

Critical dynamics of diluted relaxational models coupled to a conserved density

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We consider the influence of quenched disorder on the relaxational critical dynamics of a system characterized by a nonconserved order parameter coupled to the diffusive dynamics of a conserved scalar density (model C). Disorder leads to model A critical dynamics in the asymptotics; however, it is the effective critical behavior that is often observed in experiments and in computer simulations, and this is described by the full set of dynamical equations of diluted model C. Indeed, different scenarios of effective critical behavior are predicted.

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The critical behavior of pure systems might be changed by introducing imperfections such as dilution, defects, etc., into a critical system. Such a change is answered by the Harris criterion [1], stating that a new diluted critical behavior appears if the specific heat of the pure system is diverging. The diluted critical behavior then has a nondiverging specific heat. Since the borderline value n_c between a diverging and nondiverging specific heat at space dimensions $d=3$ lies between order parameter (OP) dimensions $n=1$ (Ising model) and $n=2$ (XY model), only the Ising case belongs to a new universality class. In consequence, this result led to the conclusion that for the critical dynamics the coupling of conserved quantities to the OP is of no relevance [2,3]. The argument was the following: For the critical dynamics of a relaxational model, it was shown [4–6] that the coupling to a conserved density (e.g., the energy density) is relevant if the specific heat diverges. Due to dilution, this is never the case, and therefore the coupling is of no relevance. Therefore, most of the papers considered only the relaxational dynamics of diluted Ising systems [7–10].

However, this argumentation is based on the asymptotic properties of the diluted model. Experimental data and computer simulations made clear that in most cases one observes nonasymptotic critical behavior, described often by dilution-dependent effective exponents (see, e.g., [11,12]). In such a case the Harris criterion does not hold and therefore one has to consider in the dynamics the coupling to the conserved density and its effects on the effective critical behavior. In addition, one is not restricted to the Ising case since already in statics the effective critical behavior for $n>1$ is different from the pure case [12].

There are two relevant parameters of model C: (i) the

static coupling γ of the OP to the conserved density and (ii) a dynamic parameter, the time scale ratio $w=\Gamma/\lambda$, where Γ is the relaxation rate of the OP and λ is the diffusion rate of the conserved density. From the renormalization group (RG) treatment of model C one knows that the one-loop order does not give reliable results due to the stability of a fixed point with the time scale ratio $w=\infty$. In two-loop (and higher) order it turns out that this fixed point is unstable and model C is characterized by strong and weak scaling regions for the dynamics at $d=3$ [5,6]. Moreover, it was shown that nonasymptotic effects are already present in model C [6]. In the following, we will consider how these aspects are influenced by disorder.

Model C [4,5] describes the relaxational dynamics of a system characterized by an n -component nonconserved OP $\vec{\varphi}_0(x,t)$ coupled to the diffusive dynamics of a conserved scalar density $m_0(x,t)$. The structure of the equations of motions is not changed by the presence of disorder. They read

$$\frac{\partial \vec{\varphi}_0}{\partial t} = -\overset{\circ}{\Gamma} \frac{\partial \mathcal{H}}{\partial \vec{\varphi}_0} + \vec{\theta}_{\varphi}, \quad \frac{\partial m_0}{\partial t} = \lambda \overset{\circ}{\nabla}^2 \frac{\partial \mathcal{H}}{\partial m_0} + \theta_m, \quad (1)$$

where $\overset{\circ}{}$ and $\overset{\circ}{}$ denote unrenormalized quantities. The stochastic forces in (1) satisfy the Einstein relations

$$\langle \theta_{\varphi_i}(x,t) \theta_{\varphi_j}(x',t') \rangle = 2\overset{\circ}{\Gamma} \delta(x-x') \delta(t-t') \delta_{ij}, \quad (2)$$

$$\langle \theta_m(x,t) \theta_m(x',t') \rangle = -2\lambda \overset{\circ}{\nabla}^2 \delta(x-x') \delta(t-t'). \quad (3)$$

