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Determination of azimuthal anchoring energy in a twisted nematic liquid crystal

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On the basis of the torque balance equation between the twisting elastic power and the torsional anchoring energy and the Jones matrix equation of twisted nematic liquid crystals (TNLCs), an improved optical method for measuring the azimuthal anchoring strength of NLCs is proposed. In the given experimental setup, the rotation of the LC layer under fixing the transmission axis of the analyzer presents optical transmission curves to give information of the real twist angle. By rotating the analyzer with the obtained real twist angle in any rotation angle of the LC layer, cell thickness is calculated. From the obtained real twist angle and cell thickness, the azimuthal anchoring strength of nematic liquid crystals (NLCs) is easily determined.

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The physical behavior of liquid crystals (LCs) is largely characterized by the surface properties and in particular the surface energy plays a fundamental role both in the surface liquid crystal physics and technical applications. Surface energy takes its origin from the reduced symmetry close to the surface and from the direct interaction between the nematic liquid crystal (NLC) and the substrate [1–3]. It usually consists of the isotropic surface tension and its anisotropic part (f_s). f_s is a scalar quantity and a local property of the interface [4]. It depends on the nematic orientation $\mathbf{L}$ and on the geometrical normal to the interface in the considered point. The surface orientation for which f_s is minimum is known as the easy axis and is indicated by $\mathbf{L}_e$. The curvature around this minimum is called the anchoring energy and is indicated by w. Very often f_s is approximated by $f_s=-(\frac{1}{2})w(\mathbf{L}\cdot\mathbf{L}_e)^2$, phenomenologically described by Rapini and Papoular [5] as anchoring energy per unit area.

In nonpolar liquid crystals, the director distribution at the interface is characterized by the polar angle with the normal z axis and by the azimuthal angle with the x axis on the surface plane. That is, there are two contributions to the surface-anchoring properties; a polar anchoring energy, constraining the out-of-plane motion of the director, and an azimuthal anchoring energy, restricting the in-plane motion of the director. If the polar angle is held fixed and equal to the easy polar angle, the surface energy becomes a function of azimuthal angle only, which is called the azimuthal anchoring energy. The azimuthal anchoring energy is known as a key factor for the bistable LC device application [6–9], the driving-voltage reduction of in-plane switching liquid crystal displays [10], and the patterning method of liquid crystal alignment [11].

Various experimental methods have been used to measure the azimuthal anchoring energy. Among these methods, the optical measurement using the polarization state of transmitted [12–18] light is relatively simple and accurate, even though the calculation of the dependence of the polarization of the transmitted light on the surface azimuthal angles requires a somewhat complex numerical analysis and, for the accurate measurement, the parameters such as entrance LC director angle and easy axis angle should be concretely defined, which is possible through a complicated analysis.

In this paper, we propose an improved optical method for measuring the azimuthal anchoring energy as well as cell thickness. By rotating the LC layer and analyzer, respectively, information of the real twist angle and cell thickness is determined. Since optical transmission curves by only twice rotation are acquired, it has the advantage that the azimuthal anchoring energy is simply and quickly calculated. In addition to that, accuracy is also reliable. The discussion of the accuracy of the proposed method is dealt with in detail in the Appendix.

In a NLC display device rubbed polymer films are widely used to obtain uniform director alignment. When the rubbing directions on both substrates in the cell have any angle to each other, the nematic liquid crystals injected in the cell also form a twist structure by the surface anchoring force. If the surface anchoring energy is weak, in the absence of an external field, the twisted structure of substrates can induce a deviation of the director at the surfaces from the easy axis and the anchoring energy can be evaluated by measuring this deviation. This deviation angle is determined by the balance between the twisting elastic power and the torsional anchoring energy.

