2i})}{2}=\frac{-(\gamma_{1i}-\gamma_{2i})[(l^2+l)-n_1+2(s^2+s-n_2)]}{2M_{ei}}\quad(i=1,2,3,4) \tag{21}$$

is a constant which is independent the magnetic moment but dependent on the quantum performance of the working fluid, $M_{e1}=\frac{C\beta_c B_1}{\mu}$, $M_{e2}=\frac{C\beta_c B_c}{\mu}$, $M_{e3}=\frac{C\beta_h B_2}{\mu}$, and $M_{e4}=C\beta_h B_h/\mu$ are, respectively, the asymptotic stationary value of the magnetic moment M in processes 1-2, 2-3, 3-4, and 4-1 when $t\to\infty$, B_h and B_c are, respectively, the asymptotic stationary value of the magnetic induction B in processes 4-1 and 2-3 when $t\to\infty$.

It is evident that the expression of time evolution can be given by

$$t=\int_{M_i}^{M_j}\frac{dM}{\dot{M}}, \tag{22}$$

where M_i and M_j are initial and final values of M along a given path $M(\beta,B)$. Comparing equations (17)–(20) and (22) gives

$$t_1=\frac{1}{2f_1}\ln\frac{\phi_1 M_3-M_{e1}}{yM_3-M_{e1}} \quad \text{(Process 1-2)}, \tag{23}$$

$$t_2=\frac{1}{2f_2}\ln\frac{M_3-M_{e2}}{\phi_1 M_3-M_{e2}} \quad \text{(Process 2-3)}, \tag{24}$$

$$t_3=\frac{1}{2f_3}\ln\frac{\phi_2 yM_3-M_{e3}}{M_3-M_{e3}} \quad \text{(Process 3-4)}, \tag{25}$$

$$t_4=\frac{1}{2f_4}\ln\frac{yM_3-M_{e4}}{\phi_2 yM_3-M_{e4}} \quad \text{(Process 4-1)}. \tag{26}$$

The cycle period time, $t=t_1+t_2+t_3+t_4$ is

$$t=\frac{1}{2f_1}\left[\ln\left(\frac{\phi_1 M_3-M_{e1}}{yM_3-M_{e1}}\right)+a_2\ln\left(\frac{M_3-M_{e2}}{\phi_1 M_3-M_{e2}}\right)+a_3\ln\left(\frac{\phi_2 yM_3-M_{e3}}{M_3-M_{e3}}\right)+a_4\ln\left(\frac{yM_3-M_{e4}}{\phi_2 yM_3-M_{e4}}\right)\right], \tag{27}$$

where $a_2=f_1/f_2$, $a_3=f_1/f_3$, and $a_4=f_1/f_4$. It may be seen from equation (27) that $t=t(M_3)$ is related to M_3.

IV. OPTIMAL POWER OUTPUT AND EFFICIENCY OF IQBESS

Based on quantum mechanics theory [27,28] and equation (4), the energy of the working substance is

$$U=\langle\hat{H}\rangle=\langle-\hat{M}_z B\rangle=-MB \tag{28}$$

and its differential form is

$$dU=-MdB-BdM. \tag{29}$$

Comparing equation (29) with differential form of the first law of thermodynamics, one can obtain the instantaneous heat flow dQ and the work dW in the following equations:

$$dQ=-BdM, \tag{30}$$

$$dW=MdB. \tag{31}$$

For processes 1-2 and 3-4, integrating equation (30) yields

$$Q_{12}=-\int_{M_1}^{M_2}BdM=-\int_{M_1}^{M_2}B_1 dM=B_1(M_1-M_2)=xB_2(y-\phi_1)M_3 \tag{32}$$

$$Q'_{34}=|Q_{34}|=\left|-\int_{M_3}^{M_4}BdM\right|=\left|-\int_{M_3}^{M_4}B_2 dM\right|=B_2(y\phi_2-1)M_3, \tag{33}$$

where $x=B_1/B_2$, Q_{12}, and Q'_{34} are the heat exchange quantities corresponding to the isomagnetic field processes 1-2 and 3-4, respectively.