Equilibrium is described by the static functional \mathcal{H} of the disordered magnetic system

$$\mathcal{H} = \int d^d x \left\{ \frac{1}{2} \dot{\vec{r}} |\vec{\varphi}_0|^2 + V(x) |\vec{\varphi}_0|^2 + \frac{1}{2} \sum_{i=1}^n (\nabla \varphi_{i,0})^2 + \frac{\dot{\vec{u}}}{4!} |\vec{\varphi}_0|^4 + \frac{1}{2} a_m m_0^2 + \frac{1}{2} \dot{\gamma} m_0 |\vec{\varphi}_0|^2 - \dot{h}_m m_0 \right\}, \quad (4)$$

where $V(x)$ is an impurity potential which introduces disorder to the system, and d is the spatial dimension. It contains a coupling $\dot{\gamma}$ to the secondary density, which can be integrated out. Thus static critical properties described by the functional (4) are equivalent to those of the functional $\mathcal{H} = \int d^d x \left\{ \frac{1}{2} \dot{\vec{r}} |\vec{\varphi}_0|^2 + V(x) |\vec{\varphi}_0|^2 + \frac{1}{2} \sum_{i=1}^n (\nabla \varphi_{i,0})^2 + (\dot{u}/4!) |\vec{\varphi}_0|^4 \right\}$. The parameters \dot{r} and \dot{u} are related to $\dot{\vec{r}}$, $\dot{\vec{u}}$, a_m , $\dot{\gamma}$, and \dot{h}_m by $\dot{r} = \dot{\vec{r}} + \dot{\gamma} \dot{h}_m / a_m$ and $\dot{u} = \dot{\vec{u}} - 3 \dot{\gamma}^2 / a_m$. \dot{r} is proportional to the temperature distance from the mean field critical temperature, and \dot{u} is positive. The properties of the random potential $V(x)$ are governed by a Gaussian distribution with width $\langle\langle V(x)V(x') \rangle\rangle = 4\Delta \delta(x-x')$, the double angular brackets mean averaging over disorder. If $\dot{\gamma} \equiv 0$, Eqs. (1) describe dynamical properties of a purely relaxational model (model A) in the presence of disorder [2].

We treat the critical dynamics of the disordered models within the field theoretical RG method [13], where the appropriate Lagrangians of the models are studied. The average over the random potential generates new terms in the Lagrangians with coupling Δ , which correspond to the static coupling terms in \mathcal{H} generated by the disorder. The renormalization of the Lagrangian leads to the RG functions, describing the critical dynamics of our models. We use the minimal subtraction scheme with dimensional regularization to calculate these functions.

For renormalization of the OP $\vec{\varphi}_0$, fourth-order couplings \dot{u}, Δ , and correlation functions with $\vec{\varphi}_0^2$ insertion, we introduce renormalization Z -factors as $\vec{\varphi}_0 = Z_\varphi^{1/2} \vec{\phi}$, $\dot{u} = \mu^\epsilon Z_\varphi^{-2} Z_u u A_d^{-1}$, $\Delta = \mu^\epsilon Z_\varphi^{-2} Z_\Delta \Delta A_d^{-1}$, and $|\vec{\varphi}_0|^2 = Z_\varphi^2 |\vec{\phi}|^2$ (μ is the scale, $\epsilon = 4-d$ and A_d is a geometric factor). Within dynamics renormalization factor for the OP kinetic coefficient $\dot{\Gamma} = Z_\Gamma \Gamma$ is introduced. The Z -factors introduced so far are enough to renormalize the diluted model A. For model C one needs to introduce additional renormalization factors. The secondary density m_0 and coupling parameter $\dot{\gamma}$ are renormalized by $a_m^{1/2} m_0 = Z_m m$ and $a_m^{-1/2} \dot{\gamma} = \mu^{\epsilon/2} Z_\varphi^2 Z_m \gamma A_d^{-1/2}$ with $Z_m^{-2}(u, \Delta, \gamma) = 1 + \gamma^2 A_\varphi^2(u, \Delta)$. The kinetic coefficient λ renormalizes as $a_m \dot{\lambda} = Z_m^2 \lambda$.