For the twist NLC doped with a chiral material, the free energy per unit area is obtained as the sum of the bulk elastic energy, f_b, and the surface anchoring energy, f_s, as in the following formula:

$$f=f_b+2f_s, \quad (1)$$

where

$$f_b=\frac{k_{22}}{2d}(\phi_t-\phi_0)^2, \quad (2)$$

$$f_s=\tfrac{1}{2}w_\phi \sin^2\phi, \quad (3)$$

k_{22} is the twist elastic constant, d is the cell thickness, ϕ_t is the real twist angle, ϕ_0 is the intrinsic pre twisted angle of the sample material given by $\phi_0=2\pi d/p$ where p is the natural pitch of the material, w_ϕ is the azimuthal anchoring

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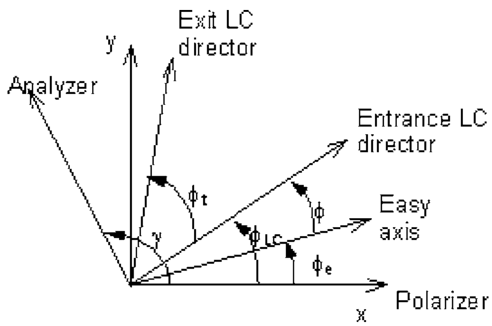

FIG. 1. Schematic illustration used to measure the azimuthal anchoring strength. ϕ_{LC} is the LC layer angle, which can be rotated. γ is analyzer angle, which can also be rotated. ϕ_t is the real twist angle.

strength, and ϕ is the deviation angle of the surface LC director from the easy axis. By minimizing the free energy, f, we obtain

$$\phi_t - \frac{2\pi d}{p} = \frac{w_\phi d}{k_{22}} \sin \phi \cos \phi. \quad (4)$$

In consequence, the azimuthal anchoring strength w_ϕ can be calculated from the measurement of the real twist angle and the cell thickness. In this work, we propose a simple method for measuring the real twist angle and the cell thickness. From the measured real twist angle and cell thickness, we can calculate the azimuthal anchoring strength by using Eq. (4).

Figure 1 shows a schematic diagram for measuring the real twist angle and the cell thickness. With the Jones matrix representation [19] for normal incident light, optical transmission T in a TNLC cell is expressed as follows (with the angles defined as in Fig. 1):

$$T = \left(\cos \delta \cos(\phi_t - \gamma) + \frac{\phi_t}{\delta} \sin \delta \sin(\phi_t - \gamma)\right)^2 + \left(1 - \frac{\phi_t^2}{\delta^2}\right)\sin^2 \delta \cos^2(\phi_t + 2\phi_{LC} - \gamma), \quad (5)$$

where the polarizer transmission axis is set at the x axis for simplicity, γ is the angle of the analyzer transmission axis, ϕ_{LC} is the sum of the azimuthal deviation angle ϕ and the easy axis angle ϕ_e, and δ is defined as $\delta = \phi_t(1+u^2)^{1/2}$ with $u = \pi \Delta n d / \lambda \phi_t$, with ϕ_t and d representing the real twist angle and the cell thickness, Δn being birefringence of LC, and λ denoting the wavelength of the incident light. Surface azimuthal anchoring strength w_ϕ can be determined by finding the azimuthal deviation angle ϕ and the cell thickness. The He-Ne laser beam impinging onto the LC layer at normal incidence was linearly polarized after passing through the polarizer. According to Eq. (5), the transmittance T can be divided into a term containing ϕ_{LC}, $T_e(\gamma, \phi_{LC})$, and that excluding ϕ_{LC}, $T_c(\gamma)$. Then, Eq. (5) can be written as

$$T = T_c(\gamma) + T_e(\gamma, \phi_{LC}). \quad (6)$$

In order to find the real twist angle and the cell thickness, the proposed method uses two rotating stages to rotate a NLC cell and an analyzer by an angle, respectively.

