

Strongly focused polarized light pulse

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We investigate theoretically the electric field of a focused light pulse carrying an inhomogeneous polarization distribution. It is found that the spectra of the polarization components are in general different, thus leading to a spatial spectral distribution that differs from the scalar case.

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With an increasing number of new applications, it becomes more and more important to understand the properties of light beams in focal regions. In the case of high aperture focusing, polarization effects become particularly important, thus leading to a number of interesting phenomena. To this end, focusing of spatially inhomogeneous polarization distributions has triggered interest, in particular radial polarization distributions, due to the fact that they may assist in obtaining a tighter focal spot [1–4]. Moreover, recent theoretical and experimental studies have demonstrated that spectral changes can take place in the focal region of continuous focused scalar waves [5,6]. However, to date no studies have investigated the spectral changes resulting from the vectorial nature of focused light. In this Brief Report we present a model for the electric field of a strongly focused pulsed laser beam carrying an inhomogeneous spatial polarization distribution. Such pulsed beams may have applications in data storage and for manipulation of charged particles.

The diffracted electric vector field of a pulsed beam in vacuum can be expressed in the complex analytical signal representation as [7–10]

$$\mathbf{E}(r, t) = \frac{1}{\pi} \int_0^{\infty} \mathbf{E}(r, \omega) \exp(-i\omega t) d\omega, \quad (1)$$

where ω is the angular frequency and $\mathbf{E}(r, \omega)$ is the electric field of a single spectral component. We are here interested in focusing systems with high angular aperture in order to obtain a tight focal spot. In order to proceed we will here make two approximations. First we will assume that the transmission through the aperture of the high aperture focusing system is independent of the wavelength. Thus, the shape and size of the incoming beam is only determined by the aperture itself. This is a reasonable approximation if the aperture (of radius a) is overfilled such that the diameter of the input beam is larger than $2a$. We will also assume that the focal length of the system is constant and independent of wavelength. This can be obtained by designing, e.g., a parabolic mirror or a lens system which focuses each wavelength at the same focal point. Using the Debye approximation, the coherent electric field vector of a single spectral component of spectral strength $S(\omega)$ can near the focal plane be expressed as [11,12]

$$\mathbf{E}(r, \omega) = -\frac{ikS(\omega)}{2\pi} \int \int_{\Omega} \frac{\mathbf{T}(s)}{s_z} \exp[ik(s_x x + s_y y + s_z z)] ds_x ds_y, \quad (2)$$

where $k = \omega/c$ is the wave number in vacuum, $s = (s_x, s_y, s_z)$ is the unit vector along a typical ray, Ω is the solid angle formed by all the geometrical rays, and $\mathbf{T}(s)$ is the vector pupil distribution which accounts for the polarization, phase and amplitude distributions at the exit pupil. We emphasize that Eq. (2) is valid only as long as the Fresnel number N is much larger than unity, i.e., $N = a^2/\lambda f \gg 1$, where f is the focal length [12]. The approach adopted here differs from previous theoretical studies on pulsed light; see, e.g., [7–10, 13–18]. The advantage of our approach is that it easily lends itself to the study of strongly focused vector pulses. However, we also emphasize that care must be taken such that the wavelength span of the pulse does not cause a violation of the requirement $N \gg 1$ (the longest wavelength must still give $N \gg 1$).

We now consider a circular symmetric focusing system (see Fig. 1). In spherical coordinates, the unit wave vector is defined as

$$s = [\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta]. \quad (3)$$

The position vector can be written as

$$r_c = [r_c \sin \theta_c \cos \phi_c, r_c \sin \theta_c \sin \phi_c, z], \quad (4)$$

where we for the rest of the paper set $r = r_c \sin \theta_c$, which represents a projection onto the focal plane. For a system of numerical aperture (NA) of $\sin \alpha$ Eq. (2) gives the following diffraction integral:

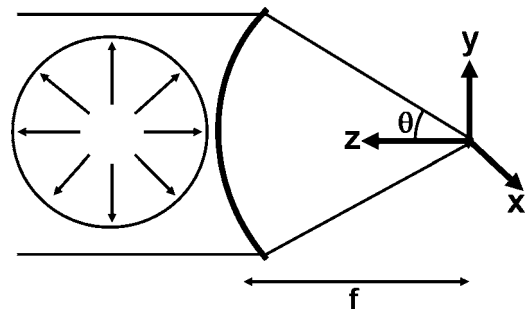


FIG. 1. Schematic drawing of the focusing geometry. The polarization distribution incident on the focusing system is radial.

$$\begin{aligned} \mathbf{E}(r, \omega) = & -\frac{ikS(\omega)}{2\pi} \int_0^\alpha \int_0^{2\pi} \mathbf{T}(\theta, \phi) \exp[ikr \sin \theta \cos(\phi - \phi_c) \\ & + ikz \cos \theta] \sin \theta d\theta d\phi, \end{aligned} \quad (5)$$

where α is the convergence semiangle and the amplitude vector for each ray is given by

$$\mathbf{T}(\theta, \phi) = C(\theta) \mathbf{P}(\theta, \phi). \quad (6)$$

Here $C(\theta)$ is the transmittance function and $\mathbf{P}(\theta, \phi)$ is the polarization distribution. In practical focusing systems, the light distribution from a spatial modulator is projected onto the exit pupil of the system, thus resulting in the angular transmittance function $C(\theta)$. We emphasize that $C(\theta)$ can be used to describe any focusing systems (with lenses and mirrors). If the incoming pulsed beam is formed into a bright ring at the exit pupil, it can be described approximately by a δ function $C(\theta) = \delta(\theta - \theta_0)$, where θ_0 is the angular location of the ring at the exit pupil. Note that such a ring may be generated by an axicon or a holographic beam shaper (see, e.g., Ref. [19] and references therein).