Defining the ζ -functions as $d \ln Z^{-1} / d \ln \mu$, where Z represents any renormalization factor, one obtains the flow equations for the renormalized static and dynamic parameters. The flow equations for u and Δ decouple from the remaining parameters and are equal to expressions obtained for any n in the diluted Ginsburg-Landau-Wilson (GLW) model [14]. For the additional static parameter γ appearing in diluted model C we have

$$l \frac{d\gamma}{dl} = \gamma \left[-\frac{\epsilon}{2} + \zeta_\varphi^2(u, \Delta) + \frac{1}{2} \gamma^2 B_\varphi^2(u, \Delta) \right]. \quad (5)$$

The flow parameter l is related to the reduced temperature and is consequently a measure for the distance to the critical

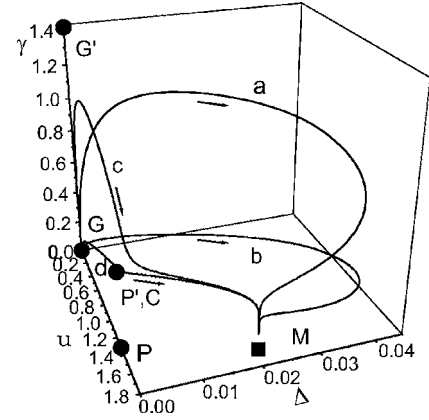


FIG. 1. Two-loop flow of static parameters for $n=1$. The stable fixed point (full square) has a finite width Δ of the impurity distribution and leads to asymptotic exponents different from the pure case (unstable fixed points are indicated by full circles). Depending on the initial values of the renormalized couplings the effective exponents are quite different.

temperature. The function $\zeta_\varphi^2(u, \Delta)$ is known from statics in the diluted model [14]. The function $B_\varphi^2(u, \Delta)$, which is defined by the additive renormalization of the specific heat within the GLW model, in the diluted case reads $B_\varphi^2(u, \Delta) = n/2 + \mathcal{O}(u^2, \Delta^2, u\Delta)$.

The flow equation for the time scale ratio [we introduce $\rho = w/(1+w)$ instead] is

$$l \frac{d\rho}{dl} = \rho(1-\rho) [\zeta_\Gamma(u, \Delta, \gamma, \rho) - \gamma^2 B_\varphi^2(u, \Delta)], \quad (6)$$

where the dynamic ζ -function ζ_Γ in two-loop order reads

$$\begin{aligned} \zeta_\Gamma(u, \Delta, \gamma, \rho) = & \zeta_\Gamma^{(C)}(u, \gamma, \rho) + 4\Delta - \frac{n+2}{3} u\Delta + 20\Delta^2 \\ & + 2\Delta\rho\gamma^2 \left[3[1 - \ln(1-\rho)] + \rho \ln \frac{\rho}{1-\rho} \right. \\ & \left. - \frac{\rho}{1-\rho} \ln \rho \right]. \end{aligned} \quad (7)$$

The corresponding explicit two-loop expression for the ζ -function of model C, $\zeta_\Gamma^{(C)}(u, \gamma, \rho) = \zeta_w^{(C)}(u, \gamma, \rho) + n\gamma^2/2$, is given in [6] [see ζ_w in Eq. (50) there]. In these terms also, the pure model A terms are included, which taken together with the last three terms in the first line of Eq. (7) recover the ζ -function for the diluted model A [2,8]. The zeros of the right-hand sides of Eqs. (5) and (6), and the corresponding equations for u and Δ give the possible fixed points.