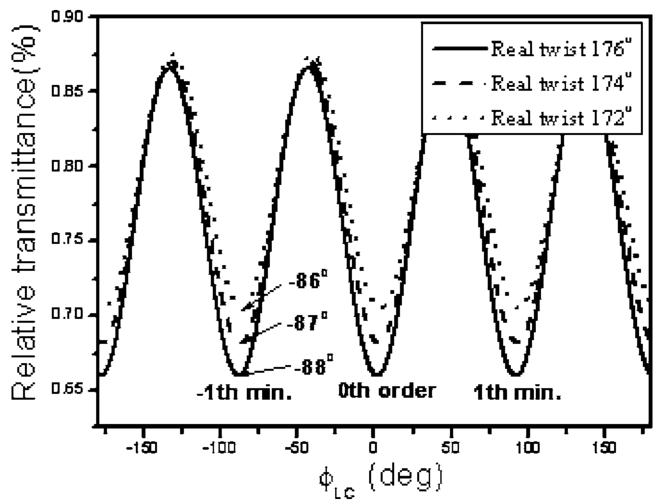

FIG. 2. Optical transmittance T versus the rotation angle ϕ_{LC} of a NLC cell. It is assumed that the analyzer angle is fixed at $\pi/2$ and Δnd is 0.4 μm.

To determine the real twist angle for a sample NLC cell with any twist angle and cell thickness, first the analyzer is set at any fixed angle. Therefore, $T_c(\gamma_o)$ becomes a constant value in Eq. (6). In such instances, when the NLC cell attached to a rotating stage is rotated by an angle (Θ), the transmittance T depends only on the term containing ϕ_{LC} in Eq. (6). By rotating the NLC cell by an angle (Θ), the rotation angle can be divided into the resolution of the electrical stepped motor. That is, a rotation angle of Θ/N is achieved by dividing the rotation angle of Θ into N steps. At each angle, transmittance values can be measured with a power meter. Figure 2 illustrates the theoretical relations between transmittance T and the rotation angle ϕ_{LC} of a NLC cell. Here, it is assumed that the analyzer angle is fixed at $\pi/2$. We increase the real twist angle by 2°, from 172° up to 176°. From Fig. 2, we can obtain ϕ_{LC} showing a minimum or maximum transmitted intensity of light, periodically at the interval of 45°. When the derivative of T with respect to LC rotation angle ϕ_{LC} is zero for a given analyzer angle, T becomes a maximum or a minimum:

$$\frac{dT}{d\phi_{LC}} = 0$$

so

$$-2\left(1 - \frac{\phi_t^2}{\delta^2}\right)\sin^2 \delta \sin 2(\phi_t + 2\phi_{LC} - \gamma_o) = 0, \quad (7)$$

where $1 - \phi_t^2/\delta^2 > 0$ because of $\delta > \phi_t$, resulting in $1 - \phi_t^2/\delta^2 \neq 0$.

From Eq. (7), we can find three optical conditions that provide $dT/d\phi_{LC}$ with zero as follows,

$$\delta = m\pi \quad or \quad 2(\phi_t + 2\phi_{LC} - \gamma_o) \neq n\pi, \quad (8a)$$

$$\delta \neq m\pi \quad or \quad 2(\phi_t + 2\phi_{LC} - \gamma_o) = n\pi, \quad (8b)$$

TABLE I. The relation between minima or maxima points and the corresponding order n. This n represents order in Eq. (9).

n	Minimum order	Maximum order
−2	⋯	−2nd
−1	−1st	⋯
0	⋯	−1st
1	⋯	0th
2	⋯	1st
3	1st	⋯
4	⋯	2nd

$$\delta = m\pi \quad and \quad 2(\phi_t + 2\phi_{LC} - \gamma_o) = n\pi \tag{8c}$$

where m and n are integers.