In order to demonstrate the applicability of the formalism we will here find the electric field in the focal region of a pulsed, radial polarization distribution. Such a distribution can be represented by [4]

$$\mathbf{P}(\theta, \phi) = \begin{bmatrix} \cos \theta \cos \phi \\ \cos \theta \sin \phi \\ -\sin \theta \end{bmatrix}.$$

Then the resulting field components can be calculated to be

$$E_x^r(r, t^*) = \frac{\cos \phi_c}{\pi c} \int_0^\alpha C(\theta) E_{r1}^*(\theta, t^*) \sin \theta \cos \theta d\theta, \quad (7)$$

$$E_y^r(r, t^*) = \frac{\sin \phi_c}{\pi c} \int_0^\alpha C(\theta) E_{r1}^*(\theta, t^*) \sin \theta \cos \theta d\theta, \quad (8)$$

$$E_z^r(r, t^*) = \frac{i}{\pi c} \int_0^\alpha C(\theta) E_{r2}^*(\theta, t^*) \sin^2 \theta d\theta, \quad (9)$$

where

$$E_{r1}^*(\theta, t^*) = \int_0^\infty \omega S(\omega) J_1\left(\frac{\omega}{c} r \sin \theta\right) \exp(-i\omega t^*) d\omega, \quad (10)$$

and

$$E_{r2}^*(\theta, t^*) = \int_0^\infty \omega S(\omega) J_0\left(\frac{\omega}{c} r \sin \theta\right) \exp(-i\omega t^*) d\omega. \quad (11)$$

Here J_n are Bessel functions of the first kind and $t^* = t - z \cos \theta / c$.

This set of two-dimensional integrals can now be used to find the electric field in the focal region. However, in order to extract results that can be interpreted analytically, let us now assume that $C(\theta) = \delta(\theta - \theta_0)$, in order to find that

$$\begin{aligned} E_{xb}^r(r, t_b^*) = & \frac{\cos \phi_c}{\pi c} \sin \theta_0 \cos \theta_0 \int_0^\infty \omega S(\omega) J_1\left(\frac{\omega}{c} r \sin \theta_0\right) \\ & \times \exp(-i\omega t_b^*) d\omega, \end{aligned} \quad (12)$$

$$\begin{aligned} E_{yb}^r(r, t_b^*) = & \frac{\sin \phi_c}{\pi c} \sin \theta_0 \cos \theta_0 \int_0^\infty \omega S(\omega) J_1\left(\frac{\omega}{c} r \sin \theta_0\right) \\ & \times \exp(-i\omega t_b^*) d\omega, \end{aligned} \quad (13)$$

and

$$E_{zb}^r(r, t_b^*) = \frac{i}{\pi c} \sin^2 \theta_0 \int_0^\infty \omega S(\omega) J_0\left(\frac{\omega}{c} r \sin \theta_0\right) \exp(-i\omega t_b^*) d\omega. \quad (14)$$

Here $t_b^* = t - z \cos \theta_0 / c$. The focused light now belongs to the class of “diffraction-free” beams discussed in Refs. [14,20–22], and there is no broadening of the light spot along the optical axis. As pointed out in Ref. [17], diffraction causes the redder frequencies to spread more than the blue frequencies, thus altering the spectrum of a localized pulsed beam upon propagation. Moreover, Refs. [5,6] demonstrated that diffraction can induce spectral changes in polychromatic, continuous focused scalar waves. Here we see that the spectrum close to the optical axis is blueshifted, whereas further away from the axis it is redshifted. However, the spectra of the polarization components differ considerably. Near the optical axis ($r \ll c/\omega \sin \theta$) we find that the spectra of the x and y components are proportional to $r\omega^2 S(\omega)$, while the spectrum of the z component goes as $\omega S(\omega)$. We also notice that whereas the z component spectrum always has its peak on the optical axis, the peaks of the x and y components are displaced from the axis such that redder frequencies spread out more. In the case of moderate angular aperture ($10^\circ < \theta < 60^\circ$) it can be estimated that the electric field components are all comparable in magnitude, and a strongly blueshifted intensity distribution near the optical axis is expected. Only in the case $\theta = 90^\circ$ can the pulsed electric field be treated as a scalar. However, in general the spectrum does indeed differ from that found in a scalar beam (where all the components are the same and governed by a zeroth-order Bessel function), but a detailed numerical study of this phenomenon is not the aim of this work. Also in a focused polychromatic continuous electromagnetic wave the spectra of the polarization components would differ considerably, and the principle demonstrated here could be extended to such cases.

In conclusion, we have presented a formalism for calculating the electric field of a strongly focused pulsed beam carrying a radial polarization distribution. Our results demonstrate that the optical frequency spectrum is not identical for the x , y , and z polarized components. The formalism presented here can also be extended to other spatially inhomogeneous polarization distributions, and will hopefully inspire more research in this area.

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