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Intrinsically anomalous roughness of admissible crack traces in concrete

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We study the roughness of postmortem cracks in concrete plates of different size. We find that the set of admissible crack paths exhibits an intrinsically anomalous roughness; nevertheless, any individual crack trace in concrete is essentially self-affine. We also find that both the local and the global amplitudes of crack traces are distributed according to a log-logistic distribution characterized by the same scaling exponent, whereas the mean-square width distribution is best fitted by the Pearson distribution, while the log-normal distribution also provides quite good adjustments and cannot be clearly rejected.

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One of the most challenging puzzles in statistical physics and materials science is the fracture phenomena [1,2]. Fracture processes are characterized by the high extent of spatiotemporal nonuniformity which results in the complex morphology of fracture surfaces and crack shapes [2,3]. Since the work of Mandelbrot *et al.* [4], many works have been dedicated to the characterization of the scaling properties of fracture patterns [5–13]. Furthermore, it was shown that crack roughness essentially affects the fracture mechanics [2,5,12,14,15].

Numerous experiments have shown that cracks exhibit self-affine scale invariance within a considerable range of length scales [5–8] up to five decades [6]. Namely, the trajectory of a crack, $z(x)$, is invariant under an anisotropic scale transformation in the sense that the rescaled trace $\lambda^{-\zeta}z(\lambda x)$ has the same statistical properties as $z(x)$, where $\lambda > 0$ is the scale factor and ζ is the (local) roughness exponent [16]. This implies that the local crack width, $w(l) = \langle\langle[z(x)-\langle z\rangle_l]^2\rangle_L\rangle_R^{1/2}$, scales with the apparent crack length, ℓ, as

$$w \propto l^{\zeta} \quad \text{for } l_0 < l < \xi_x, \tag{1}$$

where $\langle\cdots\rangle_l$ denotes the average over x in window of size $\Delta x = l$, $\langle\cdots\rangle_R$ denotes the average over different realizations, l_0 is a microscopic cutoff, and ξ_x is the horizontal correlation length, defined as $w(l>\xi_x) \propto \xi_x^{\zeta}$. Furthermore, the structure factor of self-affine trace also exhibits scaling behavior, i.e., $S(k) = \langle Z(k)Z(-k)\rangle_K \propto k^{-\theta}$, where $\theta = 2\zeta + 1$. Here $Z(k)$ is the Fourier transform of $z(x)$ and $\langle\cdots\rangle_K$ denotes the average over the interval $k \in [2\pi/\xi_x, 2\pi/l_0]$. It should be noted that for truly self-affine fractals, $l_0 = 0$ and $\xi_x = L$, where L is the system size, and so, $W(L) = w(l=L) \propto L^{\zeta}$ [16]. Other important characteristics of crack roughness are the statistical distributions of local width [17] and amplitudes $\Delta Z_m(l,L) = \max_{(0\leq x\leq l)\in L} z(x) - \min_{(0\leq x\leq l)\in L} z(x)$ [18]. It has been argued that the shape of distribution of the mean-square width can be used to distinguish between different universality classes of kinetic roughening [17,19,20]. Additionally, crack roughness can be characterized by the moments of q-order height-height correlation function $\sigma_q(l) = \langle |z(x+l)-z(x)|^q\rangle_L^{1/q} \propto l^{\zeta_q}$ [16]. For self-affine cracks $\zeta_q = \zeta$ for all q. However, in some cases the crack roughness is characterized by a nontrivial spectrum of scaling exponents $\zeta_q = f(q)$, such that $\zeta_2 = \zeta$ [21].

Self-affine roughness is commonly associated with the kinetic roughening obeying the Family–Vicsek dynamic scaling ansatz

$$W(L,t) = L^{\alpha} f(t^{1/z}/L), \tag{2}$$

where the scaling function $f(y)$ behaves as $f \propto y^{\alpha}$ with $\alpha = \zeta$, when $y \ll 1$ and $f(y)$ is a constant for $y \gg 1$. Accordingly, at the early times, $t \ll L^z$, the horizontal correlation length and global width of growing interface increase with time as $\xi_x \propto t^{1/z}$ and $W \propto t^{\alpha/z}$, respectively, where z is the dynamic exponent and $\beta = \alpha/z$ is the so-called growth exponent [16]. The roughness exponent $\alpha = \zeta$ and the dynamic exponent z characterize the universality class of the model under study [16].