The above three optical conditions that provide $dT/d\phi_{LC}$ with zero are obtained by fixing the transmission axis of the analyzer to $\pi/2$. Among these conditions, $\delta = m\pi$ of (8a) and (8c) conditions is not possible because of $m \neq \phi_t[1+(\pi\Delta nd/\lambda\phi_t)^2]^{1/2}/\pi$. As a result, the solution that can satisfy Eq. (7) is condition (8b). We obtain the simple relation between real twist angle (ϕ_t) and LC rotation angle $(\phi_{LC})_{max/min}$:

$$\phi_t = \gamma_o - 2(\phi_{LC})_{max/min} + \pi(n/2) \quad (n = integer). \tag{9}$$

Since γ_o is constant and $(\phi_{LC})_{max/min}$ is determined by experiment, we can determine the value of real twist angle ϕ_t from Eq. (9). As shown in Fig. 2, the various minima and maxima of the corresponding order can be produced. For each minimum and maximum point, we can find the corresponding order n in Eq. (9) through the numerical analysis. Table I shows the relation between minima or maxima points and the corresponding order n in Eq. (9). Figure 3 shows that when −1st minimum point is selected as $(\phi_{LC})_{min}$, the real twist angle is obtained in order $n = -1$.

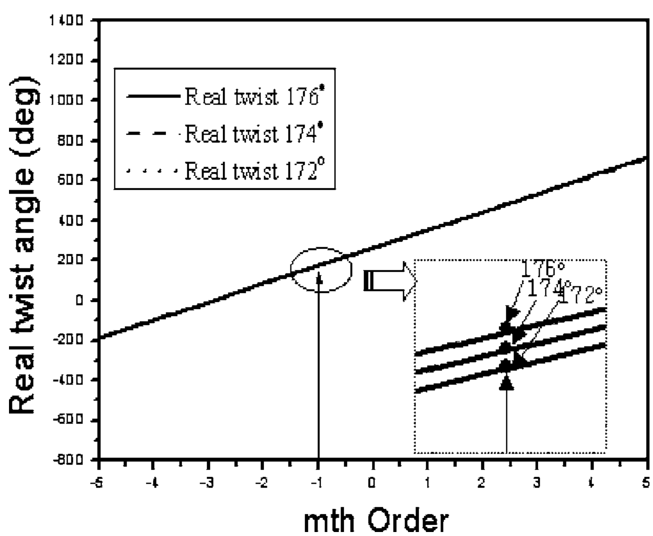

FIG. 3. Theoretical relation between −1st minimum point which satisfies $(\phi_{LC})_{min}$ and order $n = -1$.

As the next step, the cell thickness should be determined, because, after the liquid crystal cell is filled, the cell thickness and its variation across the entire panel may not be the same as they were before the filling [19]. It has an advantage that the proposed method can measure the cell thickness as well as the real twist angle. By the first step, if ϕ_t is determined in any $(\phi_{LC})_{max/min}$, the transmittance, T, depends only on the term γ as expressed in Eq. (6). In the case that we fixed ϕ_{LC} of a NLC cell to be $(\phi_{LC})_{max/min}$ and knew ϕ_t by using Eq. (9), we can obtain the cell thickness from the condition that the derivative of T with respect to analyzer rotation angle γ is zero for a given $(\phi_{LC})_{max/min}$ and ϕ_t. T becomes a maximum or a minimum:

$$\frac{dT}{d\gamma} = \Delta_1(\delta) + \Delta_2(\delta) = 0, \tag{10}$$

where

$$\Delta_1(\delta) = \cos^2\delta \sin 2(\phi_t - \gamma) - \frac{\phi_t}{\delta}\sin 2\delta \cos 2(\phi_t - \gamma),$$

$$\Delta_2(\delta) = -\frac{\phi_t^2}{\delta^2}\sin^2\delta \sin 2(\phi_t - \gamma) + \left(1 - \frac{\phi_t^2}{\delta^2}\right)\sin^2\delta \sin 2(\phi_t + 2\phi_{LC} - \gamma).$$

From the condition that Eq. (10) becomes zero, where γ is determined as the point showing a minimum or maximum transmitted intensity of light, numerically we can get a δ value, which includes the information of Δnd. As a result, if the δ value is determined, the Δnd value is also calculated from $\delta = \phi_t(1+u^2)^{1/2}$ with $u = \pi\Delta nd/\lambda\phi$. Therefore, we can calculate cell thickness from the information of Δn. In conclusion, as we obtained the real twist angle and the cell thickness, we can determine the azimuthal anchoring strength from Eq. (4).