The real heat quantity Q_h supplied by the heat source and the real heat quantity Q_c released to the heat sink are, respectively,

$$Q_h=Q_{12}+Q_r=xB_2M_3(y-\phi_1)+\dot{Q}_r t, \tag{34}$$

$$Q_c=Q'_{34}+Q_r=B_2M_3(y\phi_2-1)+\dot{Q}_r t. \tag{35}$$

The first law of thermodynamics gives that the power output P and efficiency η of the engine can be obtained in the following equations:

$$P=\frac{Q_h-Q_c}{t}=\frac{x(y-\phi_1)-(y\phi_2-1)}{\tau(M_3)} \tag{36}$$

$$\eta=\frac{Q_h-Q_c}{Q_h}=\frac{x(y-\phi_1)-(y\phi_2-1)}{x(y-\phi_1)+\dot{Q}_r\tau(M_3)} \tag{37}$$

with

$$\tau(M_3)=\frac{t(M_3)}{M_3B_2}.$$

It is clearly seen from equations (36) and (37) that both power output P and efficiency η of the engine are functions of M_3 for given parameters f_i, β_h, β_c, B_h, B_c, x, B_2, ϕ_1, ϕ_2, and $\dot{Q}_r$. The smaller the $\tau(M_3)$ is, the larger the power output P and efficiency η are. It implies that the power output P and the efficiency η are of the same extreme point. Based on the above equations, one can optimize these important performance parameters of the IQBESS. Taking the derivatives of P and η with respect to M_3 and setting them equal zero $\left(\partial P/\partial M_3=0 \text{ or } \partial\eta/\partial M_3=0 \text{ or } \partial\tau(M_3)/\partial M_3=0\right)$, one can find that when M_3 satisfies the following equation

$$M_{30}\left(\frac{\partial t}{\partial M_3}\right)\bigg|_{M_3=M_{30}}=t(M_{30}) \tag{38}$$

both the power output P and efficiency η approach optimal values

$$P=\frac{x(y-\phi_1)-(y\phi_2-1)}{\tau(M_{30})}, \tag{39}$$

$$\eta=\frac{x(y-\phi_1)-(y\phi_2-1)}{x(y-\phi_1)+\dot{Q}_r\tau(M_{30})}. \tag{40}$$

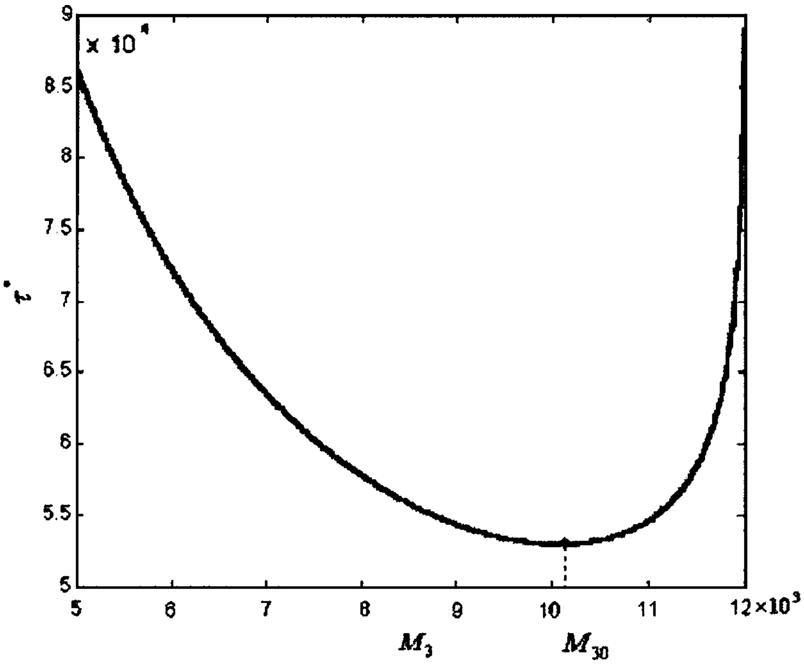

FIG. 2. The τ^*-M_3 characteristic with the minimum $\tau^*(M_{30})$ at extreme point M_{30}.