It turns out that for all values of n the stable fixed point value $\gamma^* = 0$, and the well-known asymptotic static results are reproduced. Thus, the flow in the space of the static couplings u , Δ , and γ for $n < n_c < 2$ looks like the flow for $n = 1$ (see Fig. 1), whereas for $n > n_c$ it looks like the flow for $n = 3$ (see Fig. 2). The fixed points for $n = 1$ are indicated in Table I. Only the mixed fixed point (**M** in Fig. 1) is stable. However, depending on the initial conditions, a rich cross-over behavior is observed. The same is true for $n = 3$, al-

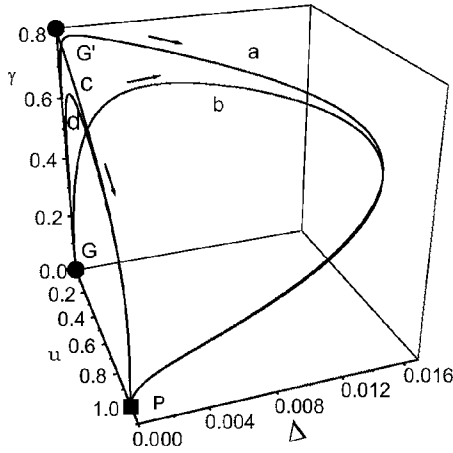


FIG. 2. Two-loop flow of static parameters for $n=3$. The stable fixed point (full square) has a zero width Δ of the impurity distribution and leads to asymptotic exponents equal to the pure case (unstable fixed points are indicated by full circles). However depending on the initial values of the renormalized couplings the effective exponents are quite different and also depend on the initial value of the width Δ .

though now the pure fixed point (**P** in Fig. 2) is stable.

The stable fixed points are found by calculating the stability matrix and its eigenvalues ω_i (i represents u , Δ , γ , or ρ). They govern the “velocity” of the flow near the fixed points. A small stability exponent at the stable fixed point indicates a slow approach of the asymptotic behavior. This is the case for all n and can be seen from Table II for $n=1$. It is the slow approach in the γ direction that characterizes the static flow within model C (see the extremely small values $\omega_\gamma=0.0018$ for $n=1$ (at the fixed point **M** in Table II) and for $n=3$ still $\omega_\gamma=0.1109$). In addition to the small values of the static stability exponents near the fixed points, one also has small values ω_ρ coming from the dynamic parameter ρ (however, only near unstable fixed points). Thus, one expects, depending on the initial values of static and dynamic parameters, a complex behavior in the nonasymptotic region. The effective dynamical exponent is of special interest here; it is found by inserting the solutions of the flow equations into the expression

$$z_{eff}(l) = 2 + \zeta_\Gamma(u(l), \Delta(l), \gamma(l), \rho(l)). \quad (8)$$

One reaches the universal asymptotic value z when the flow comes very near the stable fixed point. One observes that the

TABLE I. Static and dynamic fixed points for $n=1$. Fixed points (FPs) with $\rho^*=1$ ($w^*=\infty$) are always unstable and not shown. **M** is the stable fixed point of the diluted model C.

| FP | u^* | Δ^* | γ^* | ρ^* |
|-----------|-------|------------|------------|----------|
| G | 0 | 0 | 0 | 0 |
| G' | 0 | 0 | 1.414 | 0 |
| P | 1.315 | 0 | 0 | 0 |
| P' | 1.315 | 0 | 0.458 | 0 |
| C | 1.315 | 0 | 0.458 | 0.266 |
| M | 1.633 | 0.021 | 0 | 0 |

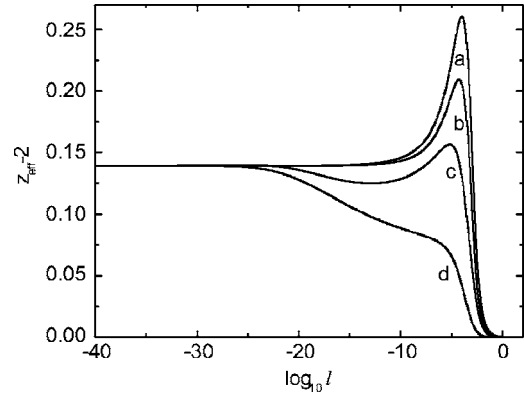


FIG. 3. Effective dynamical exponent for $n=1$ for different initial conditions. The lines correspond to the static flows shown in Fig. 1.