In order to verify the usefulness of this proposed method, the measurements are performed for LC cells. One sample cell filled with liquid crystal ZLI-1557 was fabricated with an angle of 90° between easy axes, a spacer of diameter 4.2 μm, and a chiral pitch of $p = \infty$, and two test cells filled with the same liquid crystal with an angle of 180° between easy axes and a spacer of diameter 6.5 μm was fabricated with chiral pitches of $p = 18.97$ μm and $p = 18.91$ μm using a chiral dopant S-811 (E. Merck), respectively. SE 3140 polyimide (Nissan Chemicals Co.) as alignment is used. The properties of ZLI-1557 at 293 K are as follows: $\Delta n = 0.1124$ for a red wavelength of 632.8 nm; $k_{11} = 9.5 \times 10^{-12}$ N, $k_{22} = 5.1 \times 10^{-12}$ N, and $k_{33} = 11.5 \times 10^{-12}$ N. The rubbing is made under general rubbing conditions, such that the stage speed of the rubbing system is 40 mm/s, the rotation velocity of the roller is 1000 rpm, and the rubbing depth is 5 mm.

In the experiment, as the resolution of the stepper motor used is 0.5°, we set the angle of each step to be 0.5° so that the total number of steps, N, becomes 360 for a rotation of

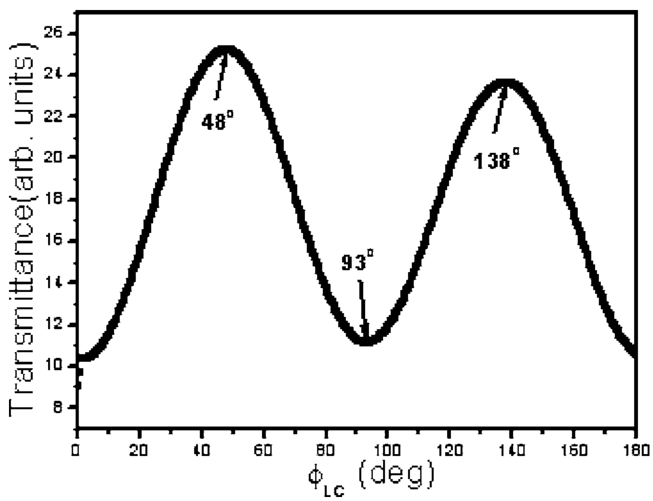

FIG. 4. Measured transmittance curve for the rotation angle (ϕ_{LC}) of the test LC cell with a chiral pitch of $p=18.97$ μm. Maximum transmittance is obtained at 48°.

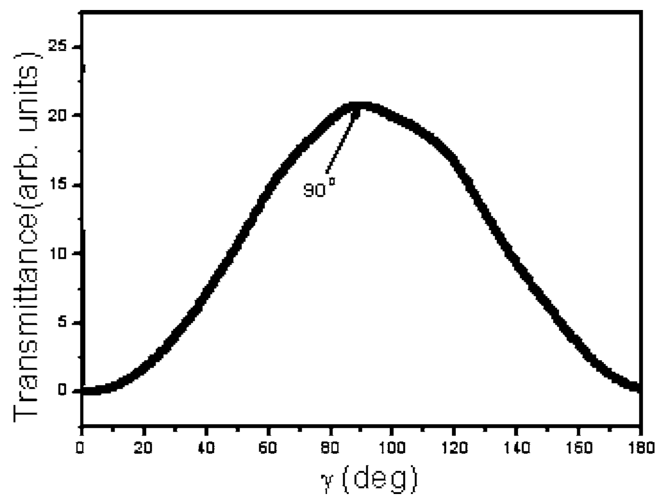

FIG. 5. Measured transmittance curve for the rotation angle (γ) of the analyzer of the test LC cell with a chiral pitch of $p=18.97$ μm. Maximum transmittance is obtained at 90°.