The parameter equation defined by equations (39) and (40) with parameter y gives the fundamental relationship between the optimal power output P and efficiency η for the given f_i, β_h, β_c, B_h, B_c, x, B_2, ϕ_1, ϕ_2, and $\dot{Q}_r$. It is the main result of this paper.

In equations (39) and (40), the constants f_i and M_e are functions of the ω_{kj}^+ and ω_{jk}^- related to the matrix elements of the density matrix $\hat{\rho}_B$. They reveal the influence of the quantum performance of the working fluid. Therefore, under a given working condition, choosing a suitable quantum working fluid to enlarge the constant f_i is very helpful for improving the thermodynamic and economic performance of the engine.

The method of this paper is still applicable to a refrigeration of ultracold system with the temperature near absolute zero. For an ultracold system, equation (16) would be replaced by equation (15). In that case, the optimal result will be very complicated.

V. DISCUSSION

The τ^* versus M_3 characteristic with $\beta_h=0.002$, $\beta_c=0.04$, $B_h=3$, $B_c=0.01$, $x=5$, $B_2=0.02$, $a_i=1$, and $\phi_1=\phi_2=1.1$ is shown in Fig. 2, where $\tau^*=2f_1B_2\tau$. It can be seen from the curve in Fig. 2 that there exists a minimum $\tau^*(M_{30})$ at extreme point M_{30} for a set of given parameters. It implies that there exist the maximum power output P and the maximum efficiency η at point M_{30} for a set of given parameters.

The optimal power output P versus efficiency η characteristics are dependent on $\dot{Q}_r$, ϕ_1 and ϕ_2 for the given f_i, β_h, β_c, B_h, B_c, x, and B_2. The larger the parameters $\dot{Q}_r$, ϕ_1, and ϕ_2 are, the smaller the power output P and efficiency η are. The dimensionless power output $P^*(P^*=p/2fB_2)$versus efficiency η curves with $\dot{Q}_r^*=1\times10^4$ $\left(\dot{Q}_r^*=\dot{Q}_r/2fB_2\right)$, $\beta_h=0.002$, $\beta_c=0.04$, $B_2=0.02$, $a_i=1$, and $\phi_1=\phi_2=1.1$ are plotted by using equations (38)–(40), as shown in Fig. 3 and 4.

It is seen from the curves in Fig. 3 and Fig. 4 that both dimensionless power output P^* and efficiency η of IQBRSS increase with the increasing of B_2 and x. It shows that the magnetic field affects strongly the performance of IQBESS. From Fig. 3 and Fig. 4, one can see clearly that there exists a maximum power output $P=P_m$ corresponding to $\eta=\eta_0$ and a maximum efficiency $\eta=\eta_{\max}$ corresponding to $P=P_0$. Obviously, for different given parameters, the maximum power output and the maximum efficiency will be different.

Equation (21) shows that the parameter f_i, which is dependent on l,s,n_1, and n_2, represents the influence of the L-S coupling. Equations (39) and (40) indicate that the power output P and efficiency η are monotonically increasing func-