stable fixed point value of the time ratio is always zero, independent of the specific heat exponent value of the pure model. Consequently, the results for model A are recovered in the asymptotics (either in the diluted universality class for $n < n_c$ or in the pure model A universality class for $n > n_c$). This combines with the fact that even in the region where dilution changes the static critical behavior, the stable fixed point value of γ^* is always zero. In consequence, the secondary density is, for all n , asymptotically decoupled and has an asymptotic dynamical exponent $z_m=2$.

The nonasymptotic behavior is, however, quite different, as can be seen from Figs. 3 and 4. Due to the static nonasymptotic behavior also the dynamics is dominated by different nonasymptotic effects. This can be seen by comparing different $z_{eff}(l)$ for different initial conditions. The curves a, b, and c in Fig. 1 and all curves in Fig. 2 reach large values of the coupling γ and/or Δ , and this leads to the typical maximum in the effective exponents independent of the initial value of ρ (for the statics see, e.g., [12]). However, an additional fixed point **P'** is present at $n=1$. This leads, for curve d in Fig. 1, almost to a plateau of z_{eff} at its value for the unstable fixed point **P'**. This plateau is more pronounced when the flow comes nearer to **P'**, where it stays longer because of the small transient exponent ω_ρ . For curve c, both effects (the maximum and the effect of fixed point **P'**) are combined, leading to the minimum in z_{eff} . Consider now the contributions to the effective dynamical critical exponent z_{eff} of different origin: (i) from the terms already present in model A (dashed curve in Fig. 5), (ii) from terms present in

TABLE II. Stability exponents according to the fixed points in Table I for $n=1$.

| FP | ω_u | ω_Δ | ω_γ | ω_ρ |
|-----------|------------|-----------------|-----------------|---------------|
| G | -1 | -1 | -0.5 | 0 |
| G' | -1 | -1 | 1 | 0 |
| P | 0.566 | -0.105 | -0.053 | 0.052 |
| P' | 0.566 | -0.105 | 0.105 | -0.053 |
| C | 0.566 | -0.105 | 0.105 | 0.041 |
| M | 0.494 | 0.194 | 0.0018 | 1.139 |

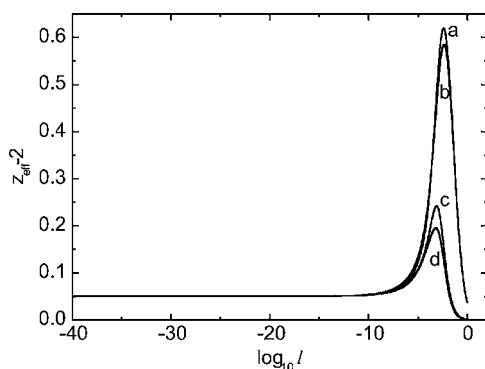


FIG. 4. Effective dynamical exponent for $n=3$ for different initial conditions. The lines correspond to the static flows shown in Fig. 1.

pure model C only (short dashed curve), and (iii) from terms present in the diluted model C only (dashed-short dashed curve). The above contributions may add up to almost the asymptotic value of the exponent, although the parameters are far away from their asymptotic values. This is an important point since the appearance of an asymptotic value in one physical quantity does not mean that other quantities have also reached the asymptotics. This is due to the different dependences of physical quantities on the model parameters. Another special feature of the diluted model C is that already in one-loop order one observes qualitatively the same behavior as in two-loop order, of course with changed values for