180°. Therefore, we can obtain 360 transmittance data with 180° rotation. More accurate data can be obtained if a stepper motor with higher resolution is used.

First, by rotation of the NLC cells attached to the stepper motor, 1st maximum transmittance was obtained at the points of 52°, 48°, and 48°, respectively. For example, the measured transmittance curve of the test cell with the chiral pitch of $p=18.97$ μm is shown in Fig. 4. From the transmittance curve, 1st maximum transmittance is obtained at the point of 48° and minimum or maximum transmittance point is repeated at the interval of 45°. Because of 1st maximum transmittance, the order n in Eq. (9) corresponds to 2, as shown in Table I. The real twist angle calculated with the maximum transmittance angle $(\phi_{LC})_{max}$ was 174°. In the same way, one can also obtain the real twist angle of 76° and 174° for the other samples, respectively.

Second, we fixed ϕ_{LC} of the NLC cell to be $(\phi_{LC})_{max/min}$ and rotated the analyzer attached to the stepper motor by 180°. Then, maximum transmittance is obtained at the points of 92.5°, 90°, and 90°, respectively. For example, the measured transmittance curve of the test cell with the chiral pitch of $p=18.97$ μm is shown in Fig. 5. For the determined real twist angle of 174° and a given LC rotation angle of 48°, the analyzer angle at which transmission T is maximum is obtained at the point of 90°, as shown in Fig. 5. Substituting these values into Eq. (10) allows us to find the Δnd that provides $dT/d\gamma$ with zero. For the sample cell with the chiral pitch of $p=18.97$ μm, the value of $\Delta nd=0.7076$ μm is obtained numerically as shown in Fig. 6. Because all the measurements are performed at 293 K, from the information of $\Delta n=0.1124$ (at 638 nm) the cell thickness of 6.3 μm is calculated. It can be found that the calculated cell thickness differs from the used spacer thickness 6.5 μm. In the same way, for the other samples, the cell thickness can be calculated to be 4.07 and 6.22 μm, respectively. The measurement uncertainty of this method is within 0.2% for the results in the cell thickness above 4 μm. (See the Appendix.) These cell thickness values were confirmed both by the optical compensation method and the rotational waveplate method [19,20].

Finally, from Eq. (4), the azimuthal anchoring strengths (w_ϕ) are calculated to be 1.38×10^{-5} J/m^2, 1.47×10^{-5} J/m^2, and 1.52×10^{-5} J/m^2, respectively. Table II shows the parameters used for the calculation and the measured results. For the comparison, it includes the results obtained by the method of Ref. [18].

In conclusion, we proposed a simple and fast method for measuring the azimuthal anchoring strength of nematic liquid crystals using the rotation of a NLC cell and an analyzer. We have found the azimuthal anchoring strength through an azimuthal deviation angle and cell thickness under general rubbing conditions, such that the stage speed was 40 mm/s, the rotation velocity was 1000 rpm, and the rubbing depth was 5 mm, for Merck LC ZLI-1557 and Nissan PI SE-3140. By numerical simulation, we also discussed the external and intrinsic errors that can occur in the measurements. By automatic control, a high resolution of rotation step, and the