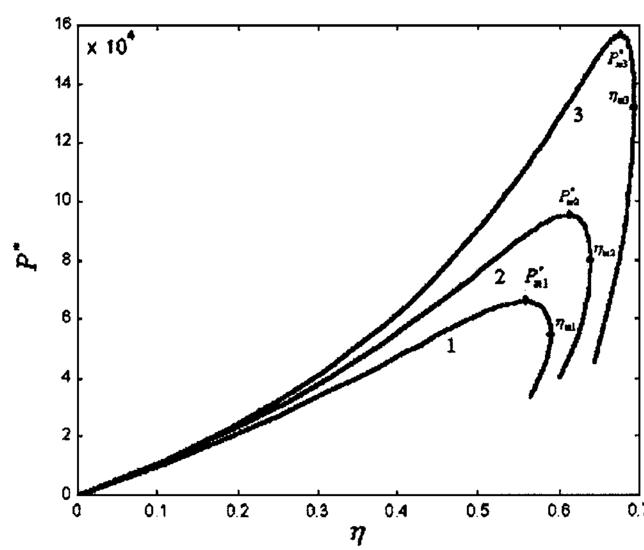

FIG. 3. The dimensionless power output P^* versus efficiency η with $x=5$. Curve 1 ($B_2=0.018$), Curve 2 ($B_2=0.02$), and Curve 3 ($B_2=0.025$) are presented.

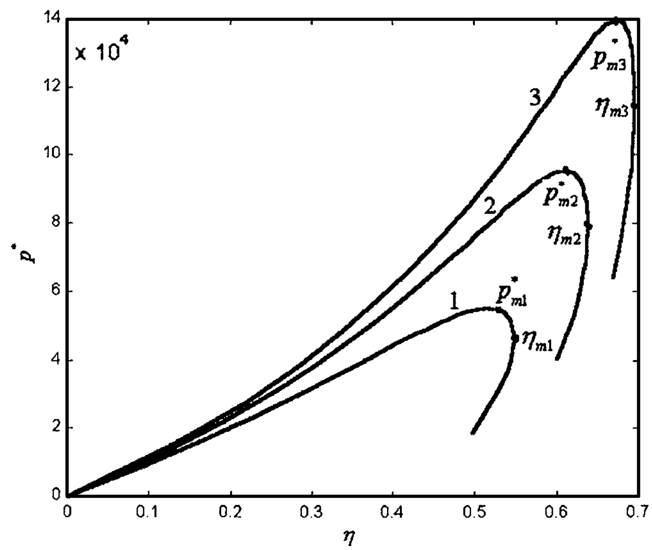

FIG. 4. The dimensionless power output P^* versus efficiency η with $B_2=0.02$. Curve 1 ($x=4.5$), Curve 2 ($x=5$), and Curve 3 ($x=5.5$) are presented.

tions of the parameters f_i, or the larger the parameter f_i is, the larger the power output P and efficiency η is. The *L-S* coupling effects strongly the optimized results of the cycle. It quantitatively affects the values of the power output P and efficiency η, but does not affect the form of P-η curves. The rate of heat leakage $\dot{Q}_r$ affects the form of curves. When $\dot{Q}_r=0$, the curve is a hyperbola, and when $\dot{Q}_r \neq 0$, the curve is a loop-shaped one.

The optimization criteria of IQBESS can been obtained from parameters P_m, η_0, η_m, and P_0, as follows:

$$P_0 \leqslant P \leqslant P_m, \eta_0 \leqslant \eta \leqslant \eta_m. \tag{41}$$

VI. CONCLUSION

In this paper, a model of an irreversible quantum Brayton engine using many noninteracting spin systems as the working substance and consisting of two irreversible adiabatic and two isomagnetic field processes is established. For the new model, the heat leakage and internal irreversibilities in two adiabatic processes is considered and the $\hat{L}$-$\hat{S}$ coupling is adopted. Based on the quantum statistical mechanics, the time evolution of the total magnetic moment M is determined by solving the generalized quantum master equation of an open system in the Heisenberg picture. The general expressions of the performance parameters of the engine are derived. The relationship between the power output P and efficiency η for the IQBESS is obtained. By using a numerical example, the performance of the IQBESS is optimized for a set of given parameters. The optimally operating region (or criteria) for IQBESS is determined in the paper. The influences of the quantum characteristic of the working fluid (f), heat leakage ($\dot{Q}_r$), the factor of internal irreversible degree in the adiabatic processes (ϕ_1, ϕ_2), and the magnetic field (B_2, x) on the performances (P and η) of the IQBESS are discussed. The analysis provides a new theoretical basis for the performance evaluation and improvement of practice IQBESS.

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