However, in many cases the interface roughening displays an anomalous scaling behavior, characterized by different roughness exponents in the local and global scales; namely $\alpha > \zeta$ [10–13,22]. Accordingly, the local fluctuations of the growing interface behave as

$$w(l \ll \xi_x, t \ll L^z) \propto l^{\zeta} t^{(\alpha-\zeta)/z} \quad \text{and } w(t \gg L^z, l_x) \propto L^{\alpha-\zeta} l_x^{\zeta}; \tag{3}$$

nevertheless the global width, $W(L,t)$, exhibits the Family–Vicsek scaling (2). Moreover, in the case of so-called superroughening, the structure factor also satisfies the Family–Vicsek ansatz with scaling exponent $\theta = 2\alpha + 1$, nevertheless, generally,

$$S \propto L^{(\alpha-\alpha_s)} k^{-(2\alpha_s+1)}, \tag{4}$$

where α_s is the spectral exponent [22]. Specifically, the intrinsically anomalous kinetic roughening is characterized by $\alpha_s = \zeta < \alpha$ [22,23]. It is pertinent to note that in this case the saturated interface exhibits self-affine invariance, so its "anomalous nature" can be detected only through the study of roughening dynamics (3) or by using test specimens of different sizes [24].

Unfortunately, in most experimental works devoted to the scaling analysis of fractures, only the local roughness of cracks was measured using the "postmortem" fractures in test specimens of standard dimensions [4–7]. So the experimental data reported in these works do not permit to distinguish between the self-affine and the intrinsically anomalous nature of crack roughening. More recently, the intrinsically anomalous roughening of cracks in quasibrittle materials was observed in experiments with woods, some kinds of paper, and mortar [10–13]. Intrinsically anomalous roughening was also observed in some numerical simulations of crack growth [25]. It was found that the global crack roughness exponent is material dependent [11–13]. With respect to the local roughness, in a number of works the local roughness exponent was conjectured to be universal, i.e., independent of the material, mechanism of fracture, and of the fracture mode [5]. Namely, it is suggested that one-dimensional (1D) cracks are characterized by $\zeta \cong 0.6$, whereas two-dimensional (2D) cracks are characterized by $\zeta \cong 0.8$ [5,6]. The universality of ζ, however, is still controversial (see Refs. [7–9,26], and references therein). There are strong experimental evidence and theoretical reasons that, at least in materials with long-range correlations in microstructure, the value of ζ is determined by the scaling properties of the material structure [8,27]. Moreover, the authors of [9] have detected a dependence of ζ on the mechanism of fracture. The dependence of ζ on the crack orientation in anisotropic materials was observed in [26]. In this respect, the authors of [28] noted that ζ depends on the strength of disorder in fractured media and the universal exponent emerges as disorder is increased [28].

In this work we studied the statistical properties of the postmortem cracks in fractured concrete slabs covering pedestrian paths in our university. The pavement was installed about three years ago. A sampling analysis of concrete has shown that the aggregate/cement ratio (by weight) is within the range of $2 \div 4$; the grain size of gravel and sands varies from 2 to 16 mm, the density of concrete is 2300 ± 500 kg/m^3, and the compressive strength is 45 ± 15 MPa.

Concrete is a quasibrittle material, and the nature of its fracture behavior continues to be the subject of intensive research [13,29,30]. A common feature of fracture in concrete is the presence of damage or process zone, which grows near the crack tip. The size of fracture process zone (its width and length) depends on both the concrete structure and the stress field, and commonly it is much larger than the typical grain size [31]. It was found that crack trajectories in concrete possess a self-affine invariance over a wide range of length scales [29,30]. Furthermore, recently it was found that the front of slowly growing crack in notched mortar beam subjected to four points bending displays intrinsically anomalous dynamic scaling [13].