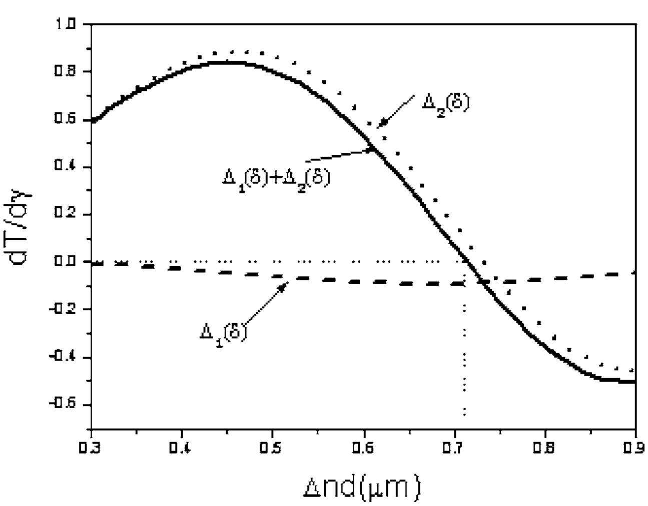

FIG. 6. $dT/d\gamma$ for the variation of Δnd values. For the test LC cell with a chiral pitch of $p=18.97$ μm, the Δnd that $dT/d\gamma$ becomes zero is 0.7076 μm.

TABLE II. Parameters used for the calculation and the measured results.

Sample	No. 1	No. 2	No. 3
Pitch (μm)	∞	18.97	18.91
k_{22} (10^{-12})	5.1	5.1	5.1
Δn (at 638 nm)	0.1124	0.1124	0.1124
ϕ(°)	7	3	3
Spacer thickness (μm)	4.2	6.5	6.5
Measured cell thickness (μm)	4.07	6.3	6.22
Real twist angle (°)	76	174	174
W_ϕ (J/m^2)	1.38×10^{-5}	1.47×10^{-5}	1.52×10^{-5}
W_ϕ (J/m^2) (Ref. [18])	1.40×10^{-5}	1.48×10^{-5}	1.50×10^{-5}

information about pretilt angle, it is confirmed that more accurate azimuthal anchoring strengths could be obtained.

ACKNOWLEDGMENTS

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APPENDIX

In this method, we can classify the causes of the error into two groups. First, there is an external error coming from the deviation of the analyzer transmission axis in the optical setup and the resolution of a stepper motor. Second, there is an intrinsic error due to pretilt angle in LC cell.

By numerical simulation, we investigated the effect of the deviation of the analyzer transmission axis with the assumption that the polarizer was precisely set at the x axis. Equation (9) can be expanded into Taylor expansion around γ_0, which is the accurate angle of the analyzer transmission axis, as follows:

$$\phi_t = \phi_t(\gamma_0) + \Delta\gamma(d\phi_t/d\gamma)_{\gamma 0} + \cdots \approx \phi_t(\gamma_0) + \Delta\gamma, \tag{A1}$$

where $\Delta\gamma$ is the deviation angle of the analyzer transmission axis.

Therefore, the error of the real twist angle occurred as much as $\Delta\gamma$. The error of the real twist angle due to $\Delta\gamma$ influences the maximum or minimum point obtained from the rotation of a NLC cell. In the case of a stepper motor with a resolution of 0.5°, even if the deviation angle is varied up to 5°, there is no change in the maximum angle as shown in Fig. 7. If the resolution of a stepper motor is increased up to 0.1°, the variation of the maximum transmittance point becomes more sensitive, but it is almost similar to that with a stepper motor with a resolution of 0.5°. However, in an actual measurement, if we control γ automatically, a precise setting to within 0.5° can be achieved. So, it can be expected that the error due to $\Delta\gamma$ may not occur. In fact, the resolution of a stepper motor is more influential to the measurement error than inaccuracy in the optical setup. For example, for the resolution of rotation angles of 0.5° and 0.1°, in the case of the LC cell with a twist angle of 180°, the measurement error is found to be within 0.02% for the resolution of 0.1°. On the other hand, in the case of using a stepper motor with resolution of 0.5°, we have to consider the measurement error within 0.2% in the cell thickness above 4 μm. Even if the measurement error is increased in the low-cell thickness be