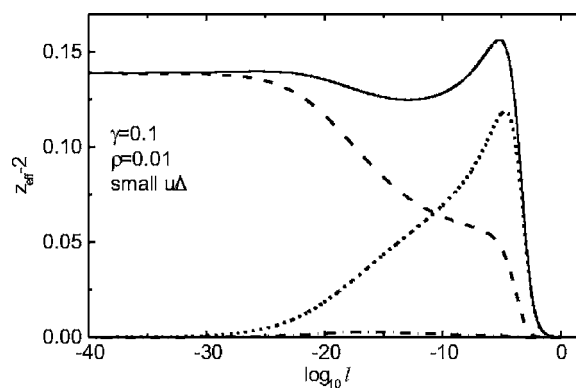


FIG. 5. Different contributions to the effective dynamical critical exponent for $n=1$. The solid line represents the total z_{eff} ; for the other lines see the text.

the exponents and the borderline value n_c , which in one loop is at $n_c=4$.

In conclusion, we remark that contrary to the general belief, the coupling of a conserved density to the order parameter is relevant for the calculation of the dynamical critical behavior of diluted systems since this coupling is important to describe nonasymptotic effects. These effects have been seen in the experiments on physical systems [12,15] as well as in Monte Carlo simulations [16].

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- [1] A. B. Harris, *J. Phys. C* **7**, 1671 (1974).
 [2] I. D. Lawrie and V. V. Prudnikov, *J. Phys. C* **17**, 1655 (1984).
 [3] U. Krey, *Phys. Lett.* **57A**, 215 (1976).
 [4] B. I. Halperin, P. C. Hohenberg, and S.-k. Ma, *Phys. Rev. B* **10**, 139 (1974).
 [5] R. Folk and G. Moser, *Phys. Rev. Lett.* **91**, 030601 (2003).
 [6] R. Folk and G. Moser, *Phys. Rev. E* **69**, 036101 (2004).
 [7] G. Grinstein, S.-k. Ma, and G. Mazenko, *Phys. Rev. B* **15**, 258 (1977).
 [8] V. V. Prudnikov and A. N. Vakilov, *Sov. Phys. JETP* **74**, 990 (1992).
 [9] K. Oerding and H. K. Janssen, *J. Phys. A* **28**, 4271 (1995).
 [10] H. K. Janssen, K. Oerding, and E. Sengerspeick, *J. Phys. A* **28**, 6073 (1995).
 [11] For the recent reviews see, e.g., R. Folk, Yu. Holovatch, and T. Yavors'kii, *Phys. Rev. B* **61**, 15 114 (2000); R. Folk and Yu. Holovatch, and T. Yavors'kii, *Phys. Usp.* **46**, 169 (2003); A. Pelissetto and E. Vicari, *Phys. Rep.* **368**, 549 (2002).
 [12] A. Perumal, V. Srinivas, V. V. Rao, and R. A. Dunlap, *Phys. Rev. Lett.* **91**, 137202 (2003); M. Dudka, Yu. Holovatch, R. Folk, and D. Ivaneiko, *J. Magn. Magn. Mater.* **256**, 243 (2003).
 [13] R. Bausch, H. K. Janssen, and H. Wagner, *Z. Phys. B* **24**, 113 (1976).
 [14] J. Kyriakidis and D. J. W. Geldart, *Phys. Rev. B* **53**, 11 572 (1996).
 [15] P. H. Barrett, *Phys. Rev. B* **34**, R3513 (1986); D. P. Belanger, B. Barago, V. Jaccarino, A. R. King, C. Lartigue, and F. Mezei, *J. Phys. (Paris), Colloq.* **49**, C8-1229 (1988); N. Rosov, C. Hohenemser, and M. Eibschütz, *Phys. Rev. B* **46**, 3452 (1992).
 [16] H. G. Ballesteros, L. A. Fernandez, V. Martin-Mayor, A. MuñozSudupe, G. Parisi, and J. J. Ruiz-Lorenzo, *Phys. Rev. B* **58**, 2740 (1998); G. Parisi, F. Ricci-Terseni, and J. J. Ruiz-Lorenzo, *ibid.* **57**, 13617 (1998).