To detect an anomalous roughness, in this work we analyzed the statistical properties of rupture lines in slabs of different sizes $L \times L$ (see Fig. 1) from different pedestrian paths. Specifically, we studied the slabs with linear sizes $L = 10$, 25, 50, 100, 200, and 300 cm. The ratio of thickness (d) to width (L) for all slabs is in the range of $0.02 \leqslant d/L < 0.1$. Furthermore, we note that in all cases the size of damage zone is larger than d. So, we assumed that the cracks may be treated as 1D traces. At least $N_L = 50$ specimens of each size were analyzed. We are not able to define specific fracture modes and loads associated with each crack. The postmortem analysis shows that cracks mainly propagate at the grain-matrix interface. The photoimage of each crack trace was

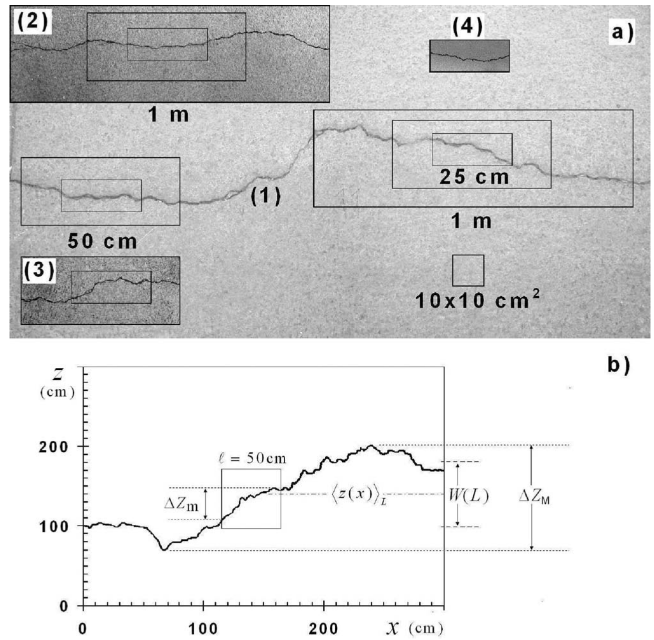

FIG. 1. (a) Photographs of cracks in the concrete slabs of different size $L=2$ (1), 1 (2), 0.5 (3), and 0.25 m (4); and (b) digitized graph of crack trace in the plate of size $L=3$ m.

digitized with the resolution of 1 mm/pixel [see Fig. 1(a)].

Figures 2 and 3 show the scaling behavior of the crack width, amplitude, and structure factor for specimens of different size, from which follows that the ensemble of postmortem cracks in concrete exhibit an intrinsically anomalous roughness characterized by

$$\zeta_q = \zeta = \alpha_S = 0.75 \pm 0.02 < \alpha = 1.35 \pm 0.02 \qquad (5)$$

for $1 \leqslant q \leqslant 5$; e.g., multiscaling was not detected. The microscopic cutoff of observed scaling is of the order of the typical grain size, $l_0 \cong 1$ cm and $\xi_x \cong L$ (see Figs. 2 and 3).

It should be pointed out that the values of local roughness exponents obtained for 320 crack traces are normally distributed with the standard deviation of 0.03; nevertheless, the large variations in loads and environmental conditions associated with studied cracks [32]. Furthermore, we note that the values of both roughness exponents coincide with those obtained for crack fronts in three-dimensional (3D) mortar beams [13], nevertheless the differences in the material structure and the fracture mode. At first glance, this result may be interpreted as support for the hypothesis of scaling universality, e.g., $\zeta = 0.8 \pm 0.05$ for 2D cracks [5,13], although the coincidence may be accidental [33]. With respect to this point, we note that experimental value $\zeta = 0.75 \pm 0.02$ agrees with the result of simulations of the two-dimensional fuse model [34], as well as with the predictions of the correlated percolation theory for 1D cracks [35]; whereas in three dimensions the fuse [36], as well as the beam lattice [37] models and the correlated percolation theory [35], predict $\zeta \cong 0.6$.