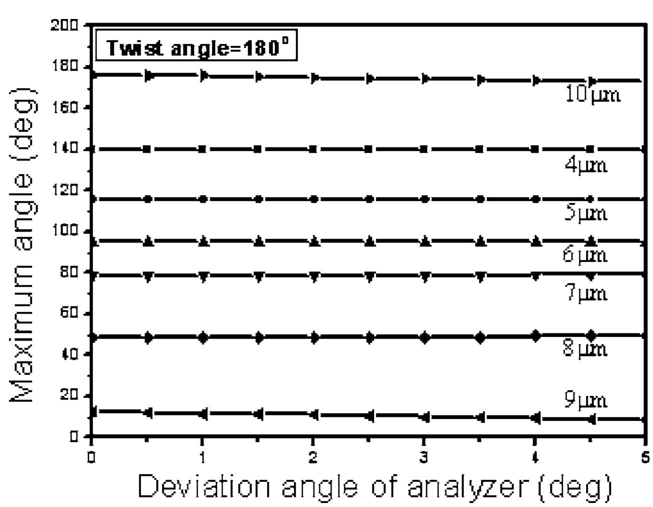

FIG. 7. Dependence of the maximum angle on the deviation of the analyzer transmission axis.

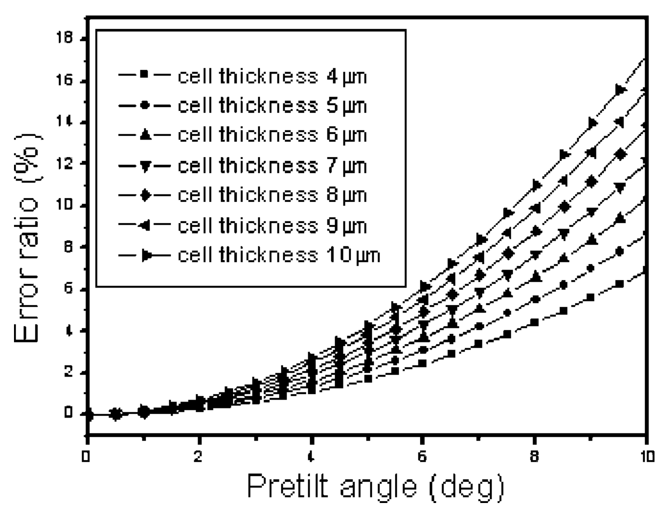

FIG. 8. Dependence of the error ratio on the pretilt angle for LC cell with a twist angle of 180°.

low about 4 μm, we can confirm that in the cell thickness above 4 μm we can obtain reliable results even with the resolution of 0.5°.

In this work we calculated the cell thickness under the assumption that the pretilt angle is zero. There can be an error due to the nonzero pretilt angle. In our sample cells, the pretilt angle measured by the crystal rotation method is approximately 5° [21]. Since the Δnd value obtained from Eq. (10) depends on the pretilt angle, the accurate cell thickness d may be varied as the deviation of it. The error ratio on the pretilt angle for a LC cell with a twist angle of 180° is shown in Fig. 8. The more the cell thickness increases, the more the error ratio increases. For example, in our sample cells, because the pretilt angle is approximately 5°, we can calculate the variation of the cell thickness. We can find the effect of the pretilt angle on the cell thickness and the azimuthal anchoring energy. In the case that pitch is ∞, because of $w_\phi \propto 1/d$, the error ratio of azimuthal anchoring energy is the same as the error of cell thickness by pretilt angle variation. In the case that pitch is finite, however, the error is extremely increased. As a result, for more accurate calculation, large pitch is needed. However, if the information about the pretilt angle in the alignment layer is known, we can solve this kind of error.

Therefore, using automatic control, a high resolution of rotation step, and the information about the pretilt angle, we confirm that more accurate azimuthal anchoring strengths can be obtained.

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