The scale-free nature of the crack roughness implies that crack growth is essentially probabilistic in nature [15]. This means that in the test specimen, under given conditions, ex-

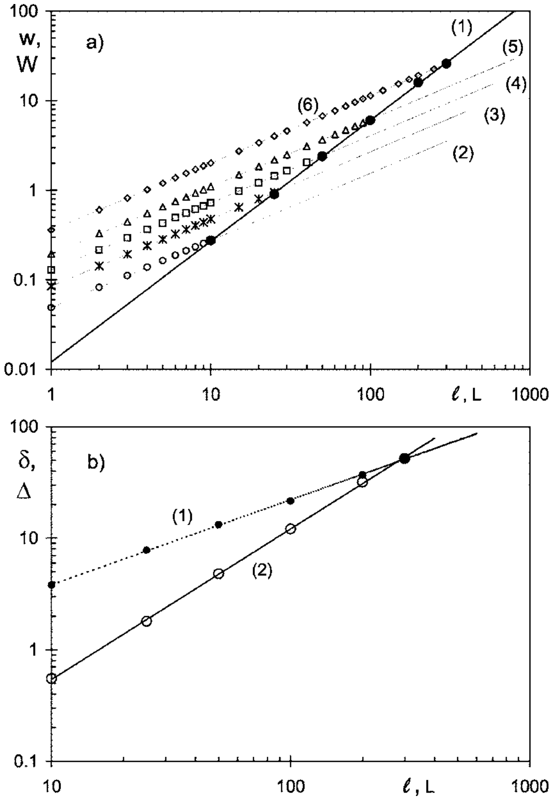

FIG. 2. (a) Log-log plots of the global (1) and the local (2–6) crack width vs l(cm) in concrete plates of size $L=10$ (2), 25 (3), 50 (4), 100 (5), and 300 cm (6); solid line slope $\alpha=1.35$, dotted lines slope $\zeta=0.75$. (b) Log-log plots of $\delta_L(l)=\langle\langle\Delta Z_m(l,L)\rangle_L\rangle_N$ vs l(cm) in plates of size $L=300$ cm (1) and $\Delta(L)=\langle\Delta Z_M(L)\rangle_N$ vs specimen size (2); solid line slope $\alpha=1.35$, dotted line slope $\zeta=0.76$.

ists a set of admissible crack paths characterized by the same local roughness exponent (see [15,38]). The statistical properties of this set can be determined from the analysis of crack traces in macroscopically identical specimens [15,18]. The ensemble of postmortem crack traces can be characterized by statistical distributions of the mean-square width $W(L)$ [17] and the global amplitudes $\Delta Z_M(L)=\Delta Z_m(l=L)$ [18].

Accordingly, we found that the distribution of the mean-square crack width is best fitted [39] by the Pearson distribution [see Fig. 4(a)],

$$f(w_L^2)=\frac{\exp(-k/y)}{k\Gamma(k+1)\langle w_L^2\rangle}\left(\frac{y}{k}\right)^{k+2}, \quad \text{where } y=\frac{w_L^2}{\langle w_L^2\rangle} \qquad (6)$$

and $\Gamma(\cdots)$ is the γ function. However, the log-normal distribution also provides quite good adjustments (p value $=0.3946$) and cannot be clearly rejected in terms of χ^2 and Kolmogorov-Smirnov statistics. Notice that both distributions have a tail heavier than exponential, but lighter than power-law distributions, as expected for self-affine traces [17,19].

At the same time, the local and the global amplitudes of cracks are more likely [39] to follow a log-logistic distribution [see Figs. 4(b) and 4(c)],

$$f(y)=my^{m-1}/Z^*[1+y^m]^2, \quad \text{where } y=(\Delta Z-z^*)/Z^*; \quad (7)$$

ΔZ is $\Delta Z_m(l)$ or $\Delta Z_M(L)$ and $Z^*=\text{median}\{\Delta Z\}-z^*$, $z^*=\min\{\Delta Z\}$, and m is the distribution tail exponent, which in the case of self-affine cracks depends on $\zeta\equiv\alpha$ [15,18]. For intrinsically anomalous cracks in concrete we find that both normalized amplitudes $\Delta z_l=\Delta Z_m/L^{\alpha-\zeta}l^{\zeta}$ and $\Delta z_g=\Delta Z_M/L^{\alpha}$ exhibit a log-logistic distribution (p value 0.6838 [40]) characterized by the same tail exponent $m=4\pm0.1$ (see Fig. 4(d) and Ref. [41]).

Intrinsically anomalous crack roughness implies that the

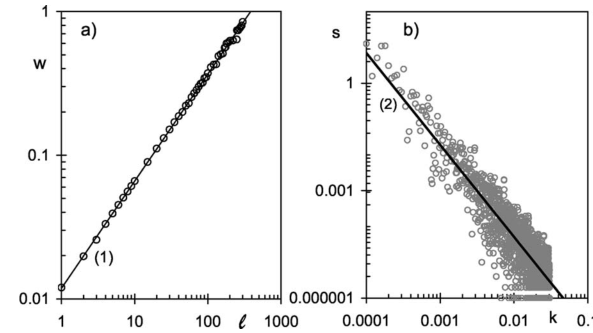

FIG. 3. Data collapse for (a) crack width, $w=w(l,L)/L^{0.6}$, and (b) power spectrum, $s(k)=S(k,L)/L^{0.6}$, for crack traces in concrete plates of different size. Straight line slopes $\zeta=0.75$ (1) and $-\theta=-2\alpha_s-1=-2.5$ (2).

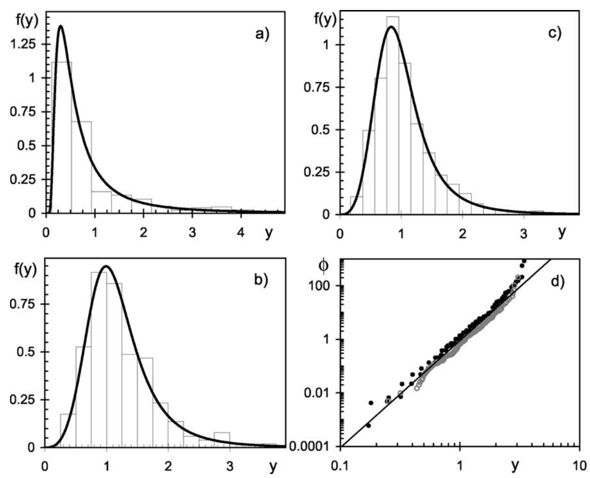

FIG. 4. (a)–(c) Conditional probability distributions of (a) the mean-square crack width and (b) the global and (c) the local amplitudes of crack traces. Bins—experimental data, lines—data fitting by (a) Pearson distribution with $k=0.76$ (p value$=0.4846$) and (c–d) log-logistic distribution (p value$=0.6838$) with (c) $m=4.053$, $\kappa=z^*/Z^*=0.0139$ and (d) $m=3.963$, $\kappa=0.0048$. (d) Log-log plot of $\phi=F(y)/[1-F(y)]$ vs y, where $F(y)$ is the cumulative distribution of the normalized global, $y=\Delta Z_M(L)/L^{\alpha}$ (full circles), and local, $y=\Delta Z_m(l,L)/l^{\zeta}L^{\alpha-\zeta}$ (circles), crack width (straight line corresponds to the log-logistic distribution with $m=4$).

statistical properties of the set of admissible crack paths in concrete are dependent on the system size; nevertheless, any individual crack possesses a self-affine invariance. This means that a crack has information about the specimen size before it fails. Surprisingly, the local and the global amplitudes of cracks exhibit the same fat-tailed statistical distribution. These observations provide a unique insight into the physics of fracture and the nature of intrinsically anomalous roughening.

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[40] The distribution can be also fitted by log-normal distribution, but with lower p value 0.3946.

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